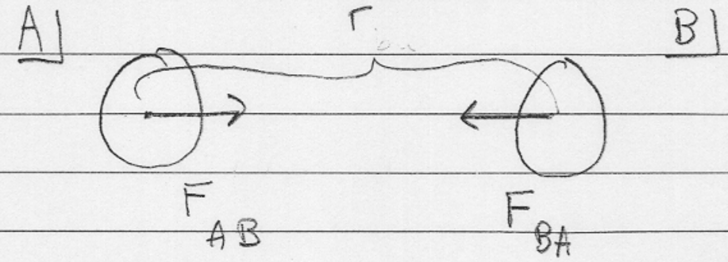
Gravity

- Newton concluded (based on earlier work) that all objects with mass attract

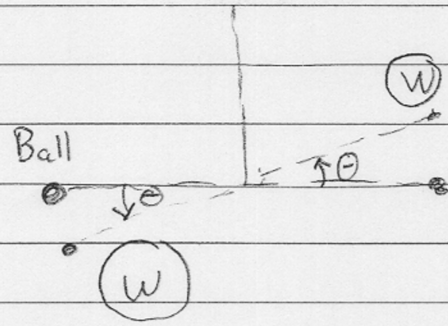


$$F_{Ba} \propto \frac{m_a m_b}{r^2} = G\frac{m_a m_b}{r^2}$$

- A hundred years later the constant was measured By Cavendish

$$G = 6.6 \times 10^{-11} \frac{Nm^2}{kg^2}$$

Outline of Cavendish Experiment

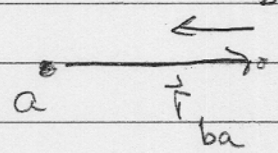


- When the weight is introduced the balls are attracted to the weights

Further Notes on Newtons Gravitation:

$$\vec{F}_{ba} = \frac{G\, m_a m_b}{r^2}\, -\hat{r}_{ba}$$

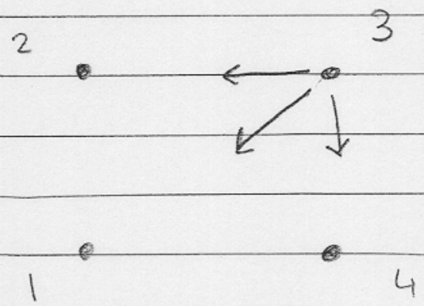
$\vec{F}_{ba} \propto -\hat{r}_{ba}$



Force on b due to a

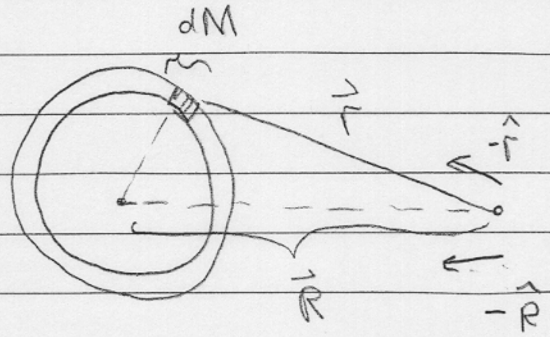
$\hat{r}_{ba}$ is a vector of unit length from $\vec{a}$ to $\vec{b}$ (i.e. the location of b relative to a)

In General, the total force is a sum:



$$F_{tot\ on\ 3} = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34}$$

Now consider a spherical Shell of mass M and determine



The gravity due to a small chunk of matter dM is

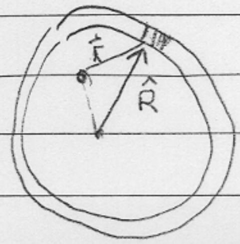
$$d\vec{F} = \frac{G\,dM\,m}{r^2}\,-\hat{r}$$

Newton invented the calculus to show (Appendix D)

$$\vec{F}_{out} = \int d\vec{F} = \frac{G\,M m}{r^2}\,-\hat{R}$$

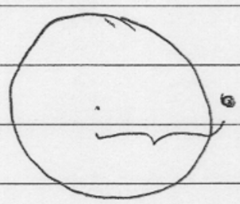
i.e. its as if all the matter was concentrated at the center

He also considered the force on the particle inside the sphere



$$\vec{F}_{in} = \int d\vec{F} = 0$$ (see App. D)

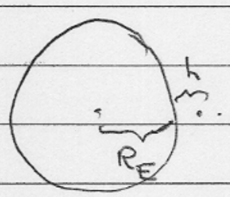
Example 1: Determime the force of gravity at the surface of the earth for an object of mass m



Solution: Treat all the mass as being at the center.

$$F_g = \frac{G M_E m}{R_E^2}$$

What is $g = 9.8 \ m/s^2$?

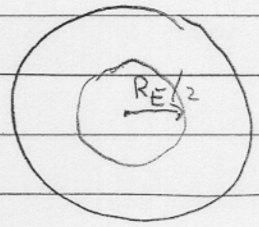


$$F_g = \frac{G M_E \cancel{m}}{(R_E + \cancel{h})^2} = \cancel{m} g$$

neglect this as small compared to R_E

$$\boxed{g = \frac{G M_E}{R_E^2}}$$

Example 2 : Determine the force of gravity on an object of mass m placed halfway from the center. Assume the density is constant



Solution: Only the matter inside the radius Radius $R_E/2$ contribute to the force. All the matter outside does not contribute.

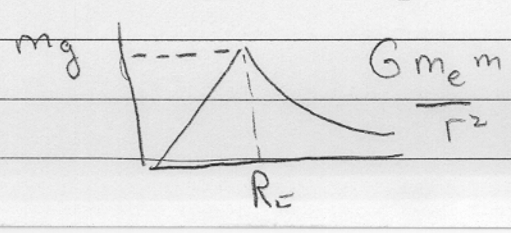
$$F_g = G\,\frac{M_{in}\, m}{(R_E/2)^2} \qquad M_{in} = \rho\, \frac{4}{3}\pi (R_E/2)^3$$

$$F_g = G \cdot \underbrace{\left(\frac{M_E}{\frac{4}{3}\pi R_E^3}\right)}_{\rho} \cdot \frac{\frac{4}{3}\pi (R_E/2)^3}{(R_E/2)^2} \qquad \rho = \frac{M_E}{\frac{4}{3}\pi R_E^3}$$

$$F_g = \frac{G M_E m}{2\, R_E^2} = \frac{g m}{2}$$ ← notice that at half the radius get half the gravity

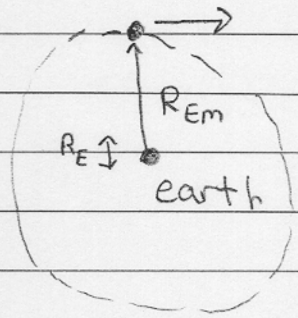
So,

$$F_g = g m \left(\frac{r}{R_E}\right) \quad \text{for} \quad r < R_E$$



How Could Anyone Ever think of this?

(1) Basic Observation:



→ There must be a force attracting the moon to the earth

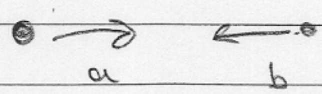
→ Objects (the apple!) are attracted to the earth why not the moon

→ The force must be proportional to mass
(to first order!)

$$F_g = \cancel{m} g = \cancel{m} a$$

otherwise acceleration would not be indep of mass, then since the force on the earth must be equal and opp to force on apple →

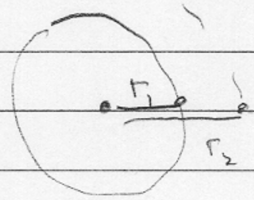
$$|F_{ab}| = |\vec{F}_{ba}| \propto m_a m_b$$



② An enormous amount of astronomical data (Kepler)

- Elliptical (almost circular) orbits of planets

- These orbits have an almost the interesting property

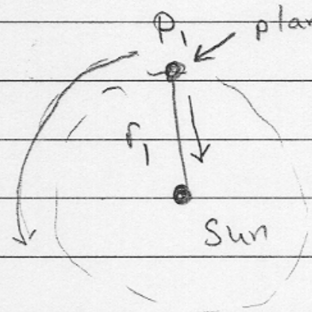


$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

T_1 = time it takes to go around sun for planet 1

r_1 = planet 1 to Sun radius

Now lets apply newtons Laws to orbits of planets



$$F_G = ma$$

$$\frac{G \cancel{m_1} M_s}{r_1^2} = \cancel{m_1} \frac{v_1^2}{r_1}$$

$$v_1 = \frac{2\pi R_1}{T}$$

So

$$\frac{G M_s}{r_1^2} = \left(\frac{2\pi r_1}{T_1}\right)^2 \frac{1}{r_1} \Rightarrow \frac{G M_s}{4\pi^2} = \frac{r_1^3}{T_1^2}$$

So since $GM_s/4\pi^2$ is constant (independent of planet)

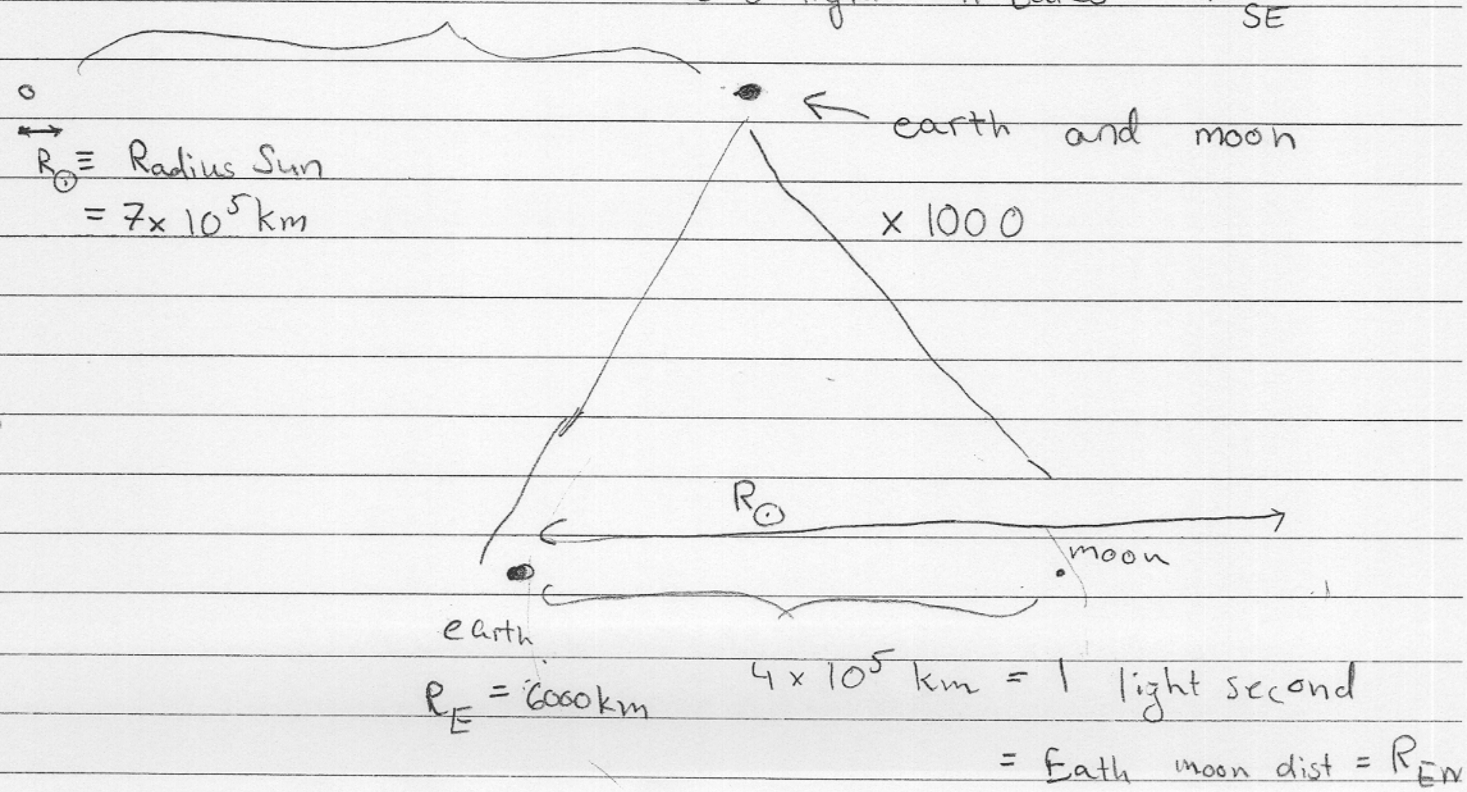
$$\frac{r_1^3}{T_1^2} = \frac{r_2^3}{T_2^2}$$

$$\left(\frac{r_1}{r_2}\right)^3 = \left(\frac{T_1}{T_2}\right)^2$$

③ Then there were <u>many</u> <u>many</u> more astronomical observations eg. trajectories of commets, tides, etc

<u>Scales</u>

1.5×10^8 km $\simeq$ 8 light minutes $= R_{SE}$



Masses

$$m_{\odot} = 2\times 10^{30}\,\mathrm{kg} \equiv \text{Sun}$$

$$m_E = 6\times 10^{24}\,\mathrm{kg} \equiv \text{Earth}$$

$$m_m = 2\times 10^{3} \equiv \text{moon}$$

$$\frac{m_E}{m_{\odot}} \simeq 10^{-6} \qquad \frac{m_m}{m_E} \simeq \frac{1}{3}\times 10^{-2}$$

Lengths and

$$\frac{R_{\odot}}{R_{ES}} \sim 10^{-3} \equiv \frac{\text{Radius of sun}}{\text{Sun earth distance}}$$

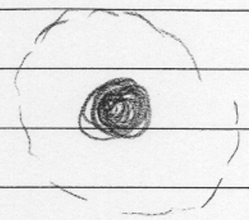
$$\frac{R_{\odot}}{R_{EM}} \sim 1 \equiv \frac{\text{Radius of Sun}}{\text{Earth moon dist}}$$

$$\frac{R_E}{R_{EM}} \sim 50 \equiv \frac{\text{Radius of Earth}}{\text{Earth moon dist}}$$

$$\frac{R_m}{R_E} \sim \frac{1}{6} \equiv \frac{\text{Radius of moon}}{\text{Radius of earth}}$$

Weightlessness

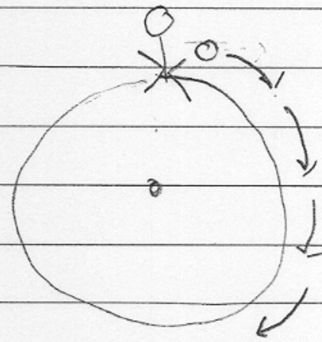
Problem: Consider a sattelite of mass m orbitting at twice the earths radius. What is the force of gravity



$$F_g = G\frac{M_E m}{(2R_E)^2} = \frac{1}{4}\overbrace{\left(\frac{GM_E}{R_E^2}\right)}^{g} m$$

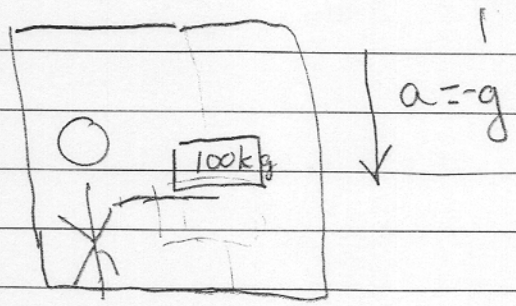
$$F_g = \frac{mg}{4} \leftarrow \text{ not zero!!}$$

So why is everything "weightless" in space:

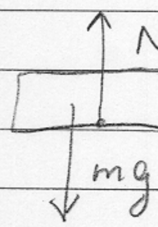


- Suppose our astronaught throws something a ball
- It falls but moves forward at the same time. Then the process continues again
- So the astronaught and ball are "falling" around the earth

Example: Suppose you are in an elevator in free fall

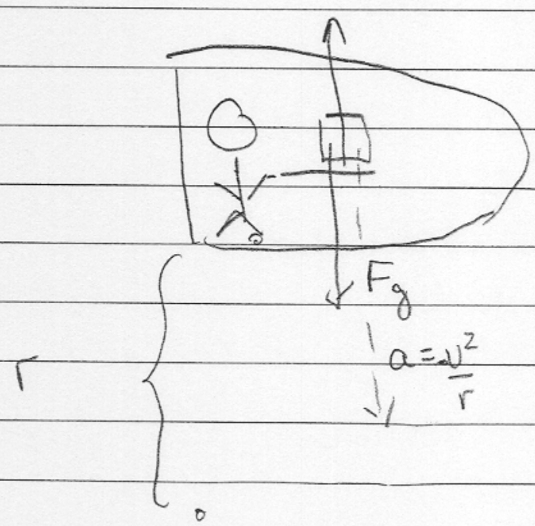


Free Body diagram:



$$N - m\cancel{g} = ma = -mg$$

So, $N - \cancel{m}\cancel{g} = -\cancel{m}\cancel{g} \Rightarrow N = 0$



To be in orbit at height r we have

$$F_g = -m\frac{v^2}{r}$$

Now look at Free body:

$$N - \cancel{F_g} = \cancel{-mv^2/r}$$

$$N = 0$$