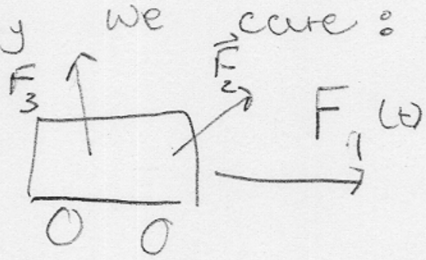
Last Times:

① Work

$$W_F = \int \vec{F} \cdot d\vec{r} = \vec{F} \cdot \Delta\vec{r} \quad \text{(For constant force)}$$

② Why we care:

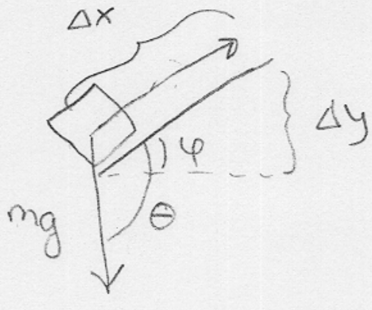


Three arbitrary forces F_1, F_2, F_3

We calculate to work done by each, $W_1\ W_2\ W_3$

$$W_1 + W_2 + W_3 = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Along the way we showed:



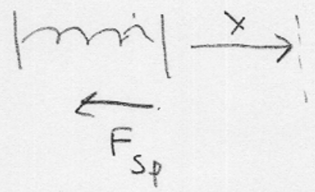
$$W_g = mg\,\Delta x \cos\theta$$

$$= mg\,\Delta x \cos(90 + \varphi)$$

$$W_g = -mg\,\underbrace{\Delta x \sin\varphi}_{\Delta y}$$

$$W_g = -mg\,(y_2 - y_1)$$

$$\vec{F} = -k \cdot \vec{x}$$

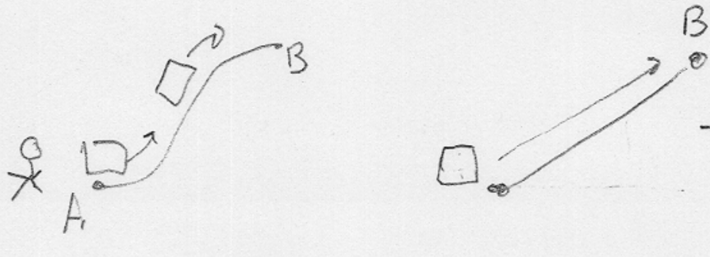


$$W_{sp} = -\frac{1}{2} k x^2$$

Today we will further classify forces:

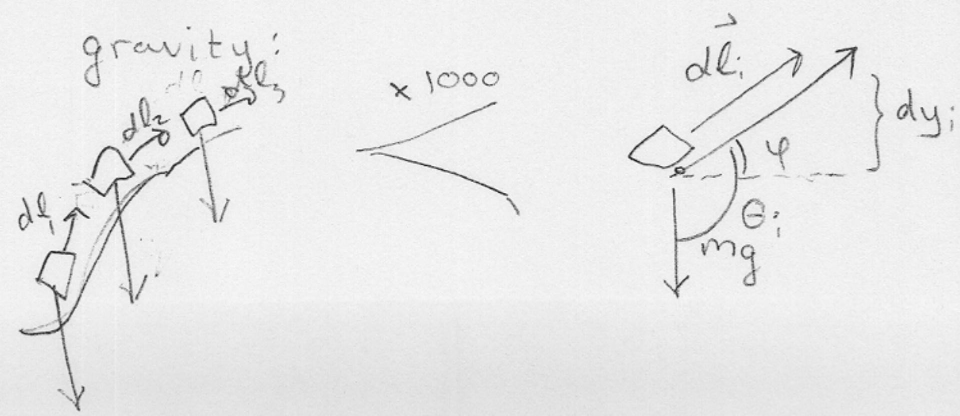
<u>Conservative</u>	<u>Dissipative</u>
gravity	friction
springs	air resistance
electromagnetic	

Mathematically: A force is conservative iff the work done in going from $A \leftrightarrow B$ is independent of the path in going from $A \leftrightarrow B$.



$$\rightarrow \quad W_g = -mg\,(y_B - y_A)$$

Take gravity:



Then

$$dW_i = mg\, dl_i \cos\theta_i = -mg\, dy_i$$

So

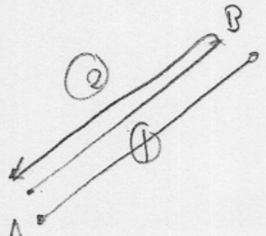
$$W = \sum W_i = \sum_i -mg\, dy_i = -\int_A^B mg\, dy$$

$$W = -mg\,(y_B - y_A)$$

So Same as for the straight path!

The Difference @ Friction:

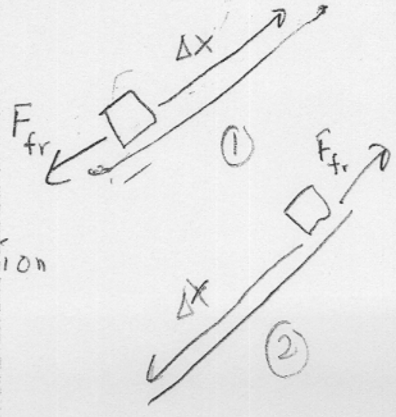
gravity



$$\omega = \omega_{①} + \omega_{②}$$

$$\omega = -mg\,(y_B - y_A) + -mg\,(y_A - y_B)$$

$\omega = 0 \Rightarrow$ For a conservative force the work done in a closed loop is zero!



Friction

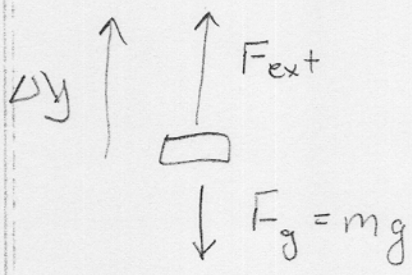
$W_1 < 0$

This is not the case for friction

$W_2 < 0$

$$W = W_1 + W_2 < 0$$

Potential Energy (Conservative Forces only



$F_{ext} = mg$

$$W_{ext} = mg\,\Delta y \cos 0^\circ = mg\,\Delta y$$

$$W_g = mg\,\Delta y \cos 180^\circ = -mg\,\Delta y$$

By doing work on the book we have increased the ability gravity to do work, we have Stored the E. When the book is released this energy gets coverted to kinetic energy

Then define the change in potential energy

$$\Delta U = U_2 - U_1 = W_{ext} = -W_G$$

$$\Delta U = U_2 - U_1 = mg(y_2 - y_1)$$

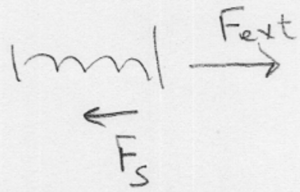
So

$$U = mgy$$

But we really only care about changes in potential energy

$$\Delta U = U_2 - U_1$$

So far only gravity



$W_{ext} > 0$

$W_{spring} < 0$

By increasing the length of spring you increase the ability of the spring to do work

$$\Delta U_s = +W_{Ext} = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2$$

$$\Delta U_{sp} = -W_{sp} = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2$$

$$U_{sp} = \frac{1}{2} k x^2$$

General:

$\Delta U = W_{ext}$ required to bring about a change of coordinates

$$\Delta U = -W = -\int_{x_0}^{x} \vec{F} \cdot dx$$

Relation to forces: 1D

$$U(x) - U(x_0) = -\int_{x_0}^{x} F\, dx$$

$$\frac{dU(x)}{dx} - 0 = -\frac{d}{dx} \int_{x_0}^{x} F\, dx$$

$$\frac{dU}{dx} = -F \Rightarrow \boxed{F(x) = -\frac{dU}{dx}}$$

Example

$U = +\frac{1}{2} k x^2$

$F_{sp} = -kx$

In 3-D we have

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$F_x = -\frac{\partial U}{\partial x} \qquad F_y = -\frac{\partial U}{\partial y} \qquad F_z = -\frac{\partial U}{\partial z}$$

known as the minus gradient of U

Solving Problems with Energy

$$\underbrace{W_{TOT}}_{-\Delta U} + W_{fr} = \Delta K$$

So

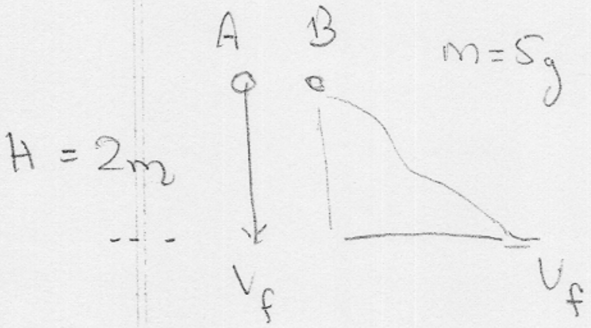
$$W_{fr} = \Delta K + \Delta U$$

Now if there is no friction

$$\Delta K + \Delta U = 0$$

I.e, the total change in Kinetic + potential energy is zero

$\rightarrow$ e



Question 1: Which one is going faster at the Bottom

Qestion 2: Who gets to the bottom first

$$\cancel{W_{fr}}^{\,0} = \Delta K + \Delta U$$

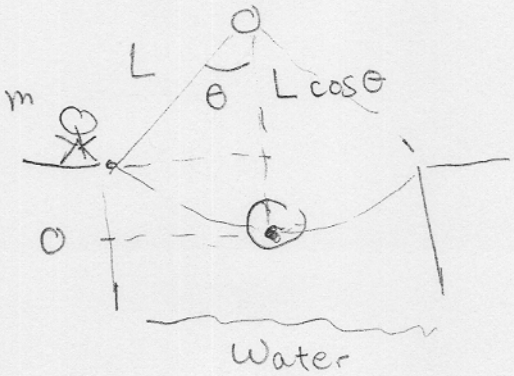
$$= K_f - \cancel{K_i}^{\,0} + U_f - U_i$$

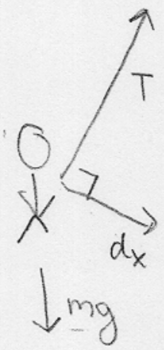
$$U_i - U_f = K_f$$

$$mgH - mg(0) = \frac{1}{2} m V_f^2$$

$$\sqrt{2gH} = V_f$$

Ex2





Whats the force on the Childs arms at the bottom?

Solution:

First determine how fast the child is going

Fundamental $\rightarrow$ $$\omega_T + \underbrace{\omega_g}_{-\Delta U} = \Delta K$$

$$dW_T = \vec{T} \cdot d\vec{x}$$

$$= 0 \leftarrow$$ Tension is $\perp$ to dx

definition of PE $\rightarrow$ $$\omega_T = \Delta K + \Delta U_g$$

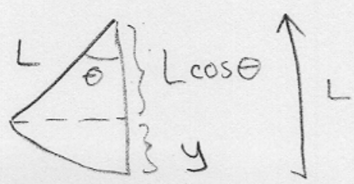
= Tensions and normals do no work!

$$0 = \Delta K + \Delta U_g$$

$$0 = K_f - \cancel{K_i}^{\,0} + \cancel{U_f}^{\,0} - mgy$$

$$mgy = \frac{1}{2} m v_f^2$$

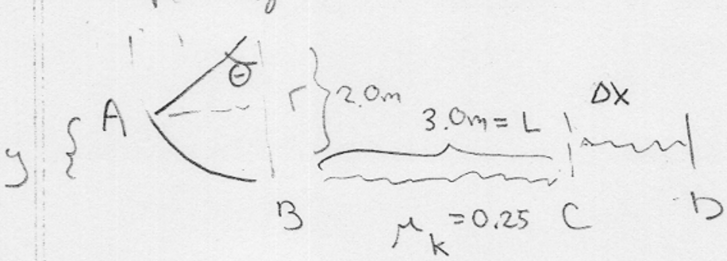
$$mg(L - L\cos\theta) = \frac{1}{2} m v_f^2$$



$$\sqrt{2gL(1-\cos\theta)} = v_f$$

Last Example @ Friction

$m = 1\,kg$



$$W_{fr} = \Delta K + \Delta PE$$

$\star 1$

$$-\mu_k N L = \frac{1}{2} m \cancel{v_f^2} - \frac{1}{2} m \cancel{v_i^2} + \cancel{U_{si}} - U_{sp,f} + U_{gi} - \cancel{U_{gf}}$$

$$-\mu_k m g L = mgy - \frac{1}{2} k x^2$$

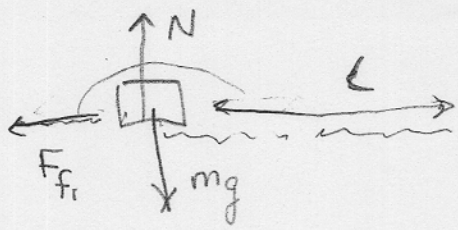
$\star 2 \longrightarrow$

$$\frac{1}{2} k x^2 = m g\, r\,(1 - \cos\theta) + \mu_s m g L$$

$$k = \frac{2mg}{x^2}\left[r(1-\cos\theta) + \mu_s L\right]$$

Notes

$\star 1$



$$N - mg = m\cancel{a_y}$$

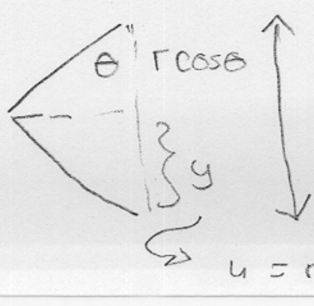
$$N = mg$$

$$W_{fr} = \vec{F}_{fr} \cdot \vec{\Delta x} =$$

$$= \mu m g L \cos 180^\circ$$

$$= -\mu_k m g L$$

$\star$



$$y = r - r\cos\theta = r(1-\cos\theta)$$