Problems:

9.16, 9.17, 10.2, 10.3, 10.6, 10.8, 10.9

- Consider two spin 1/2 fermi dirac particles (electrons say) in a harmonic oscillator potential Write down the energies for the first 8 excited states. Write down the coordinate space wave function for the third excited state.
- Now suppose these "electrons" in the box were spinless bosonic particles. Write down the energies for the first 8 excited states and the the coordinate space wave function for the third excited state.

Fermions

The wave function of two fermions is in general a function of the coordinates r_1 and r_2 of the two fermions

$$\Psi(r_1, r_2) \tag{1}$$

and satisfies

$$\Psi(r_2, r_1) = -\Psi(r_1, r_2) \tag{2}$$

We will consider on the case of non interacting fermions in a potential.

- In this case the wave function is

  $$\Psi_{ab}(r_1, r_2) \propto \psi_a(r_1)\psi_b(r_2) - \psi_a(r_2)\psi_b(r_1) \tag{3}$$

  where the single particle wave functions obey the single particle Scrödinger equation

  $$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + V(r)\right]\psi_a(r) = E_a\psi_a(r)\,. \tag{4}$$

  For instance for particles in a box

  $$\begin{aligned} \psi_a(r) &= \sqrt{\frac{2}{L}}\sin\left(\frac{\pi a}{L}\right) && (5)\\ E_a &= \frac{\hbar^2\pi^2}{2mL^2}\,a^2 \qquad \text{with} \qquad a = 1, 2, 3, \ldots && (6) \end{aligned}$$

- The energy is a sum of the energies of the two particle energies

  $$E_{\rm tot} = E_a + E_b \tag{7}$$

  but it is impossible to say that one particle has energy E_a and one particle has energy E_b. Both particles have energy E_a and E_b

- The quantum numbers a and b must be different. This is the pauli exclusion principle. Not two particles can occupy the same state.
- In general several particles in a potential (take the box for example) states of the are found by filling up all orbitals one by one with each quantum number. The energy is a sum of the single particle energies. For instance for three particles in the box, the lowest state has orbital $(1, 2, 3)$ filled and the energy is

  $$\begin{aligned} E_{\rm tot} &= E_1 + E_2 + E_3 && (8)\\ &= \frac{\hbar^2\pi^2}{2mL^2}\left(1^2 + 2^2 + 3^2\right) && (9) \end{aligned}$$

  and the first excited state is $(1, 2, 4)$

  $$\begin{aligned} E_{\rm tot} &= E_1 + E_2 + E_4 && (10)\\ &= \frac{\hbar^2\pi^2}{2mL^2}\left(1^2 + 2^2 + 4^2\right) && (11) \end{aligned}$$

Bosons

The wave function of two fermions is in general a function of the coordinates r_1 and r_2 of the two fermions

$$\Psi(r_1, r_2) \tag{12}$$

and satisfies

$$\Psi(r_2, r_1) = +\Psi(r_1, r_2) \tag{13}$$

We will consider on the case of non interacting fermions in a potential.

- In this case the wave function is

  $$\Psi_{ab}(r_1, r_2) \propto \psi_a(r_1)\psi_b(r_2) + \psi_a(r_2)\psi_b(r_1) \tag{14}$$

  where the single particle wave functions obey the single particle Scrödinger equation

  $$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + V(r)\right]\psi_a(r) = E_a\psi_a(r)\,. \tag{15}$$

- The energy is a sum of the energies of the two particle energies

  $$E_{\text{tot}} = E_a + E_b \tag{16}$$

  The quantum numbers a and b may be the same.

- In general several particles in a potential (take the box for example) states of the are found by filling up all orbitals one by one with each quantum number which can be the same. For instance for three particles in the box, the lowest state is orbital $(1, 1, 1)$

  $$\begin{aligned} E_{\text{tot}} &= E_1 + E_1 + E_1 \qquad (17)\\ &= \frac{\hbar^2\pi^2}{2mL^2}\left(1^2 + 1^2 + 1^2\right) \qquad (18)\end{aligned}$$

  and the first excited state is $(1, 1, 2)$

  $$\begin{aligned} E_{\text{tot}} &= E_1 + E_1 + E_2 \qquad (19)\\ &= \frac{\hbar^2\pi^2}{2mL^2}\left(1^2 + 1^2 + 2^2\right) \qquad (20)\end{aligned}$$

Statistical Mechanics

- For a system interacting in with a heat bath, the probability that the system will have energy E is

  $$P(E) \propto e^{-\frac{E}{k_BT}} \tag{21}$$

  We will usually use this for a single particle interacting with the bath.

- For an ideal gas, the particles velocities are distributed according to the Maxwell-Boltzman velocity distributions

  $$P(v)\underbrace{d^3\mathbf{v}}_{dv_x dv_y dv_z} = \left(\frac{M}{2\pi k_BT}\right)^{3/2} e^{-\frac{Mv^2}{2k_BT}}\, d^3\mathbf{v} \tag{22}$$

  This is the probability to find a particle with velocity $\mathbf{v}$ in a cell of size dv_x, dv_y, dv_z. If you do not care about the direction then we can integrate over the sphere in velocity space to find

  $$P(v)\,dv = \left(\frac{M}{2\pi k_BT}\right)^{3/2} e^{-\frac{Mv^2}{2k_BT}}\, 4\pi v^2\, dv \tag{23}$$

  This is the probability that an ideal gas will have velocity between v and $v + dv$

- The number of particles per unit volume with velocity between v and $v + dv$ is found by multiplying this probability with the total number of particles per volume N/V

$$n(v)\, dv = \frac{N}{V} \left(\frac{M}{2\pi k_B T} \right)^{3/2} e^{-\frac{\frac{1}{2} M v^2}{k_B T}}\, 4\pi v^2\, dv \tag{24}$$

- We will often work with momentum instead of velocity. Since $\mathbf{p} = m\mathbf{v}$ we have

$$P(p) d^3\mathbf{p} = \frac{1}{(2\pi M k_B T)^{3/2}} e^{-\frac{p^2}{2M k_B T}}\, d^3\mathbf{p} \tag{25}$$

  and

$$P(p)\, dp = \frac{1}{(2\pi M k_B T)^{3/2}} e^{-\frac{p^2}{2M k_B T}}\, 4\pi p^2\, dp \tag{26}$$