Problems:

$$1.26, 1.29, 1.33, 1.34, 2.14, 2.19 \tag{1}$$

- (very easy!) The book writes the lorrentz transformation as (neglecting y' and z')

$$t' = \gamma(t - vx/c^2) \tag{2}$$
$$x' = \gamma(x - vt) \tag{3}$$

Show that Eq. (5) is just another way to write this. I prefer Eq. (5) because when you learn what a matrix is, you will see this can be written

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix} \tag{4}$$

Lorentz Transformations

1. An event at space time point (t, x, y, z) will appear at a different space time point (t', x', y', z') according to an observer moving with velocity v in the positive x direction.

$$\begin{aligned} ct' &= \gamma\,(ct) - \gamma\beta\, x \\ x' &= -\gamma\beta\,(c\,t) + \gamma\, x \\ y' &= y \\ z' &= z \end{aligned}$$

For an observer moving in the negative x direction the same formula holds with the replacement $\beta \to -\beta$

2. This should be compared to *Classical Galilean* transformations where the space time point (t, x, y, z) appears at a different space point but the same time point (t', x', y', z')

$$\begin{aligned} t' &= t \\ x' &= x - vt \\ y' &= y \\ z' &= z \end{aligned}$$

3. For a given observer in a fixed frame the normal rules of classical physics apply.

4. The Lorentz transformation leaves the following unchanged

$$(\Delta x')^2 - (c\Delta t')^2 = (\Delta x)^2 - (c\Delta t)^2 \tag{5}$$

Other results

1. A space ship moving with velocity $\mathbf{u} = (u_x, u_y, u_z)$ in the "lab" frame will be moving with a different velocity $\mathbf{u}'$ according to an observer moving with speed v in the positive x direction. $\mathbf{u}'$ is related to $\mathbf{u}$

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} \tag{6}$$
$$u'_y = \frac{u_y}{\gamma(1 - u_x v/c^2)} \tag{7}$$
$$u'_z = \frac{u_z}{\gamma(1 - u_y v/c^2)} \tag{8}$$

If the observer is moving to the left then the same formula applies with the substitution $v \to -v$. If the space-ship is moving to the left then u_x is negative in this formula.

2. A source emits light waves with frequency f_o. According to an observer moving directly toward the source with speed v the source has a frequency which is blue shifted

$$f = f_o \sqrt{\frac{1 + v/c}{1 - v/c}} \tag{9}$$

For an observer moving away from the source make the replacement $v \to -v$.

Dynamics

1. The momentum of a particle moving with velocity $\mathbf{u}$ is

$$\mathbf{p} \equiv \gamma m \mathbf{u} \qquad \text{with} \qquad \gamma \equiv \frac{1}{\sqrt{1 - u^2/c^2}} \tag{10}$$

2. The energy of a particle moving with velocity $\mathbf{u}$ is

$$E \equiv \gamma m c^2 \qquad \text{with} \qquad \gamma \equiv \frac{1}{\sqrt{1 - u^2/c^2}} \tag{11}$$

3. The rest energy is the energy of a particle when it is not moving, and the kinetic energy is the energy minus the rest energy

$$E_{\text{rest}} = mc^2 \qquad K = \gamma m c^2 - mc^2 \tag{12}$$

4. The energy and momentum and velocity given above are related by the formulas

$$E^2 = (cp)^2 + (mc^2)^2 \qquad \mathbf{u} = \frac{\mathbf{p}}{E} \tag{13}$$

For a photon we have $E = cp$ and $u = c$ (mass is zero)

5. Total energy and momentum (the sum of the energy's and momenta of all the particles) are always conserved before and after the collision

$$E_{\text{tot}} = E'_{\text{tot}} \qquad \mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}} \tag{14}$$

6. If the energy and momentum $(E, c\mathbf{p})$ of a particle according to one observer is known, then according to an observer moving to the right with speed v, the energy and momentum of the particle is $(E', c\,\mathbf{p}')$ with

$$E' = \gamma\, E - \gamma\beta\,(cp_x) \tag{15}$$
$$cp'_x = -\gamma\beta\, E - \gamma\,(cp_x) \tag{16}$$
$$cp'_y = cp_y \tag{17}$$
$$cp'_z = cp_z \tag{18}$$

with $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ and $\beta = v/c$. Of course for a left moving observer we have $v \to -v$. We multiply c times momentum so that $c\,p$ has the same units of energy. The change of frames mixes energy and momenta.