Problems:

$$4.9, 4.16, 4.20, 4.28, 4.31, 4.34 \tag{1}$$

- Show that the only combination of $\hbar$, m_e and c which has the dimension of length is the compton wavelength.

$$\lambda = \frac{\hbar}{m_e c} \tag{2}$$

  Determine λ numerically using only the quantities below.

- Using the numbers given below and the fact that the mass of the proton is $m_p c^2 \approx 2000\, m_e c^2 \approx 938 MeV$, determine the proton compton wavelength (i.e. m_e is repaced by m_p). This is the fuzziness scale of the proton. Show that the size of a typical nucleus is about 25 proton compton wavelengths.

- Using only the numbers below, estimate the potential energy of two electrons separated by 1 nm in units of eV.

- Below we have derived a formula for an electron orbiting a proton which says that

$$\frac{v_1}{c} = \alpha \simeq \frac{1}{137} \tag{3}$$

  When an electron orbits Z protons as in a nucleus, show that

$$\frac{v_1}{c} = Z\alpha \tag{4}$$

  Approximately how many protons are needed in a nucleus before the inner electron is no longer non-relativistic.

- This problem will derive the Bohr radius in another way
  - Show (using the fact that the $p = mv$) that the momentum of an electron in the first orbit

$$p = \frac{\hbar}{a_0} \tag{5}$$

  - Show for an electron in circular orbit that

$$KE = -\frac{1}{2} PE \tag{6}$$

  - Show that the kinetic energy of the electron can be written

$$KE = \frac{1}{2} m v^2 = \frac{p^2}{2m} \tag{7}$$

    and use this to find a formula for the kinetic energy in terms of $\hbar, m, a_0$ (Answer: $\hbar^2/(2ma_0^2)$)
  - What is the potential energy of an electron circling the nucleus with radius a_o.
  - Use the above relation between KE and PE to determine a_o. Verify that this is the same expression for a_o that you started with when deriving Eq. (5). The whole set-up is self consistent.

- A diffraction grating has 100,000 lines per inch (2.54 cm) . Determine the deflection angle of the first maximum for UV light $\lambda = 200\,\text{nm}$, green light $\lambda = 540\,\text{nm}$, infrared light $\lambda = 750\,\text{nm}$.

Physical Constants

1. 2π is annoying often use

$$\hbar = \frac{h}{2\pi} \qquad \lambda = \frac{\lambda}{2\pi} \tag{8}$$

2. The quantity is very useful

$$\hbar c = 197 \text{eV nm} \tag{9}$$

3. The fine structure constant is a pure number and is is the only dimensionless quantity that can be made out $\hbar, m_e$ and e

$$\alpha = \frac{k_C e^2}{\hbar c} \simeq \frac{1}{137} \tag{10}$$

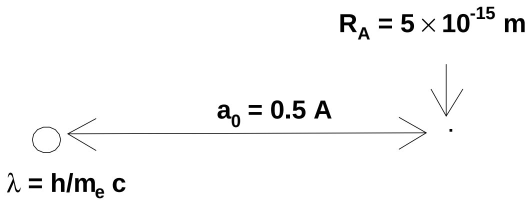
4. The mass of the electron is

$$m_e c^2 \simeq 0.5 MeV \tag{11}$$

5. The Bohr radius is

$$a_0 = \frac{\hbar}{m_e c} \frac{1}{\alpha} \simeq 0.5 \mathring{A} \tag{12}$$

The picture of the atom is the following (the circle is the electron and the dot is the nucleus)



Bohr Model

1. Electrons move about the nucleus in circular orbits determined by Newtons Laws
2. Only certain orbits are stable, in these orbits the angular momentum of the electorn is

$$L = m_e v r = n\hbar n = 1, 2, 3, \ldots \tag{13}$$

3. For these orbits (labelled by n) we have

$$\frac{v_n}{c} = \alpha \left[\frac{1}{n}\right] \Leftrightarrow \text{velocity} \tag{14}$$

$$E_n = -\frac{1}{2} m_e c^2 \alpha^2 \left[\frac{1}{n^2}\right] \Leftrightarrow \text{Energy} \tag{15}$$

$$= -13.6\text{eV} \left[\frac{1}{n^2}\right] \tag{16}$$

$$r_n = a_0 \left[n^2\right] \Leftrightarrow \text{radius} \tag{17}$$

$$= \frac{\hbar}{m_e c\, \alpha} \left[n^2\right] \tag{18}$$

4. You should feel comfortable deriving these results.
5. Light of a given frequency is emitted as the atom makes a transition from one n (say $n = 2$) to another (say $n = 1$). If light is emitted, the change in energy $\Delta E = E_f - E_i$ of the atom is negative sense the atom lowers its energy by emitting light energy which makes up the change. The frequency of the light which is emitted is given by energy conservation.

$$hf = (E_i - E_f) \tag{19}$$

6. The above formulas are for a single electron running around a single proton. When a single electron runs around Z protons the formulas become

$$
\begin{aligned}
L = m_e v r &= n\hbar \qquad n = 1, 2, 3 \ldots && (20) \\
\frac{v_n}{c} &= Z\alpha \left[\frac{1}{n}\right] \Leftrightarrow \text{velocity} && (21) \\
E_n &= -\frac{1}{2} m_e c^2 (Z\alpha)^2 \left[\frac{1}{n^2}\right] \Leftrightarrow \text{Energy} && (22) \\
&= -13.6\text{eV} Z^2 \left[\frac{1}{n^2}\right] && (23) \\
r_n &= \frac{a_0}{Z} \left[n^2\right] \Leftrightarrow \text{radius} && (24) \\
&= \frac{\hbar}{m_e c\, Z\alpha} \left[n^2\right] && (25)
\end{aligned}
$$

7. When the mass of the proton is taken into account (the above formulas assume $m_p = \infty$) the mass of the electron should be replaced by the reduced mass μ

$$\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_p} \tag{26}$$

This is not that important but is needed for problem 4.34.

Experiments

1. Diffraction grating can separate light into its spectrum. The angle of deflection is

$$\sin(\theta) = m\frac{\lambda}{d} \qquad m = 0, 1, 2, 3 \ldots \tag{27}$$

Individual atoms will absorb and emitt characteristic frequencies.

2. Assuming the nucleus is a point like charge (charge +Z), Rutherford predicted what fraction of the alpha particles (charge +2) sent in would be deflected into an angle between ϕ and $\phi + \Delta\phi$.

$$\Delta n \propto \frac{\Delta\phi}{\sin^4(\phi/2)} \tag{28}$$

He estimated that this formula would not work when the Kinetic energy of the α particles was sufficient to penetrate the nucleus.

$$K = \frac{1}{2} m_\alpha v_\alpha^2 = k\frac{(Ze)(2e)}{d_{\text{min}}} \tag{29}$$