Last Times

1. **Work**
   \[ w = \int_{F}^{2} F \cdot dx \]
   \[ dW = F \cdot dx \]

2. **Potential Energy**
   \[ \Delta U = U_{2} - U_{1} = -\int w \]
   Or
   \[ dU = -F \, dx \]
   \[ F = -\frac{dU}{dx} \quad \text{(more examples today!)} \]

3. **Work and Kinetic energy**
   \[ w_{fr} + w_{ext}^{\text{ext}} + w_{g}^{\text{grav}} + w_{sp}^{\text{spr}} = K_{2} - K_{1} \]
   \[ w_{fr} + w_{ext} = \Delta K + \Delta U_{g} + \Delta U_{sp} \]
   So if no friction or external forces
   \[ 0 = \Delta K + \Delta U_{\text{tot}} \Rightarrow K_{1} + U_{1} = K_{2} + U_{2} \]
Energy Diagrams

\[ U(y) \]

\[ y \]

\[ h \]

\[ y_{max} \]

\[ E \]

\[ \frac{E}{\dot{y}} \]

\[ mgy \]

\[ y \]

\[ y_{max} \]

Comments:

1. \[ U(y) = mg \ y \]
   \[ F_g = - \frac{dU}{dy} = -mg \] down the slope

2. \[ K_1 + U_1 = K_2 + U_2 = \text{Const} = E_{\text{tot}} \]
   \( E \) we draw this a straight line

3. Notice that at any moment
   \[ K = E - U(x) \]
   \[ \text{kinetic} \]
   \[ \text{energy at position} \]
   \[ x \]
   \[ = \text{Difference between the energy line and the potential} \]
The "turning point" happens when the potential energy equals the total energy, i.e.

\[ \frac{1}{2} m u_0^2 + mg h = E = mg y_{\text{max}} \]

\[ \frac{u_0^2}{2g} + h = y_{\text{max}} \]

**Ex 2**

![Graph showing potential energy function](image)

**Comments**

1. \( U(x) = \frac{1}{2} k x^2 \Rightarrow F = -\frac{dU}{dx} = -kx \)
2. No force in equilibrium position (slope of pot = 0)
3. \( E = \frac{1}{2} m u_0^2 + \frac{1}{2} k x_0^2 \)
Turning points happen when

\[ \frac{1}{2} k x_{\text{max}}^2 = E \]

\[ \frac{1}{2} k x_{\text{max}}^2 = \frac{1}{2} m v_0^2 \quad \Rightarrow \quad x_{\text{max}} = \left( \frac{m v_0^2}{k} \right)^{\frac{1}{2}} \]
Ex3 - Gravitational Potential of the Earth and Escape Velocity

Consider the work done by gravity:

\[ W_{12}^g = \int_{r_1}^{r_2} F \cdot dr \]

\[ W_{12}^g = \int_{r_1}^{r_2} G \frac{M_{EM}}{r^2} \, \frac{dr}{r} \]

\[ W_{12}^g = \left. \frac{GM_{EM}}{r} \right|_{r_1}^{r_2} + GM_{EM} \frac{1}{r_1} - GM_{EM} \frac{1}{r_2} \]

So the change in potential energy

\[ U_2 - U_1 = -W_{12}^g = -\frac{GM_{EM}}{r_2} + \frac{GM_{EM}}{r_1} \]

From this conclude

\[ U(r) = -\frac{GM_{EM}}{r} + C \]

The traditional choice is \( C = 0 \) (arbitrary)

(no potential at \( r \to \infty \))
So suppose shoot a rocket straight up

\[ E = \frac{1}{2}mv_0^2 + \frac{-GM_{\text{earth}}m}{RE} \]

if \( E < 0 \) then there will be an \( \Gamma_{\text{max}} \) where all the energy is potential, the rocket will then plummet back.

On the other hand if \( E > 0 \),

then the rocket will go a great distance and move with kinetic energy \( K_f \)

\( E = 0 \) \( v_0 \) is just enough to escape the earth's pull
This

\[ \frac{1}{2}mu_0^2 - G\frac{M_E m}{r_E} = 0 \]

\[ u_0 = \left( \frac{2M_E G}{r_E} \right)^{1/2} \]

\[ v_0 \approx \sqrt{2} \times 10^4 \text{ m/s} \sim \text{mach 3-4} \]

Other examples of Energy diagrams

\[ U(r) \]

\[ \text{Where } \frac{dU}{dr} = 0 \]
Momentum

So for single particles we will now move to systems of particles (bullet vs. wrecking ball).

1. \[ \vec{p} = m \vec{v} \]

2. \[ \Sigma F = m \vec{a} \quad \text{Actually this isn't quite right} \]

\[ \Sigma F = m \frac{d\vec{v}}{dt} = \frac{d}{dt} (m \vec{v}) = \frac{d\vec{p}}{dt} \]

In general the sum of all forces acting on a body is

\[ \Sigma \vec{F} = \frac{d\vec{p}}{dt} \quad \text{matters if} \]

Momentum Conservation and multiple bodies

\[ \vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B \]

\[ m \vec{v}_A + m \vec{v}_B = m \vec{v}'_A + m \vec{v}'_B \]
This is also a general consequence of Newton's laws.

\[ \vec{F}_{BA} = -\vec{F} = \text{force of } B \text{ on } A \]
\[ \vec{F}_{AB} = -\vec{F} = \text{force of } A \text{ on } B \]

Then from Newton's laws:

\[ \vec{F} = \frac{d\vec{p}_A}{dt} \]
\[ -\vec{F} = \frac{d\vec{p}_B}{dt} \]

Adding these equations, we find:

\[ 0 = \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = \frac{d}{dt} (\vec{p}_A + \vec{p}_B) \]

The total momentum is conserved.

Example: Estimate the recoil velocity of a shot gun.

Before the shot, the system is at rest.

\[ P_{\text{before}} = 0 = P_{\text{after}} = m_g v_g + m_B v_B \]

So,

\[ v_g = -v_B \frac{m_B}{m_g} \approx -500 \text{ m/s} \]

\[ \frac{0.005}{10^{-4}} \]