PHY 251 Fall 2008: homework problem set 4

3.10: We divide power by energy per photon:

$$\frac{100 \times 10^3 \text{ J/s}}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(94 \times 10^6 \text{ s}^{-1})} = 1.6 \times 10^{30} \text{ photons/sec}$$

so we'd be hard put to see individual radio-wavelength photons...

3.12: The energy per photon is $E = hc/\lambda$ so the photon flux is P/E or

$$\frac{10 \text{ J/s}}{hc/\lambda} = \frac{10 \text{ J/s}}{[(1240 \text{ eV} \cdot \text{nm})/(589.3 \text{ nm})] \cdot (1.602 \times 10^{-19} \text{ J/eV})} = 2.97 \times 10^{19} \text{ photons/sec}$$

3.15: From $E_k = h\nu - \phi$ we find that we need a frequency of $\nu = \phi/h = (4.2 \text{ eV})/(4.136 \times 10^{-15} \text{ eV} \cdot \text{s}) = 1.02 \times 10^{15}$ Hz, or a wavelength of $\lambda = c/\nu = 2.99 \times 10^8/1.02 \times 10^{15} = 2.93 \times 10^{-7}$ m or 293 nm. If we instead use $\lambda = 200$ nm light, we have a photon energy of $hc/\lambda = (1240 \text{ eV} \cdot \text{nm})/(200 \text{ nm})$ or 6.2 eV, leaving 2 eV of kinetic energy or a 2 Volt stopping potential.

3.16: From $E_k = h\nu - \phi$ we can write

$$\phi = h\nu - E_k = \frac{hc}{\lambda} - E_k = \frac{1240 \text{ eV} \cdot \text{nm}}{300 \text{ nm}} - 2.23 \text{ eV} = 1.90 \text{ eV}$$

3.20: We can write the following equations, realizing that the work function is the same in the two situations:

$$\begin{aligned} E_{k,1} &= \frac{hc}{\lambda} - \varphi \\ E_{k,2} &= 2\frac{hc}{\lambda} - \varphi. \end{aligned}$$

If we subtract the first equation from the second, we have $E_{k,2} - E_{k,1} = (hc)/\lambda$ or

$$\lambda = \frac{hc}{E_{k,2} - E_{k,1}}.$$

We can then substitute this in the first equation and solve for the work function φ:

$$\varphi = \frac{hc}{\lambda} - E_{k,1} = hc\frac{E_{k,2} - E_{k,1}}{hc} - E_{k,1} = E_{k,2} - 2E_{k,1} = 4.00 - 2 \cdot 1.00 = 2.00 \text{ eV}.$$

3.22: Start with $F = ma$

$$qvB = m\frac{v^2}{r} \text{ leading to } qBr = mv$$

which gives

$$v = \frac{qBr}{m} = \frac{1.6 \times 10^{-19}\dot{2} \times 10^{-5} \cdot 0.2}{9.11 \times 10^{-31}} = 7.03 \times 10^5 \text{ m/s}$$

from which we can say that

$$\beta = v/c = (7.03 \times 10^5)/(3 \times 10^8) = 2.33 \times 10^{-3}.$$

The non-relativistic approximation is a good one. Thus the kinetic energy of the electron is

$$\frac{1}{2}mv^2 = \frac{1}{2}mc^2\beta^2 = \frac{1}{2} \cdot (511 \times 10^3 \text{ eV}) \cdot (2.33 \times 10^{-3})^2 = 1.39 \text{ eV}$$

Then from $E_k = h\nu - \phi$ we have

$$\phi = \frac{hc}{\lambda} - 1.39 = \frac{1240}{450} - 1.39 = 1.37 \text{ eV}$$

3.25: The Compton wavelength shift is

$$\lambda' - \lambda_0 = \frac{hc}{mc^2}(1 - \cos\theta) = \frac{1240 \text{ eV} \cdot \text{nm}}{511 \times 10^3 \text{ eV}}(1 - \cos 30^\circ) = 3.25 \times 10^{-4} \text{ nm}$$

Let's get the energy of the scattered photon:

$$\begin{aligned}
\lambda' &= \lambda_0 + (\lambda' - \lambda_0) \\
\frac{hc}{\lambda'} &= \frac{hc}{\lambda_0 + (\lambda' - \lambda_0)} \\
E' &= \frac{1}{\lambda_0/hc + (\lambda' - \lambda_0)/hc} = \frac{1}{1/E_0 + (\lambda' - \lambda_0)/hc} \\
E' &= \frac{1}{1/300 \times 10^3 \text{ eV} + (3.25 \times 10^{-4} \text{ nm})/(1240 \text{ eV} \cdot \text{nm})} = 278 \times 10^3 \text{ eV}
\end{aligned}$$

or 278 keV. The kinetic energy of the scattered electron is then 300-278=22 keV.

3.28: By conservation of energy, the energy of the incident photon must be 120+40=160 keV which means its wavelength is

$$\lambda_0 = \frac{hc}{E_0} = \frac{1240 \text{ eV} \cdot \text{nm}}{160 \times 10^3 \text{ eV}} = 7.75 \times 10^{-3} \text{ nm}.$$

The angle of the Compton photon can be found from

$$\begin{aligned}
\lambda' - \lambda_0 &= \frac{h}{m_e c}(1 - \cos\theta) \\
\frac{hc}{E'} - \frac{hc}{E_0} &= \frac{hc}{m_e c^2}(1 - \cos\theta) \\
\frac{m_e c^2}{E'} - \frac{m_e c^2}{E_0} &= 1 - \cos\theta \\
\cos\theta &= 1 + \frac{m_e c^2}{E_0} - \frac{m_e c^2}{E'} \\
\theta &= \text{arccos}\left(1 + \frac{511}{160} - \frac{511}{120}\right) = \text{arccos}(-0.0646) = 93.7^\circ.
\end{aligned}$$

The recoil angle of the electron can be found from conservation of momentum. In the $\hat{y}$ direction, we have a net momentum of zero before and therefore after the collision, so (referring to Serway Fig. 3.24) we have

$$\begin{aligned} p_{\lambda'} \sin\theta &= p_e \sin\phi \\ \frac{h}{\lambda'} \sin\theta c p_{\lambda'} \sin\theta &= c p_e \sin\phi \end{aligned}$$

i.e. with $E = cp$ for a photon

$$\sin(\phi) = \frac{E_{\lambda'}}{cp_e} \sin(\theta) \tag{1}$$

Now the energy and momentum are related for an electron

$$E^2 = (m_e c^2)^2 + (cp_e)^2 \tag{2}$$

so

$$(cp_e) = \sqrt{E^2 - (m_e c^2)^2} \tag{3}$$
$$= \sqrt{(511 + 40)^2 - (511)^2} \tag{4}$$
$$= 206\text{keV} \tag{5}$$

Putting $E_{\lambda'} = 120\text{keV}$ and $\theta = 93.7$ and $cp_e = 206\text{keV}$ we have

$$\phi = 33.4^\circ. \tag{6}$$

3.36: The incident photon must have an energy of $E_0 = E' + E_k = 80 + 25$ keV, so its wavelength is

$$\lambda_0 = \frac{hc}{E_0} = \frac{1240 \text{ eV} \cdot \text{nm}}{105 \times 10^3 \text{ eV}} = 11.8 \times 10^{-3} \text{ nm}$$

We can also find $\lambda' = hc/E' = 15.5 \times 10^{-3}$ nm. We then have

$$\begin{aligned} \lambda' - \lambda_0 &= \frac{hc}{mc^2}(1 - \cos\theta) \\ \frac{\lambda' - \lambda_0}{hc} mc^2 &= 1 - \cos\theta \\ \cos\theta &= 1 - \frac{\lambda' - \lambda_0}{hc} mc^2 \\ \theta &= \cos^{-1}\Big[1 - \frac{\lambda' - \lambda_0}{hc} mc^2\Big] \\ &= \cos^{-1}\Big[1 - \frac{15.5 \times 10^{-3} - 11.8 \times 10^{-3}}{1240} 511 \times 10^3\Big] = 121^\circ \end{aligned}$$