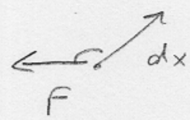
Last Times

① Dot Product $\vec{A} \cdot \vec{B} = AB\cos\theta$

$= A_{\parallel} B$

② Work

$dW_F = \vec{F} \cdot d\vec{x}$



$F\,dx\,\cos\theta$

③ F_{fr}, F_{sp}, F_g Work & Kinetic

Total work done on object $= W_{fr} + W_{sp} + W_g = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$= \Delta KE$

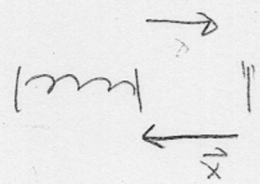
④ $\underbrace{W_{fr}}_{\text{non-conserv}}$, $\underbrace{W_{spring} \,,\, W_{grav}}_{\text{conservative}}$

non-conserv or dissipative

conservative - indep of path

$F = -k\vec{x}$

$W_{sp} = -\frac{1}{2}kx^2$



Work done by spring

$\Delta U_s = -W_{sp} = \boxed{\frac{1}{2}kx^2 = \Delta U_s}$

④ Potential Energy

$$\underbrace{W_{fr}}_{\text{non-consv}} + \underbrace{W_{sp} + W_g}_{\text{consv}} = \Delta KE$$

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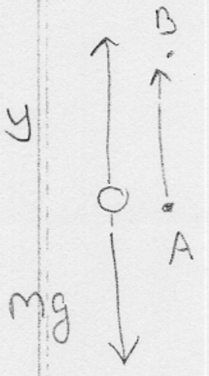


$$W_g = -mg\,y$$

gravity did negative work
i.e. we did work against gravity

$$\Delta U_g = -W_g \quad \text{or} \quad \boxed{\Delta U = -W}$$

Differentially: $dU = -dW \Rightarrow$

$$dU = -\vec{F}\,d\vec{x}$$

$$\boxed{-\frac{dU}{dx} = F}$$

⑤ Where we are

$$W_{fr} + \underbrace{W_{sp}}_{-\Delta U_s} + \underbrace{W_g}_{-\Delta U_g} = \Delta K$$

$$\boxed{W_{fr} = \Delta K + \Delta U_{TOT}}$$

When no friction energy is conserved

$$\boxed{0 = \Delta K + \Delta U_{TOT}}$$

Differential

(6) $dW = dK + dU$

$F_{fr}\, dx = mv\,dv + dU$

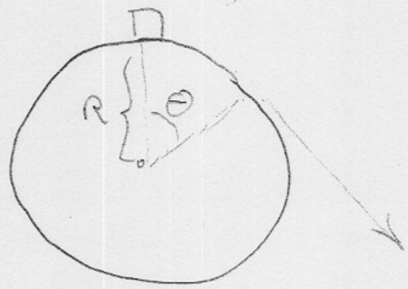
$K = \frac{1}{2}mv^2$

$dK = mv\,dv$

Then

Example:

m, g



$[m] \sim kg$

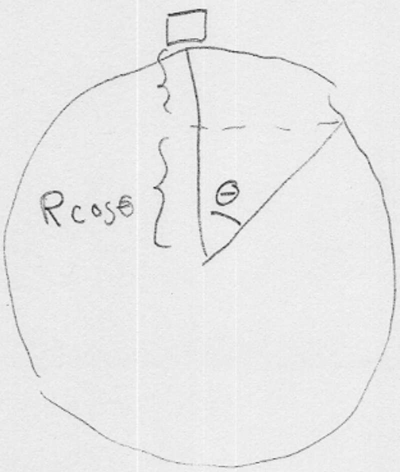
$[R] \sim m$

$[g] \sim m/s^2$

Q: At what angle does it fly off?

$\theta \approx \#$ (there are no dimensionless numbers out of g, m, R

Sol: First determine its speed



$$0 = \Delta KE + \Delta PE$$

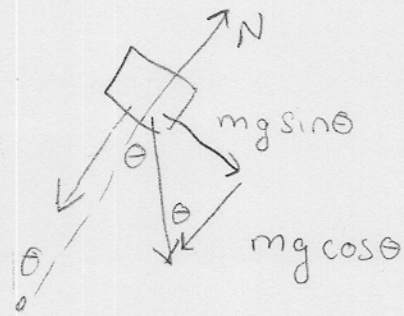
$$0 = \frac{1}{2} m v_f^2 - \cancel{\frac{1}{2} m v_i^2}^{\,0} + mg y_f - mg y_i$$

$$0 = \frac{1}{2} m v_f^2 + mgR\cos\theta - mgR$$

$$mgR - mgR\cos\theta = \frac{1}{2} m v^2$$

$$2gR(1 - \cos\theta) = v^2$$

Then



$$N - mg\cos\theta = -m\frac{v^2}{R}$$

Now just before free $N = 0$

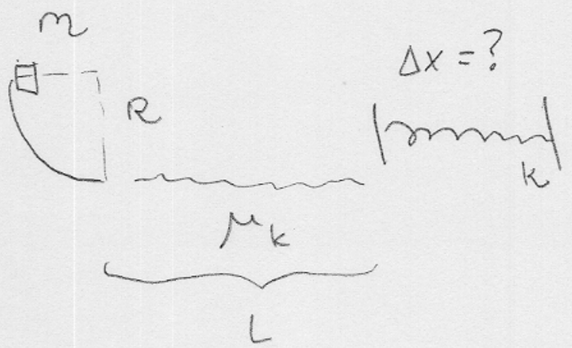
So

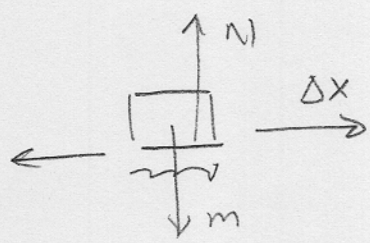
$$\cancel{m}g\cos\theta = \frac{\cancel{m}}{\cancel{R}}\,2g\cancel{R}(1-\cos\theta)$$

$$3\cos\theta = 2$$

$$\cos\theta = \frac{2}{3} \qquad \theta \simeq 48^\circ$$

Ex2



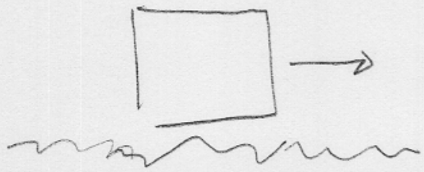


$$W_{fr} = \Delta K + \Delta U$$

$$-\mu_k mgL = \cancel{\tfrac{1}{2}mv_f^2}^{\,0} - \cancel{\tfrac{1}{2}mv_i^2}^{\,0} + U_f - U_i$$

$$-\mu_k mgL = \frac{1}{2}kx^2 - mgR \Rightarrow \left[\frac{2(mgR - \mu_k mgL)}{k}\right]^{\frac{1}{2}} =$$

Energy Conservation:



$$W_{fr} = \Delta K_{block} + \Delta U_{block}$$

negative so $\Delta K_{block} + \Delta U_{block}$

However energy is not being lost rather it is being transferred to the many molecules of the ground. We are not observing these molecules so from our perspective enery is being lost.

$$0 = \Delta K_{block} + \Delta U_{blok} + [\Delta E_{\text{of environent/groun}}]$$

$-W_{fr}$

Power

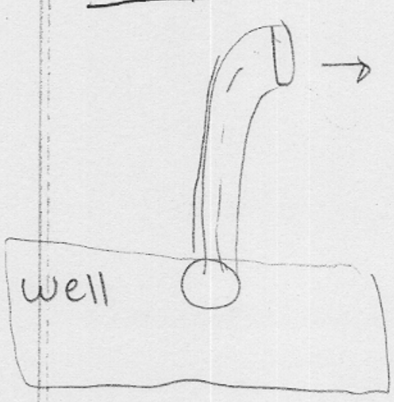
$$P = \frac{\Delta W}{\Delta t} \Rightarrow P = \frac{dW}{dt} \qquad [P] = J/s$$

Power exerted by a force

Now, $dW = \vec{F} \cdot d\vec{x}$, So

$$P = \frac{\vec{F} \cdot d\vec{x}}{dt} = \boxed{\vec{F} \cdot \vec{v} = P}$$

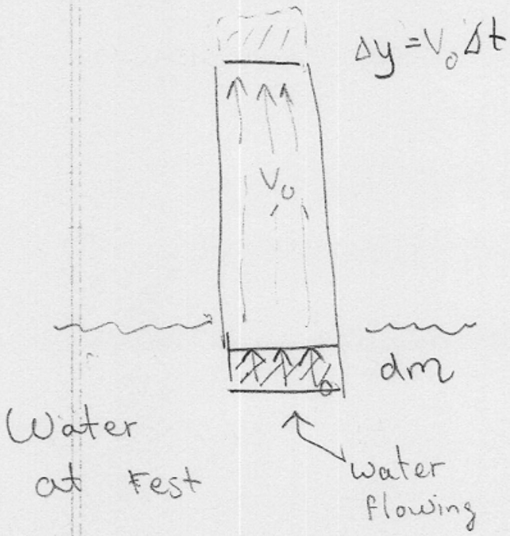
Example:



A pump raises water from a well of depth 20m and discharges it at $6 m/s = v_0$. It pumps out $10 kg/s = dm/dt$

What is the power of the moto in the pump?

Solution



consider a time interval Δt

$$\rho = \frac{m}{L}$$

Now:

$$\Delta m = \rho\ \Delta y$$

$$\Delta m = \frac{m}{L}\ \Delta y$$

$$L \frac{\Delta m}{m} = \Delta y$$

$$\Delta W_{pump} = \Delta K + \Delta U$$

$$\Delta W = \frac{1}{2} \Delta m\, V_0^2 + m g \Delta y$$

$$\Delta W = \frac{1}{2} \Delta m\, V_0^2 + \cancel{m} g \frac{L}{\cancel{m}} \Delta m$$

$$\frac{\Delta W}{\Delta t} = \frac{1}{2} \frac{\Delta m}{\Delta t} V_0^2 + gL \frac{\Delta m}{\Delta t}$$

$$P = 2180\ \frac{J}{s}$$

$L = 20m$

$g = 9.8 m/s^2$

$\frac{\Delta m}{\Delta t} = 10\ kg/s$

$v_0 = 6\ m/s$