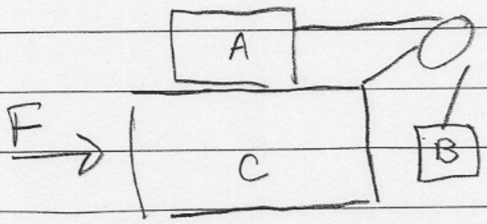
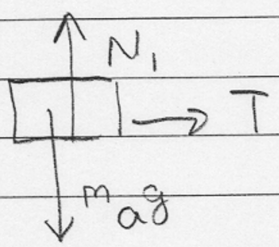
4.59

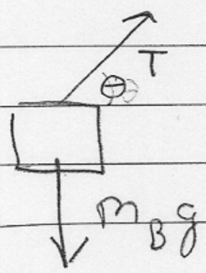


Free Body

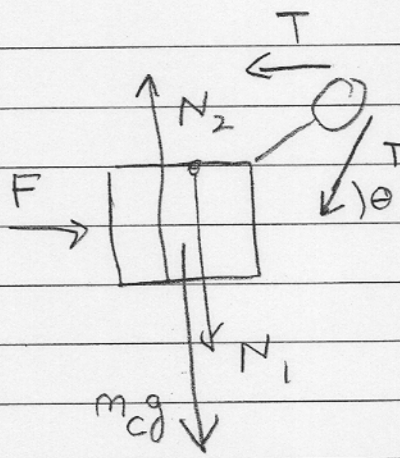
(A)



(B)



(C)



Newton laws in x

(A) $T = m_a a_{ax}$ $\qquad$ $N_1 - m_a g = \cancel{m_a a_{ya}}^{0}$

(B) $T\cos\theta = m_B a_{Bx}$ $\qquad$ $T\sin\theta - m_B g = \cancelto{0}{a_{By}}\, m_B$

(C) $F - T\cos\theta - T = m_c a_{cx}$ $\qquad$ $-T\sin\theta - N_1 + N_2 = \cancelto{0}{m_c a_{cy}}$

Now

$$\underbrace{a_{Bx} = a_{cx} = a_{ax} = a}_{\substack{\text{all x-accel the same} \\ \text{no relative x motion}}} \quad \text{and} \quad \underbrace{a_{ay} = a_{by} = a_{cy} = 0}_{\substack{\text{not moving up} \\ \text{and down}}}$$

So the 3x equations become and one y-equation

$T = m_a a$ $\qquad$ $T\sin\theta - m_B g = 0$

$T\cos\theta = m_B a$

$F = T + T\cos\theta + m_c a$

Unknowns: T, a, θ, F $\qquad$ Eqs $= 4$ ✓

Now working we first eliminate T

(1) $m_a \cancel{a} \cos\theta = m_B \cancel{a}$ $\qquad$ (2) $m_a a \sin\theta = m_B g$

or $\cos\theta = m_B / m_a$

(3) $F = m_a a + m_a a\cos\theta + m_c a$

$$F = a\,(m_a + m_a \cos\theta + m_c)$$

$$= a\,\left(m_a + \cancel{m_a}\,\frac{m_b}{\cancel{m_a}} + m_c\right)$$

$$F = a\,(m_a + m_b + m_c) \longleftarrow \text{Obvious!}$$

Now using

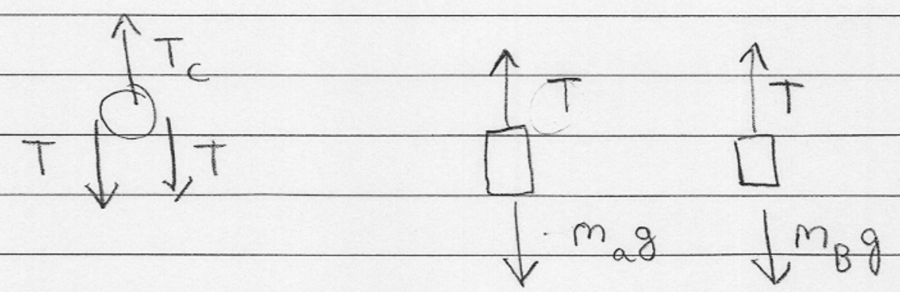
$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1 - m_b^2/m_a^2}$$

We use (2) to get $\quad a = \dfrac{m_B g}{m_a \sin\theta} = \dfrac{m_B}{m_a\sqrt{1-m_B^2/m_a^2}}$

So $\qquad\qquad = \dfrac{m_B}{\sqrt{m_a^2 - m_B^2}}$

$$F = \frac{m_B}{\sqrt{m_a^2 - m_B^2}}\,(m_a + m_b + m_c)$$

4.54



So Newtons laws

$$T_c - 2T = m_p \cancel{a_p}^{0} \qquad T - m_a g = m_a a_a \qquad T - m_B g = m_B a_B$$

Now with $a_a + a_b = 0$ $\quad a_a = -a_b = a$ we get

$$T - m_a g = m_a a \qquad T - m_B g = m_B(-a)$$

$$\frac{T}{m_a} - g = a \quad \Rightarrow \quad T - m_B g = -m_B\left(\frac{T}{m_a} - g\right)$$

$$T\left(1 + \frac{m_B}{m_a}\right) = 2 m_B g$$

$$T = \frac{2 m_B g}{1 + m_B/m_a}$$

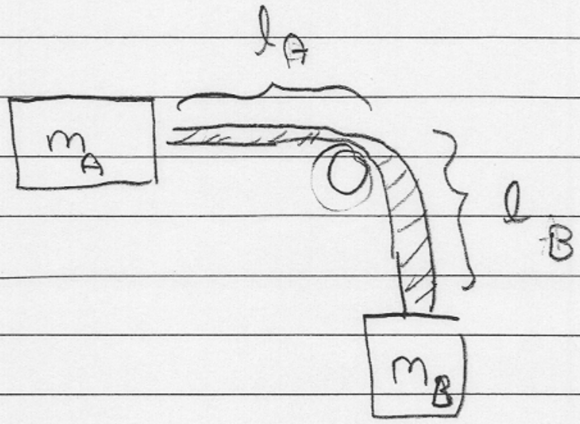
So

$$T_c = 2T = 4\frac{m_B g}{1 + m_B/m_a} = 4\frac{m_B m_a}{m_a + m_B} g$$

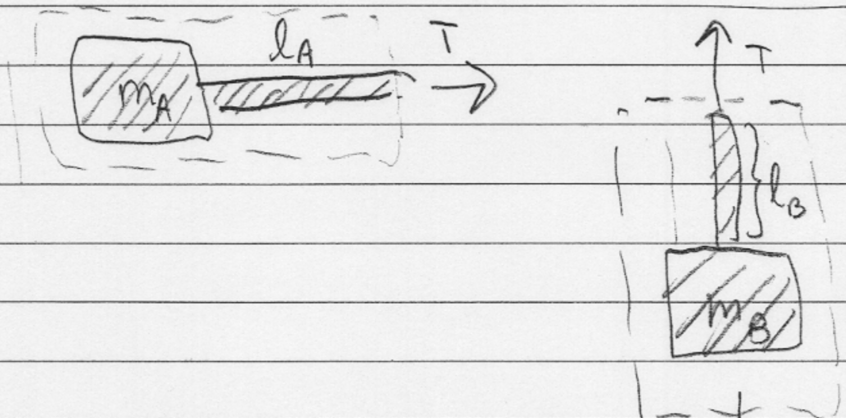
$m_a = 1.2$
$m_B = 3.2$
$g = 9.8$

$T = 34.2\,N$

4.53



Solution draw a fictious box around relevant parts



$$\left(m_B + \overbrace{m_c \frac{l_B}{l_B + l_A}}^{\text{This is the mass of the "B" cord}}\right) g$$

So

$$T = \left(\overbrace{m_A + m_c \frac{l_A}{l_A + l_B}}^{\text{mass of a + cord mass}}\right) a_A \qquad T - \left(m_B + m_c \frac{l_B}{l_B + l_A}\right) g$$

$$= \left(m_B + m_c \frac{l_B}{l_B + l_A}\right) a$$

Notation $L = l_A + l_B$ $\qquad x_A = \frac{l_A}{l_B + l_A}$ = fraction of part a

Also $a_a = -a_B = a$ $\qquad 1-x = \frac{l_B}{l_B + l_A}$ = fraction of part b

So:

$T = (m_a + m_c x)\, a$ and $T - (m_B + m_c(1-x))\, g = [m_B + m_c(1-x)] \times (-a)$

So

$$-(m_a + m_c x)\, a + (m_B + m_c(1-x))\, g = [m_B + m_c(1-x)](+a)$$

$$m_B + m_c(1-x)\, g = [m_B + m_c(1-x) + m_a + m_c x] \cdot a$$

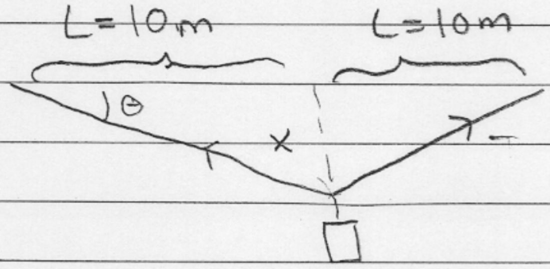
So

$$\frac{m_B + m_c(1-x)\, g}{m_a + m_B + m_c} = a$$

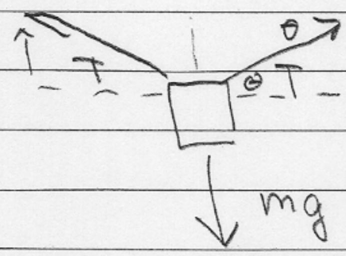
$$\boxed{\frac{\left(m_B + m_c \frac{l_B}{l_A + l_B}\right) g}{m_A + m_B + m_c} = a}$$

This is the $\frac{\text{~~m~~Weight of B part}}{\text{Total mass}} = a$

Problem 4.56 - see web page



Draw a FBD



$$2T\sin\theta = mg$$

$$\theta = \sin^{-1}\frac{mg}{2T}$$

$$\theta = \sin^{-1}\left(\frac{71\ kg\ 9.8\ m/s^2}{2\cdot 2600\ N}\right)$$

So:

$$\theta \simeq 0.12\ rad$$

$$\tan\theta = \frac{x}{L}$$

$$x = L\tan\theta = 10m \cdot \tan(0.12) \simeq 1.2$$

For part b we note that for $\theta \ll 1$

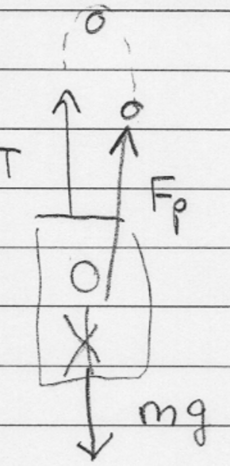
$$\theta \simeq \sin\theta \simeq \tan\theta = \frac{x}{L} \quad \text{so} \quad \sin\theta = \frac{mg}{2T} \simeq \frac{x}{L}$$

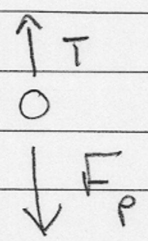
So $x \simeq \frac{Lmg}{2T}$ so if x is $\rightarrow$ over

So if x is four times smaller the tension is approximately four times larger

4.32

FBD of person:





FBD of rope end:

$$T - F_p = \cancel{m} \overset{0}{a}$$

So $F_p = T$

Here $T + F_p - mg = \cancel{m\overset{0}{a}}$ ← no accel

$$2T = mg$$

$$F_p^0 \equiv T = mg/2$$

Then if the force is increased by 1.18

$$T + 1.18 F_p^0 - mg = ma \qquad T = 1.18 F_p^0$$

$$2 \cdot 1.18 F_p^0 - mg = ma$$

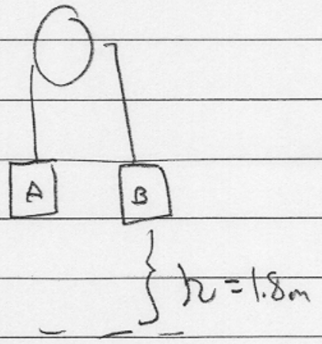
$$\cancel{2}\, 1.18 \frac{mg}{\cancel{2}} - mg = ma$$

$$\boxed{0.18 g = a}$$

4.58

First we determine the accel.

See Lecture 6+7



$$a_B = g\left(\frac{m_a - m_B}{m_a + m_B}\right)$$

$$a_B = (9.8 m/s^2)\left(\frac{2.2 kg - 3.6 kg}{2.2 kg + 3.6 kg}\right)$$

$$a_B = -2.3655 m/s^2$$

The speed of Block a when B hits ground is v_1

$$v_1^2 = \cancel{v_0^2} + 2a_a \Delta x \qquad \Delta x = h = 1.8m$$

$$a_a = -a_B = +2.36 m/s^2$$

$$v_1 = \sqrt{2a_a h} = 2.92 m/s$$

After this time a continues to rise a dist Δx

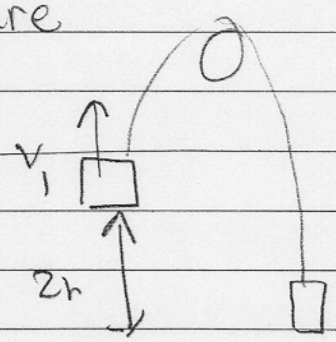
$$v_f^2 = v_0^2 + 2a\Delta x$$

$$0 = v_1^2 + -2g\Delta x$$

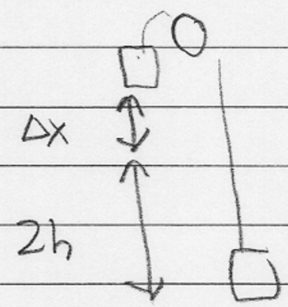
$$\frac{v_1^2}{2g} = \Delta x \Rightarrow \frac{(2.92 m/s)^2}{2 \cdot 9.8 m/s^2} = 0.435 m$$

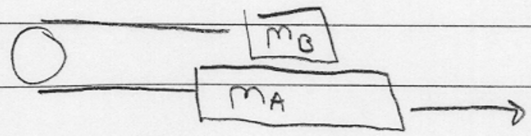
So Total height $= 2h + \Delta x = 4.035$

Picture



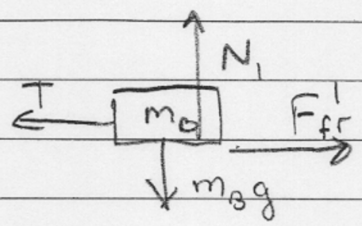
B hits ground

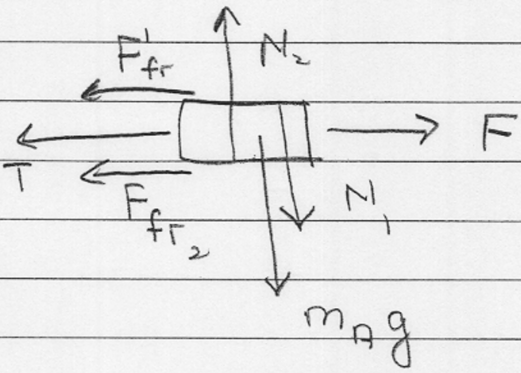




Draw FBD

B





So in y-direc:

$$N_1 - m_B g = \overset{0}{\cancel{m_B a_{By}}} \qquad N_2 - N_1 - m_a g = \overset{0}{\cancel{m_a a_{ay}}}$$

$$N_1 = m_B g \qquad N_2 = (m_a + m_b) g$$

X|

$$-T + \overbrace{\mu_s m_B g}^{F'_{fr}} = \overset{0}{\cancel{m_B a_{BX}}} \qquad -T - \overbrace{\mu_s m_B g}^{F'_{fr}} - \overbrace{\mu_s (m_a + m_B) g}^{F^2_{fr}} + F = \cancel{m_a}$$

There is no x-accel we have not broken free from static fric yet.

$T = \mu_s m_B g$, so $\quad F = T + \mu_s m_B g + \mu_s (m_B + m_a) g$

$$F = \overbrace{\mu_s m_B g}^{T} + \mu_s m_B g + \mu_s (m_a + m_B) g$$

$$F = 3\mu_s m_B g + \mu_s m_a g$$

$$F = 82N$$

For part B we know the force $F_* = 1.1F = 90.2N$
y-direction is the same $a_{ax} = -a_{Bx} = a$

$$-T + \mu_k m_B g = -m_B a \qquad -T - \mu_k m_B g - \mu_k (m_a + m_B) g + F_*$$

$$m_B a + \mu_k m_B g = T \qquad = m_a a$$

Find

$$-m_B a + \mu_k m_B g - \mu_s m_B g - \mu_k (m_a + m_B) g + F_* = m_a a$$

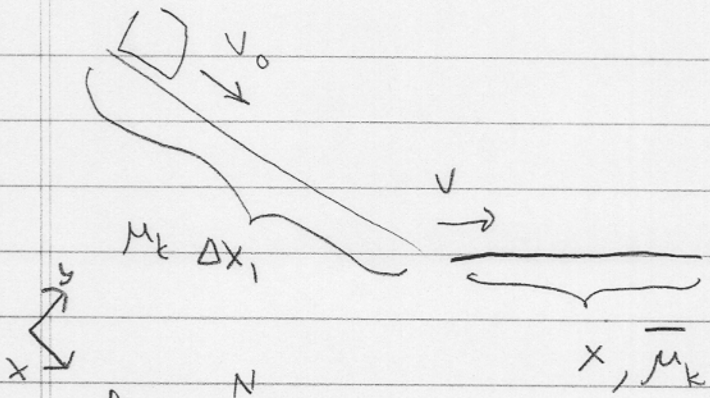
So

$$a = \frac{1}{(m_a + m_b)} \left[F_* - (3\mu_k m_B g + \mu_k m_a g) \right] ✓$$

Or

$$a = \frac{3m_B g + m_a g}{m_a + m_b} \left[1.1\mu_s - \mu_k \right] \text{ since } F_* = 1.1\mu_s (3m_B g + m_a g$$

Snowboard



$x = ?$

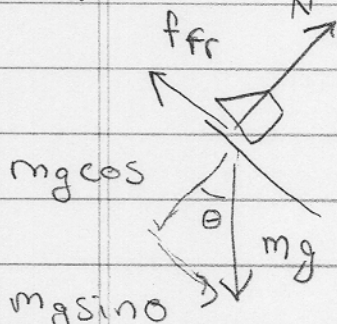
$x_1 = 110\text{m}$

$\mu_k = 0.18$

$\bar{\mu}_k = 0.15$

$v_0 = 5.0\text{m/s}$

$\theta = 28°$



$x \downarrow$

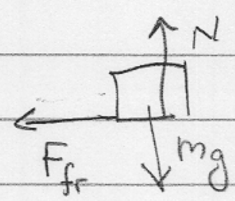
$$mg\sin\theta - \mu_k N = ma_x \qquad N - mg\cos\theta = m\cancel{a_y}^{\,0}$$

$$\cancel{m}g\sin\theta - \mu_k \cancel{m}g\cos\theta = \cancel{m}a$$

$$g(\sin\theta - \mu_k\cos\theta) = a_{slope}$$

For the flat part we set $\theta \to 0$

$$-\bar{\mu}_k g = a_{flat}$$



$$v^2 = v_0^2 + 2a_{slope}\Delta x_1$$

$$v^2 = v_0^2 + 2g(\sin\theta - \mu_k\cos\theta)\Delta x_1$$ ← velocity just before flat

For the second part

$$\cancel{v_f^2}^{0} = v_0^2 + 2a_{flat}x$$

$$\frac{+v^2}{+2\bar{\mu}_k g} = x$$

$$\frac{v_0^2 + 2g(\sin\theta - \mu_k\cos\theta)x_1}{2\bar{\mu}_k g} = x$$

$$\frac{v_0^2}{2\bar{\mu}_k g} + \frac{x_1(\sin\theta - \mu_k\cos\theta)}{\bar{\mu}_k} = x$$