PHY 251 : homework 3

2.9: We have

$$E_k = 5m_0c^2 = (\gamma - 1)m_0c^2 \qquad \Rightarrow \quad \gamma = 6 \qquad \Rightarrow \qquad \beta = \sqrt{1 - 1/\gamma^2} = \sqrt{1 - 1/36}$$

or $\beta = \sqrt{35}/6 = 0.986$. Its total energy is $E = E_0 + E_k = \gamma m_0 c^2 = 6 \cdot 511 \times 10^3$ eV or 3.07 MeV.

2.15: The kinetic energy of an electron with $m = 511 \times 10^3$ eV/c^2 is qV so we have

$$\begin{aligned} qV &= (\gamma - 1)mc^2 \\ \frac{qV}{mc^2} &= \gamma - 1 \\ \gamma &= 1 + \frac{qV}{mc^2} = 1 + \frac{5 \times 10^4 \text{ eV}}{511 \times 10^3 \text{ eV}} = 1.0978. \end{aligned}$$

The speed can then be found from

$$\beta = \sqrt{1 - 1/\gamma^2} = \sqrt{1 - 1/1.0978^2} = 0.412.$$

If we do this classically we find

$$\begin{aligned} qV &= \frac{1}{2}mv^2 = \frac{1}{2}mc^2\beta^2 \\ \beta &= \sqrt{\frac{2qV}{mc^2}} = \sqrt{\frac{2 \cdot 5 \times 10^4}{511 \times 10^3}} = 0.442 \end{aligned}$$

The difference is then about 6.8%. This will probably affect the size of the electron beam on the TV screen because the electron focusing optics will not be quite set right. A bigger scanned electron spot on the phosphor will make the image appear fuzzier (pixels will blur into each other). It will also affect the brightness of the image because the amount of light given out by the phosphor is proportional to the energy of the electrons slamming into it. It will *not* affect the color of the image because that's determined by the dopant atoms in the phosphor; some phosphors are designed to glow red when excited, others blue, others green. . .

2.16: The problem is to show the invariance of $E^2 - (cp)^2$. Then we have

$$E' = \gamma\, E - \gamma\beta\,(cp) \qquad (1)$$

$$cp' = -\gamma\beta\, E + \gamma\,(cp) \qquad (2)$$

So

$$E'^2 - (cp')^2 = (\gamma\, E - \gamma\beta(cp))^2 - (-\gamma\beta\, E + \gamma(cp))^2 \qquad (3)$$

$$= (\gamma^2 - \gamma^2\beta^2)E^2 + (\gamma^2\beta^2 - \gamma^2)(cp)^2 \qquad (4)$$

Then using the fact that

$$\gamma^2 - \gamma^2\beta^2 = \frac{1-\beta^2}{1-\beta^2} = 1 \tag{5}$$

we have

$$E'^2 - (cp')^2 = E^2 - (cp)^2 \tag{6}$$

2.21: Conservation of energy gives

$$(E_K + m_e c^2) + m_e c^2 = 2E_\gamma$$

because we go from an electron with kinetic energy E_K and the mass-energy of the electron plus the positron (same mass as electron) to two photons of identical energy E_γ. Since $m_e = 511$ keV/c^2 we have $E_\gamma = (1.000 + 0.511 + 0.511)/2 = 1.011$ MeV. Note for the electron we have

$$\begin{aligned} E_K &= (\gamma - 1) m_e c^2 \\ \gamma &= 1 + \frac{E_k}{m_e c^2} = 1 + \frac{1.000}{0.511} = 2.957 \end{aligned}$$

from which we can find $\beta = \sqrt{1 - 1/\gamma^2} = 0.941$. At the same time, conservation of momentum gives

$$\begin{aligned} p_e &= 2p_\gamma \cos\theta \\ \gamma m_e v &= 2\frac{E_\gamma}{c}\cos\theta \\ \gamma\beta m_e c^2 &= 2E_\gamma \cos\theta \\ \cos\theta &= \frac{\gamma\beta m_e c^2}{2E_\gamma} \\ \theta &= \cos^{-1}\Big(\frac{\gamma\beta m_e c^2}{2E_\gamma}\Big) = \cos^{-1}\Big(\frac{2.957 \cdot 0.941 \cdot 0.511}{2 \cdot 1.011}\Big) = \cos^{-1}(0.703) = 45.3^\circ \end{aligned}$$

2.22: We know that the pions have equal and opposite momenta because the kaon was at rest prior to the decay. We know $p = \gamma m v = qBR$ for each pion so we have

$$\gamma v = \frac{qBR}{m} \qquad \text{or} \qquad \gamma\beta = \frac{qBRc}{mc^2} = \frac{1.602 \times 10^{-19} \cdot 2.0 \cdot .344 \cdot 3 \times 10^8}{(140 \times 10^6 \text{ eV}) \cdot (1.602 \times 10^{-19} \text{ J/eV})} = 1.43$$

Given $\gamma\beta$, we can find β:

$$\begin{aligned} \text{Let } x &= \gamma\beta \\ x^2 &= \frac{\beta^2}{1-\beta^2} \\ x^2 - x^2\beta^2 &= \beta^2 \\ \beta^2(1 + x^2) &= x^2 \\ \beta &= \frac{x}{\sqrt{1+x^2}} = \frac{1}{\sqrt{1 + 1/x^2}} = \frac{1}{\sqrt{1 + 1/(\gamma\beta)^2}} \end{aligned}$$

Therefore we have

$$\beta = \frac{1}{\sqrt{1+1/(\gamma\beta)^2}} = \frac{1}{\sqrt{1+1/(1.43)^2}} = 0.82$$

We can then find $\gamma = 1/\sqrt{1-\beta^2} = 1.75$ so that we can write the momentum of each pion as

$$p = \gamma m v = \gamma m c^2 \beta / c = 1.75 \cdot 140 \text{ MeV} \cdot .82/c = 201 \text{ MeV}/c.$$

The total energy of each pion is $\gamma mc^2 = 1.75 \cdot 140$ MeV=245 MeV. The mass of the kaon is then equal to the sum of the total energy of the pions or 490 MeV/c^2.

2.31: Momentum conservation reads

$$\gamma_u m u - \gamma_u \frac{m}{3} u = \gamma_V M V \tag{7}$$

where M and V are the mass and velocity of the final particle, $\gamma_u = 1/\sqrt{1-(u/c)^2}$, $\gamma_V = 1/\sqrt{(1-(V/c)^2}$ Energy conservation reads

$$\gamma_u mc^2 + \gamma_u \frac{m}{3} c^2 = \gamma_V M c^2 \tag{8}$$

So

$$\gamma_V M = \gamma_u \frac{4}{3} m \tag{9}$$

which makes momentum conservation read

$$\gamma_u \frac{2}{3} m u = \gamma_u \frac{4}{3} m V \tag{10}$$

$$\frac{1}{2} u = V \tag{11}$$

So we can then find M

$$M = m \frac{4}{3} \frac{\gamma_u}{\gamma_V} \tag{12}$$

$$= \frac{4}{3} m \sqrt{\frac{1-(u/2c)^2}{1-(u/c)^2}} \tag{13}$$

$$= \frac{2}{3} m \sqrt{\frac{4-(u/c)^2}{1-(u/c)^2}} \tag{14}$$