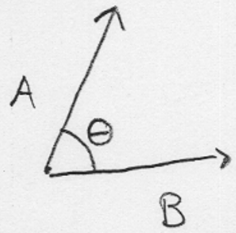
Last Time



$$\vec{A} \cdot \vec{B} = A\,B\cos\theta$$

Notation: $\vec{A}$ denotes vector

A w/out arrow means $|\vec{A}|$

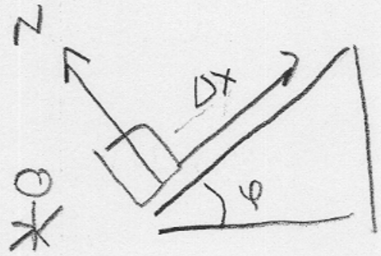
or $\sqrt{\vec{A}\cdot\vec{A}} = A$

Wrote:

$$\vec{A} \cdot \vec{B} = A_\parallel B = A_x B_x + A_y B_y$$

Work

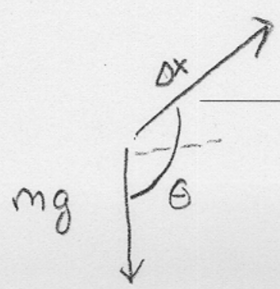
$$W = \vec{F} \cdot \vec{\Delta x} = F_\parallel \Delta x = F\,\Delta x\cos\theta$$



Analyzed Work

$$W_n = N\,\Delta x\cos\theta$$

$$= 0 \qquad \text{Since } \theta = 90^\circ$$

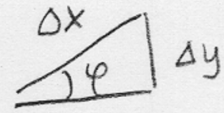


$$W_g = mg\,\Delta x\cos\theta < 0$$

Since $\theta > 90^\circ$

Now

$$W_g = mg\,\Delta x \cos(90^\circ + \varphi)$$

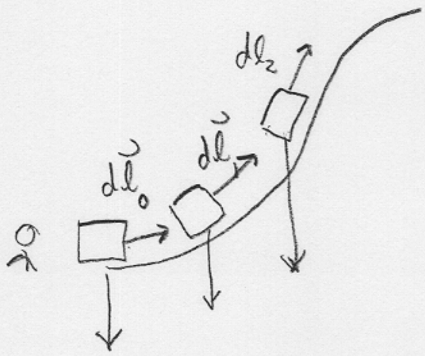


$$= -mg\,\underbrace{\Delta x \sin\varphi}_{\Delta y}$$

$$\boxed{W_g = -mg\,(y_2 - y_1)}$$

A very gereral formula wich we will se more of today

What if F not constant or Θ not constant



$$W = \sum_i F_i \cos\theta_i \, d\ell = \int_a^b F(\ell) \cos\theta(\ell) \, d\ell$$

Alternatively

$$W = \sum_i \vec{F} \cdot \vec{\Delta \ell} = \int_a^b \vec{F} \cdot d\vec{\ell}$$

This is known as a line integral

More math than needed:

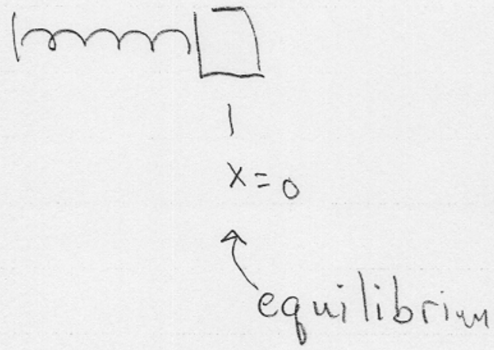
$$\vec{\ell} = x(\ell)\,\hat{i} + y(s)\,\hat{j}$$

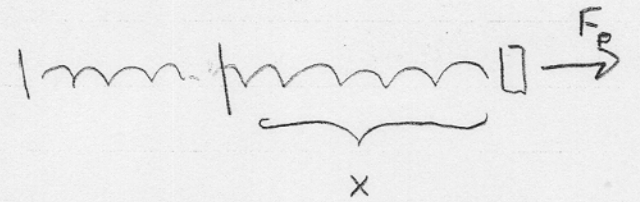
$$d\vec{\ell} = \left(\frac{dx}{d\ell}\,\hat{i} + \frac{dy}{d\ell}\,\hat{j} \right) d\ell$$

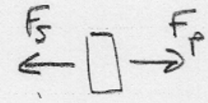
$$\vec{F} = F_x(\ell)\,\hat{i} + F_y(\ell)\,\hat{j}$$

$$\boxed{W = \int \vec{F} \cdot d\vec{\ell} = \int_a^L \left(F_x \frac{dx}{d\ell} + F_y \frac{dy}{d\ell} \right) d\ell}$$

Springs







Generally the more you stretch it the more it pulls

$$\vec{F}_p = k\vec{x}$$

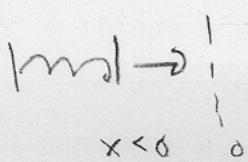
spring constant

So since $F_s + F_p = m\cancel{a}^{\,0}$

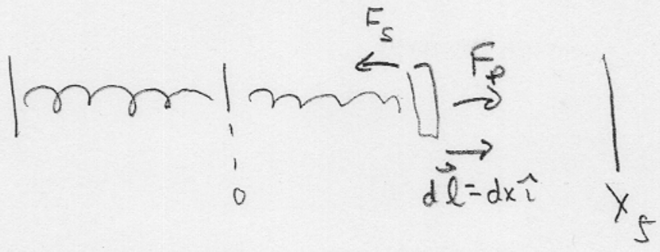
$$\vec{F}_s = -k\vec{x} \qquad F_s = |\vec{F}_s| = kx$$

indicates force is opposite displacem

If you push in



$$\vec{F} = -k\vec{x} = k(-\vec{x})$$



Whats the work done moving the spring to x_f

$$W_p = \int_a^b \vec{F} \cdot d\vec{l} = \int F_p\,\hat{\imath} \cdot dx\,\hat{\imath}$$

work done by puller

$$= \int F_p\,dx = \int_0^{x_f} kx\,dx = \frac{1}{2}kx_f^2$$

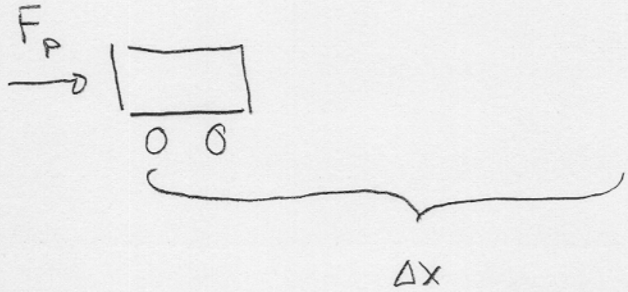
Notice

$$W_s = \int_0^{x_f} \vec{F}_s \cdot d\vec{l} = \int |F_s|\,(-\hat{\imath}) \cdot dx\,\hat{\imath} = -\int_0^{x_f} kx\,dx$$

work done by spring

$$= -\frac{1}{2}kx_f^2$$

Work and Kinetic Energy (Why we care)



Can relate the work done to the change in speed without talking about time

$$F_p = ma \qquad a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\overbrace{\frac{dx}{dt}}^{v}$$

$$F_p = m\,v\,\frac{dv}{dx}$$

$$\underbrace{\int_i^f F_p\,dx}_{\text{Work}} = \int_i^f m\,v\,dv$$

$$W_p = \frac{1}{2}mv^2\Big|_i^f = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

We call:

$$K = \frac{1}{2}mv^2 \equiv \text{the kinetic energy}$$

So

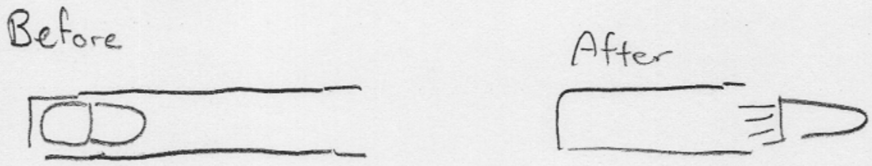
$$W_p = \Delta K = K_f - K_i$$

We treated one force for many forces $F_1, F_2, \ldots$ each force would do work and $W_1, W_2, \ldots$

$$W_1 + W_2 + \ldots = \Delta KE = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

Example:

Estimate the work done when launching a bullet:

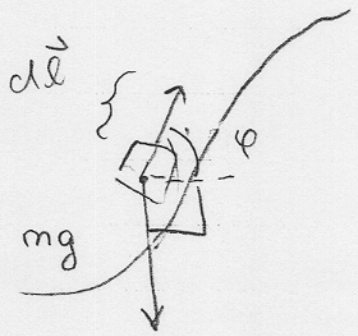


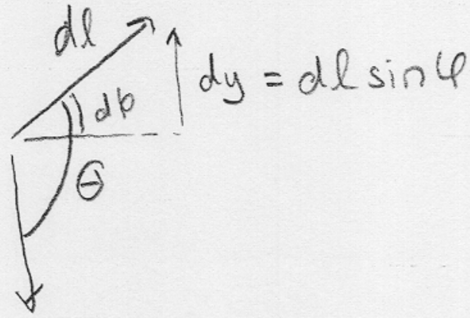
$$W = \Delta K = K_f - K_i = \frac{1}{2} m v_f^2 - 0$$

$m \simeq 5g \quad v_f \simeq 500 m/s$

$$W = \frac{1}{2}(0.005 kg)(5 \times 10^2 \tfrac{m}{s})^2 = 625 J$$

Proof





$$dW = \vec{F}_g \cdot d\vec{l} = mg\cos\theta \, dl = -mg\,dy$$

$$\cos\theta \, dl = \cos(90^\circ + \varphi)\,dl = -\sin\varphi \, dl = -dy$$

Now

$$W = \int_a^b dW = \int_a^b -mg\,dy = -mg(y_2 - y_1)$$

$$\boxed{W_g = -mg(y_2 - y_1)}$$

So consider a closed loop

$$W_g^{Total} = \overbrace{-mg(y_2 - y_1)}^{\text{work up}} + \overbrace{-mg(y_1 - y_2)}^{\text{Work down}}$$

$$W_g^{Total} = 0$$