**Problems:**

$$2.9, 2.15, 2.21, 2.22, 2.16, 2.31 \tag{1}$$

1. A proton with mass $m_p = 938\text{MeV}/c^2$ is observed with momentum components $p_x = 1\text{GeV}/c$ and and $p_y = 0.5\text{GeV}/c$.
   - What is the angle that the proton makes with the $x - axis$?
   - What is the energy of the proton in this frame?
   - According an observer moving to the left on the $x - axis$ (i.e. the negative $x$ direction) with speed $\frac{12}{13}c$ what is the energy and momentum components of the proton?
   - What is the angle that the proton makes w.r.t. the $x - axis$ in this frame?
2. Suppose that the current is measure to be $I \pm \Delta I$ and the resistance is measured to be $R \pm \Delta R$. Then starting with Eq. (3) show that the uncertainty in the voltage ($V = IR$) is

$$\frac{\Delta V}{V} = \sqrt{\left(\frac{\Delta I}{I}\right)^2 + \left(\frac{\Delta R}{R}\right)^2} \tag{2}$$

**Error Analysis**

- If you have have two items $a \pm \Delta a$ and $b \pm \Delta b$ , then the uncertainty in a function of $a$ and $b$

$$F = f(a, b) \pm \Delta F$$

is

$$\Delta F = \sqrt{(\Delta F)_a^2 + (\Delta F_b)^2} \tag{3}$$

where

$$(\Delta F)_a = f(a + \Delta a, b) - f(a, b) \approx \frac{\partial f}{\partial a}\Delta a \tag{4}$$

$$(\Delta F)_b = f(a, b + \Delta b) - f(a, b) \approx \frac{\partial f}{\partial b}\Delta b \tag{5}$$

- Special case (1) : For

$$F = a + b \qquad \text{we have} \qquad \Delta F = \sqrt{(\Delta a)^2 + (\Delta b)^2}$$

- Special case (2) : For

$$F = ab \qquad \text{we have} \qquad \frac{\Delta F}{F} = \sqrt{\left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta b}{b}\right)^2}$$

**Dynamics**

1. The momentum of a particle moving with velocity $\mathbf{u}$ is

$$\mathbf{p} \equiv \gamma m \mathbf{u} \qquad \text{with} \qquad \gamma \equiv \frac{1}{\sqrt{1 - u^2/c^2}} \tag{6}$$

2. The energy of a particle moving with velocity $\mathbf{u}$ is

$$E \equiv \gamma mc^2 \qquad \text{with} \qquad \gamma \equiv \frac{1}{\sqrt{1 - u^2/c^2}} \tag{7}$$

3. The rest energy is the energy of a particle when it is not moving, and the kinetic energy is the energy minus the rest energy

$$E_{\text{rest}} = mc^2 \qquad K = \gamma mc^2 - mc^2 \tag{8}$$

4. The energy and momentum and velocity given above are related by the formulas

$$E^2 = (cp)^2 + (mc^2)^2 \qquad \mathbf{u} = \frac{\mathbf{p}}{E} \tag{9}$$

   For a photon we have $E = cp$ and $u = c$ (mass is zero)

5. Total energy and momentum (the sum of the energy's and momenta of all the particles) are always conserved before and after the collision

$$E_{\text{tot}}^{\text{before}} = E_{\text{tot}}^{\text{after}} \qquad \mathbf{p}_{\text{tot}}^{\text{before}} = \mathbf{p}_{\text{tot}}^{\text{after}} \tag{10}$$

6. If the energy and momentum $(E, c\mathbf{p})$ of a particle according to one observer is known, then according to an observer moving to the right with speed $v$, the energy and momentum of the particle is $(E', c\,\mathbf{p}')$ with

$$E' = \gamma\, E - \gamma\beta\,(cp_x) \tag{11}$$

$$cp'_x = -\gamma\beta\, E + \gamma\,(cp_x) \tag{12}$$

$$cp'_y = cp_y \tag{13}$$

$$cp'_z = cp_z \tag{14}$$

   with $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ and $\beta = v/c$. Of course for a left moving observer we have $v \to -v$. We multiply $c$ times momentum so that $c\,p$ has the same units of energy. The change of frames mixes energy and momenta.