Last Times

① Work

$$W_{12} = \int_1^2 \vec{F} \cdot d\vec{x}$$

$$dW = \vec{F} \cdot dx$$

② Potential Energy

$$\Delta U = U_2 - U_1 = -W_{12}$$

or

$$dU = -F\,dx$$

$$F = -\frac{dU}{dx}$$ (more examples today!)

③ Work and Kinetic energ

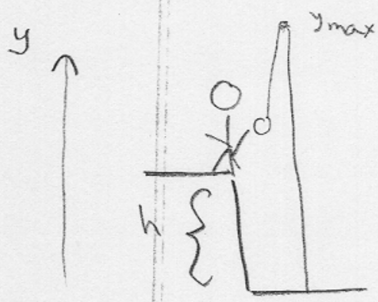
$$W_{12}^{fr} + W_{12}^{ext} + \underbrace{W_{12}^{g}}_{-\Delta U_g} + \underbrace{W_{12}^{ss}}_{-\Delta U_{sp}} = K_2 - K_1$$

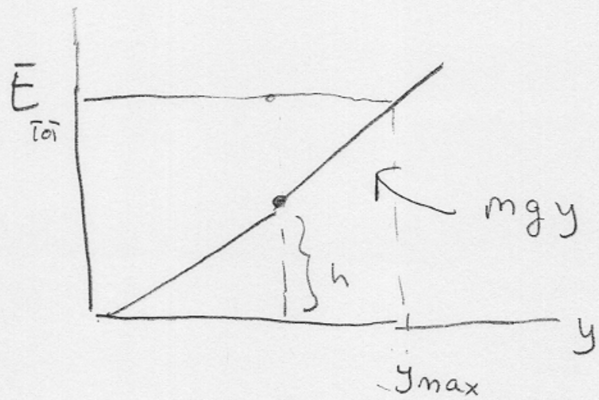
$$W_{fr} + W_{ext} = \Delta K + \Delta U_g + \Delta U_{sp}$$

So if no frict or external forces

$$0 = \Delta K + \Delta U_{TOT} \Rightarrow \boxed{K_1 + U_1 = K_2 + U_2}$$

Energy Diagrams $U(y)$





Comments

① $U(y) = mgy$

$$F_g = -\frac{dU}{dy} = -mg$$ ← down the slope

② $K_1 + U_1 = K_2 + U_2 = \text{Const} = E_{TOT}$

↖ we draw this a straight line

③ Notice that at any moment

$$K = E - U(x)$$

kinetic energy at position x

pot. energy at pos x

= Difference between the energy line and the potential

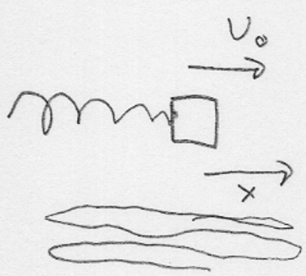
④ The "turning point" happens when the potential energy equals the total energy

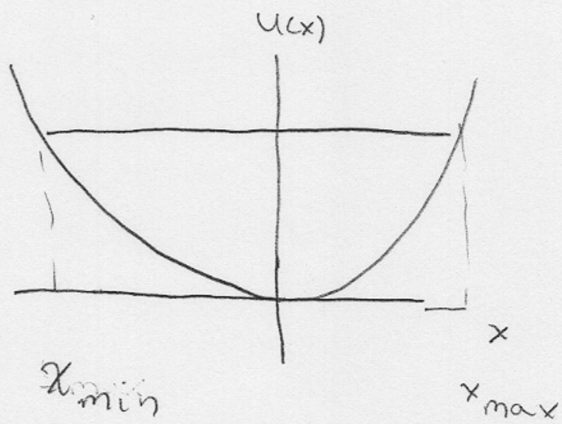
i.e.

$$\frac{1}{2} m v_0^2 + mgh = E = mg\, y_{max}$$

$$\frac{v_0^2}{2g} + h = y_{max}$$

Ex 2





Comments

① $U(x) = \frac{1}{2} k x^2 \Rightarrow F = -\frac{dU}{dx} = -kx$

② No force in equilibrium position (slope of pot $= 0$)

③ $E = \frac{1}{2} m v_0^2 + \cancel{\frac{1}{2} k x_0^2}^{\,0}$

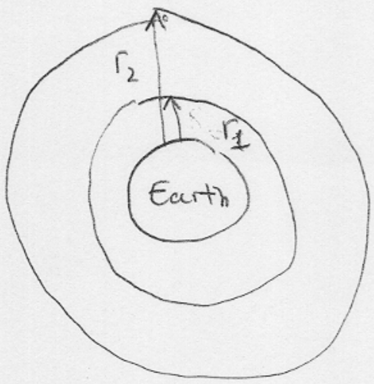
(4) Turning points happen when

$$\frac{1}{2} k x_{max}^2 = E$$

$$\frac{1}{2} k x_{max}^2 = \frac{1}{2} m v_0^2 \implies x_{max} = \left(\frac{m v_0^2}{k} \right)^{1/2}$$

Ex3 - Gravitational Portential of the earth and escape velocity

Consider the work done by gravity:



$$W^g_{12} = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

$$W^g_{12} = \int_{r_1}^{r_2} \frac{G M_E m}{r^2} (-\hat{r}) \cdot dr\, \hat{r}$$

$$W^g_{12} = -\int_{r_1}^{r_2} \frac{G M_E m}{r^2}\, dr$$

$$W^g_{12} = + \frac{G M_E m}{r_2} - \frac{G M_E m}{r_1}$$

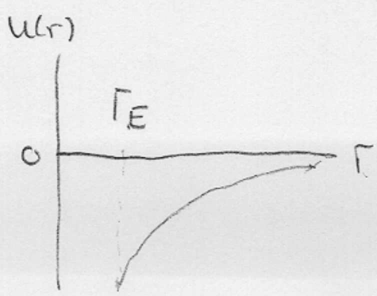
So the change in potential energy

$$U_2 - U_1 = - W^g_{1\to 2} = - \frac{G M_E m}{r_2} + \frac{G M_E m}{r_1}$$

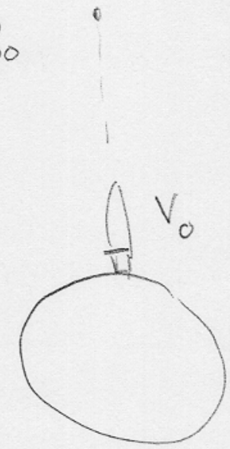
From this conclude

$$U(r) = - \frac{G M_E m}{r} + C$$

The traditional Choice is $C = 0$ (arbitrary) (no potental at $r \to \infty$)

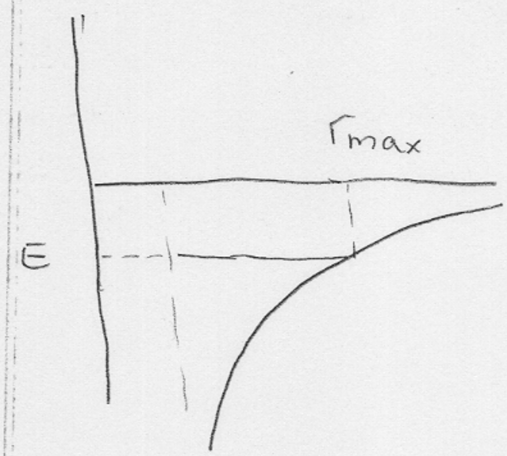


So



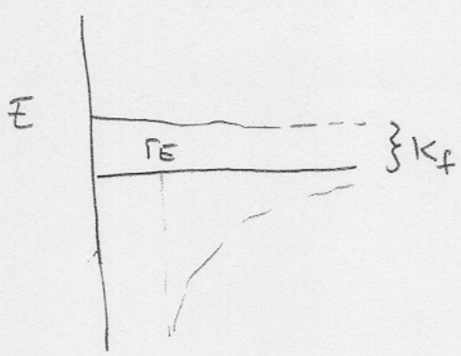
Suppose shoot a rocket straight up

$$E = \frac{1}{2} m v_0^2 + \frac{-GM_E m}{R_E}$$



if $E<0$ then there will be an r_{max} where all the energy is potential, the rocket will then plummet back

On the other hand If $E>0$,



then the rocket will go a great distance and move with kinetic energy K_f

$E=0$ v_0 is just enough to escape the earths pull

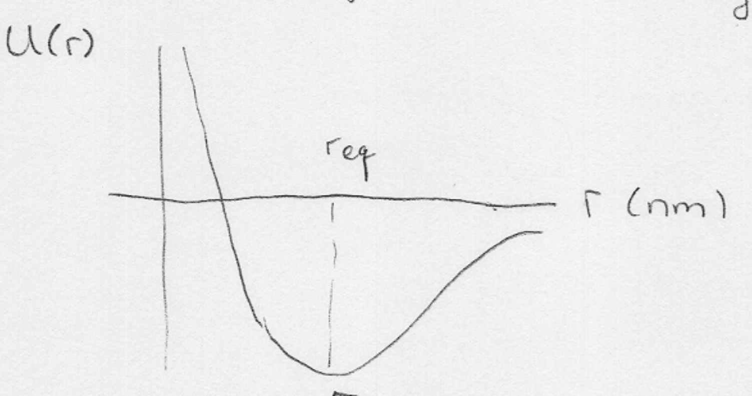
This

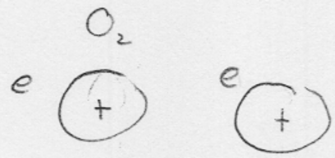
$$\frac{1}{2}mv_0^2 - \frac{G M_E m}{r_E} = 0$$

$$v_0 = \left(\frac{2 M_E G}{r_E}\right)^{1/2}$$

$$v_0 \simeq 1.12 \times 10^4 \, m/s \sim \text{mach } 3\text{-}4$$

Other examples of Energy diagrams





U(r)

where $\frac{dU}{dr} = 0$

Momentum

So For Single partices we will now move to systems of particles

① $\vec{p} = m\vec{v}$ (Bullet vs. wrecking ball)

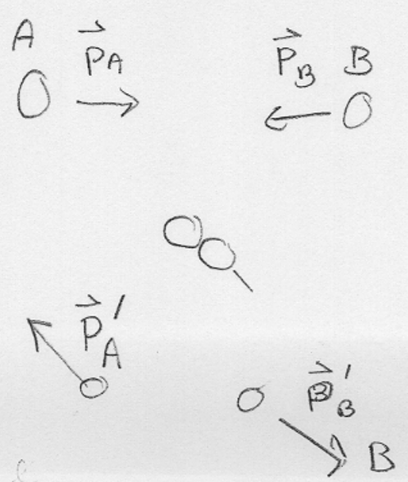
② $\sum \vec{F} = m\vec{a}$ ← actually this isn't quite right

$$\sum_i F = m\frac{d\vec{v}}{dt} = \frac{d\,m\vec{v}}{dt} = \frac{d\vec{p}}{dt}$$

In general the sum of all forces acting on a body is

$$\sum \vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$ ← matters if

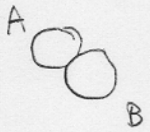
Momentum Conservation and multiple bodies

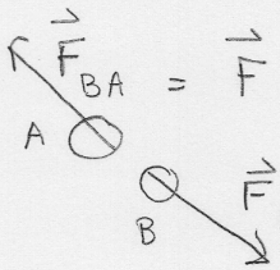


$$\vec{P}_A + \vec{P}_B = \vec{P}_A' + \vec{P}_B'$$

$$m_A\vec{V}_A + m_B\vec{V}_B = m_A\vec{V}_A' + m_B\vec{V}_B'$$

This is also a general consequence of Newton Laws





$\vec{F}_{BA} = \vec{F}$ = force of B on A

$\vec{F}_{AB} = -\vec{F}$

= Force of A on $\vec{B}$

Then from Newton Laws

$$\vec{F} = \frac{d\vec{p}_A}{dt}$$

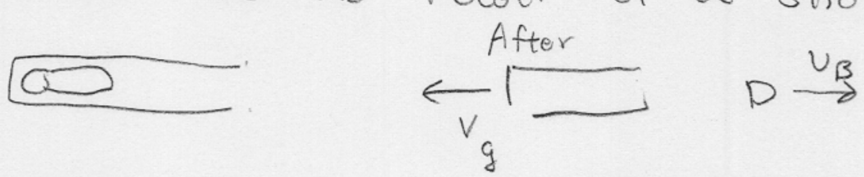
$$-\vec{F} = \frac{d\vec{p}_B}{dt}$$

Adding these equations we find

$$0 = \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = \frac{d}{dt}(\vec{p}_A + \vec{p}_B)$$

The total momentum is conserved

Example: Estimate the recoil velocity of a shot gun



$$P_{Before} = 0 = P_{After} = m_g v_g + m_B v_B$$

So $$v_g = -v_B\, m_B/m_{gun} \simeq -500\,m/s\ \frac{0.005}{10\cdot k} \simeq -0.25\,m$$