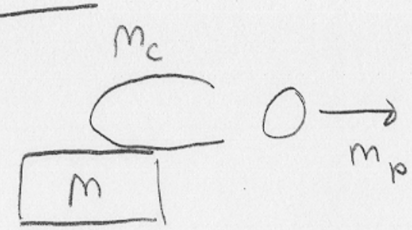
Problem 1



① Suppose that the cannon/block recoils @ velocity v, then the pellet moves @ velocity $v + u_0$, v is negative

$$(m_c + M)v + m_p(v + u_0) = 0$$

$$v = -\frac{m_p u_0}{(m_c + m_p + M)}$$

u_0 is the relative speed

Problem 1

(2) Using E-consverve

$$\frac{1}{2}(m_c + M)\, V^2 = \frac{1}{2} k x_{max}^2$$

$$\sqrt{\frac{m_c + m}{k}}\ V = x_{max}$$

$$-\sqrt{\frac{m_c + m}{k}} \cdot \frac{m_p u_0}{(m_c + m + m_p)} = x_{max}$$

(3) The period is

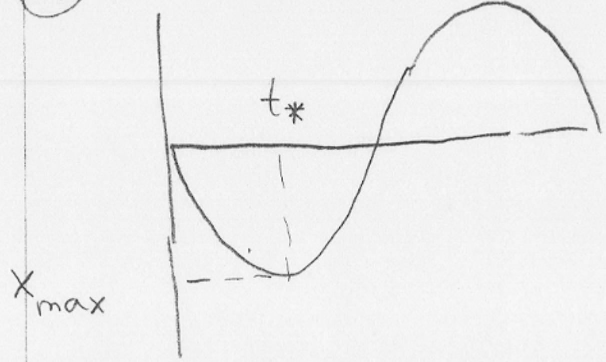
$$T = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{(m + m_c)}{k}}$$

Problem 1

So from graph $t_* = \frac{T}{4}$

$$t_* = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{(m+m_c)}{k}}$$

(4)



(5) The acceleration of the cannon is:

$$x = \frac{V_0}{\omega_0}\sin(\omega_0 t)$$

$$\ddot{x} = -\omega_0^2 x = -\omega_0^2 \frac{V_0}{\omega_0}\sin(\omega_0 t)$$

Problem 1

$$a_{max} = -\omega_0 v_0$$

So

$$|F_{max}| = m_c \omega_0 v_0$$

$$|F_{max}| = m_c \sqrt{\frac{k}{(m+m_c)}} \frac{m_p u_0}{(m+m_c+m_p)}$$

Problem 2

(1)

$$I = \frac{1}{2} mR^2 \cdot 2 + \frac{1}{2} m \left(\frac{R}{2}\right)^2$$

$$I = mR^2 + \frac{mR^2}{8} = \frac{9}{8} mR^2$$

(2) From energy conservation

$$3mgL\sin\theta = \frac{1}{2}(3m)\, v_{cm}^2 + \frac{1}{2} I\omega^2$$

$$3mgL\sin\theta = \frac{1}{2}(3m)\, v_{cm} + \frac{1}{2}\left(\frac{9}{8}mR^2\right)\left(\frac{v_{cm}}{R}\right)^2$$

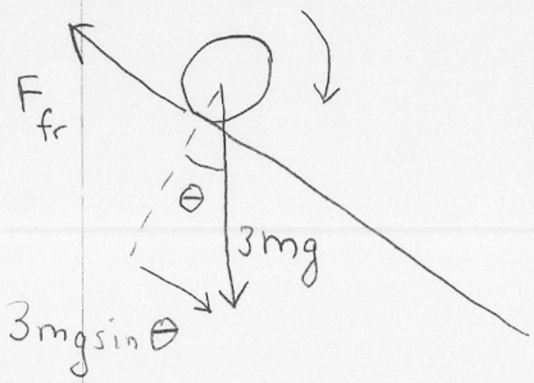
$$\cancel{3}\cancel{m}gL\sin\theta = \cancel{m}v_{cm}^2 \left(\frac{\overset{1}{\cancel{3}}}{2} + \frac{\cancel{9}3}{16}\right)$$

So $$\frac{gL\sin\theta}{\left(\frac{1}{2} + \frac{3}{16}\right)} = v_{cm}^2$$

Problem 2

$$V_{cm} = \sqrt{\frac{16}{11} g L \sin\theta}$$

From



$$F_{fr} - 3mg\sin\theta = 3m a_{cm}$$

$$-F_{fr} R = I\alpha$$

Now

$a_{cm} = \frac{\alpha}{R}$ for rolling @/out slipping

Problem 2

$$-F_{fr} R = \frac{9}{8} mR^2 \frac{a_{cm}}{R}$$

$$-F_{fr} = \frac{9}{8} m a_{cm}$$

$$-\frac{9}{8} m a_{cm} - 3mg\sin\theta = 3m a_{cm}$$

$$-3\cancel{m}g\sin\theta = \left(3+\frac{9}{8}\right)\cancel{m} a_{cm}$$

$$-\frac{3}{3+9/8} g\sin\theta = a_{cm}$$

$$-\frac{1}{1+3/8} g\sin\theta = a_{cm}$$

$$\boxed{-\frac{8}{11} g\sin\theta = a_{cm}}$$

Problem 2

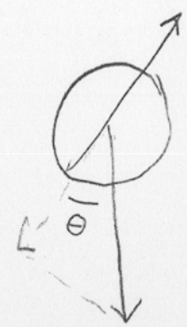
So $\quad \Delta x = \frac{1}{2} a t^2$

$$-L = \frac{1}{2} a \Delta t^2$$

$$\sqrt{-2\frac{\Delta L}{a}} = \Delta t$$

$$\sqrt{\frac{22}{8}\frac{L}{g\sin\theta}} = \Delta t$$

4.



$$-3mg\cos\theta + N = 0$$

$$N = 3mg\cos\theta$$

$$F_{fr}^{max} = \mu_s N$$

$$|F_{fr}^{max}| = \mu_s \, 3mg\cos\theta$$

$$|F_{fr}| = \frac{9}{8} m |a_{cm}|$$

$$= \frac{9}{8} m \left(\frac{8}{11} g\sin\theta\right)$$

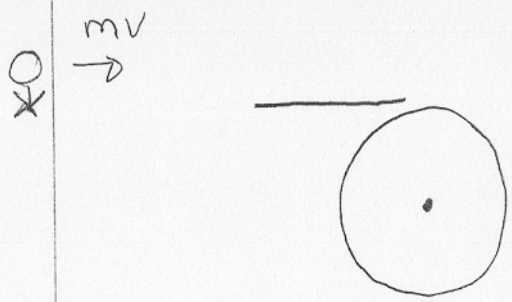
So θ_{max}

$$\mu_s \cancel{3}\cancel{m}\cancel{g}\cos\theta_{max} = \frac{3\cancel{9}}{11}\cancel{m}\cancel{g}\sin\theta_{max}$$

$$\frac{11}{3}\mu_s = \tan\theta$$

$$\tan^{-1}\left(\frac{11}{3}\mu_s\right) = \theta_{max}$$

Problem3



Using L consveration

$$L_{bef} = mvb = mvR$$

$$L_{aft} = \left[\frac{1}{2}(4m)R^2 + mR^2\right]\omega$$

$$L_{aft} = (3mR^2)\omega$$

$$\cancel{m}vR = 3\cancel{m}R^2\omega$$

$$\frac{v}{3R} = \omega$$

Problem 3

So, $T = \frac{2\pi}{\omega} = 2\pi \cdot \frac{3R}{V} = \frac{6\pi R}{V}$

(2) Then

$$I_i \omega_i = I_f \omega_f$$

$$3mR^2 \omega_i = \left(\frac{1}{2} 4mR^2\right) \omega_f$$

$$\frac{3}{2} \omega_i = \omega_f$$

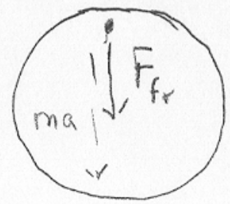
$$\frac{3}{2} \cdot \frac{V}{3R} = \omega_f$$

$$\frac{V}{2R} = \omega_f \qquad t_2 = \frac{2\pi}{\omega_f} = \frac{2\pi}{V/2R}$$

$$t_2 = 4\pi R/V$$

Problem 3

(3)



$$F_{fr} = ma$$

$$F_{fr} = m\omega^2 r$$

$$I_r \omega_r = I_o \omega_o$$

$$\left[\frac{1}{2}(4m)R^2 + mr^2\right]\omega_r = (3mR^2)\omega_o$$

$$(2\cancel{m}R^2 + \cancel{m}r^2)\omega_r = 3\cancel{m}R^2\omega_o$$

$$\omega_r = \frac{3R^2\omega_o}{2R^2 + r^2}$$

Problem 3

So

$$\omega_r = \frac{\cancel{3} R^2 (v / \cancel{3} R)}{2R^2 + r^2}$$

$$\omega_r = \frac{R v}{(2R^2 + r^2)}$$

$$F_{fr} = m \omega^2 r$$

$$= m \left(\frac{Rv}{2R^2 + r^2} \right)^2 r$$

$$F_{fr} = m \frac{v^2}{R} \quad \frac{r/R}{(2 + (r/R)^2)^2}$$