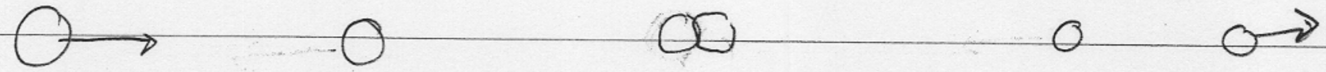
① Last Time - Momentum Conservation

Collision Some forces Fly apart



Lets examine the forces

- The potential energy must be a function of $U(x_1 - x_2)$

$$F_1 = -\frac{dU}{dx_1} \quad \text{while} \quad F_2 = -\frac{dU}{dx_2} = -\left(-\frac{dU}{dx_1}\right) = -F_1$$

Then

$$\vec{F}_1 = \frac{d\vec{p}_1}{dt}$$

$$F_2 = \frac{dp_2}{dt}$$

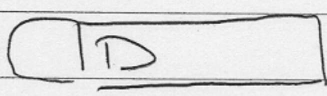
$$\Rightarrow \vec{F}_1 + \vec{F}_2 = 0 = \frac{d}{dt}(\vec{p}_1 + \vec{p}_2)$$

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1{}' + \vec{p}_2{}'$$

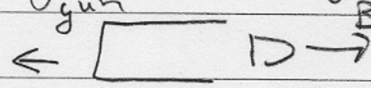
Example

So Estimate the recoil due to a shot gun.

Before



v_{gun} $v_B \simeq 600\,m/s$

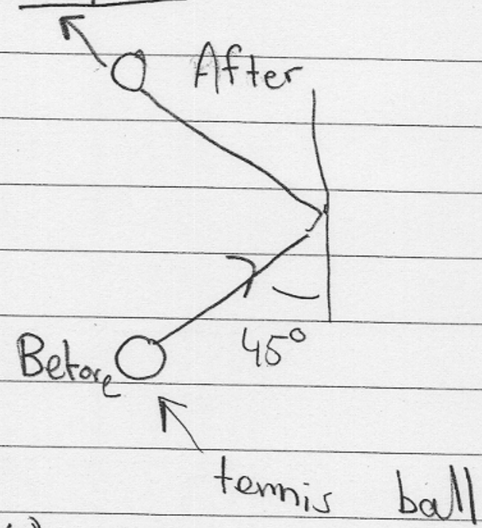


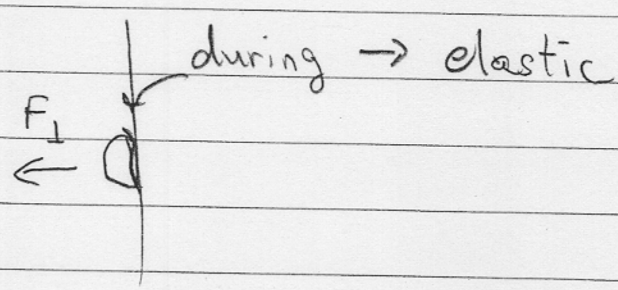
$$p_{Before} = 0 = \underbrace{m_g v_g + m_B v_B}_{p_{after}} \Rightarrow v_g = -\frac{m_g v_B}{m_B}$$

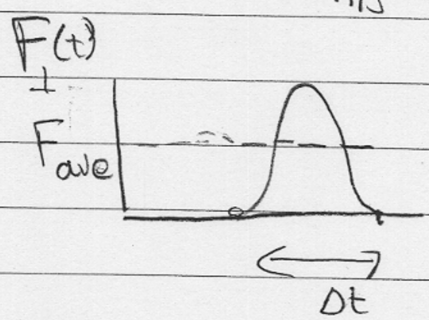
$$= -\frac{(0.005\,kg)\cdot 600\,m/s}{(10\,kg)}$$

$$v_g \simeq 0.3\,m/s$$

Impulse







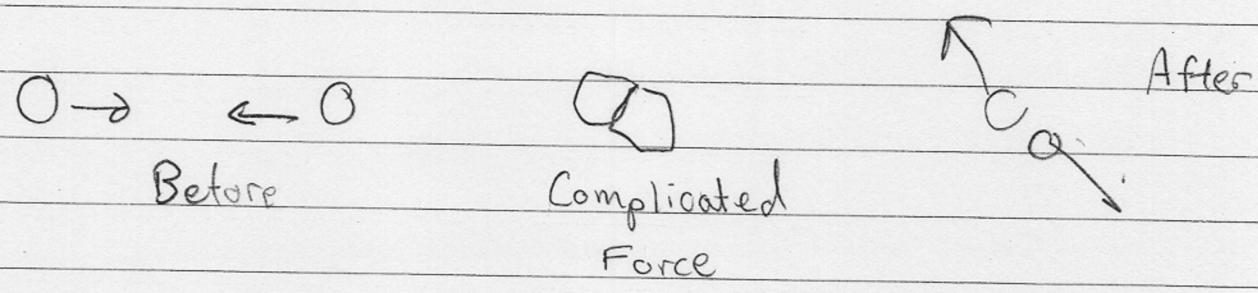
$$F_{ave} = \frac{1}{\Delta t} \int F(t)\, dt$$

Then impulse or momentum transfer:

$$\Delta p \equiv \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \frac{d\vec{p}}{dt}\, dt = \int \vec{F}(t)\, dt$$

$$\Delta p = \int \vec{F}(t)\, dt = F_{ave}\, \Delta t$$

Collisions



If the internal dynamics has no friction (no unobserved components) then the energy is conserved

$$K_I = K_F$$

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

(elastic collisions only)

On the other hand if a significant fraction of the energy is converted to heat we would have

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 + \text{heat}$$

Kinetic energy not conserved (inelastic)

This E-cons (elastic) and p-consv,

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$$

are the equations we need

One Dimensional Elastic Collisions; A useful formula

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \qquad (\mathrm{I})$$

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2 \qquad (\mathrm{II})$$

From I + II

$$m_A (v_A - v_A') = m_B (v_B - v_B') \qquad (\mathrm{III})$$

$$m_A [v_A^2 - (v_A')^2] = m_B [v_B^2 - v_B'^2]$$

$$\underbrace{m_A (v_A - v_A')} (v_A + v_A') = \underbrace{m_B (v_B - v_B')} (v_B + v_B')$$

same by eq III

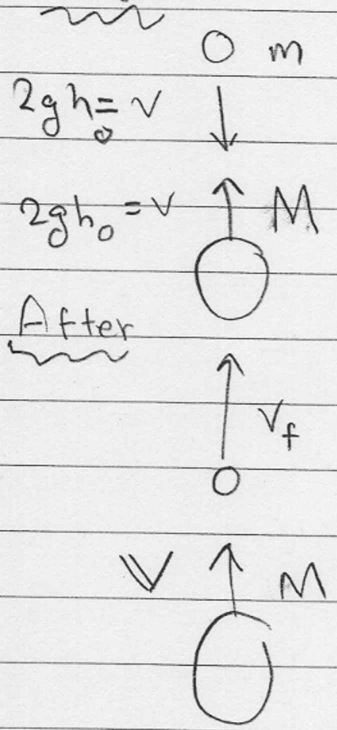
$$v_A + v_A' = v_B + v_B'$$

$$\boxed{v_A - v_B = -(v_A' - v_B')} \quad \leftarrow \text{1D - EconsV}$$

can be used instead of E-consv in 1D
relative velocity of a and B
changes sign upon collision

Example: (Demonstrated) What is the recoil velocity of the small ball

Before



$$-mv + Mv = mv_f + M\mathbb{V} \quad \text{(p-consV)}$$

$$-v - v = \mathbb{V} - v_f \quad \text{(1D-Econs)}$$

Knowns: m, M, v

Unknowns: v_B, v_T

$$(M - m)v = mv_f + M\mathbb{V}$$

$$\mathbb{V} = v_f - 2v$$

$$(M-m)v = mv_f + M(v_f - 2v)$$

$$(M-m)v + 2Mv = (m+M)v_f$$

$$\frac{(3M-m)v}{(m+M)} = v_f$$

$$3v = v_f \quad \text{for } M \gg m$$

So

$$h = \frac{v_f^2}{2g} = 9\left(\frac{v^2}{2g}\right) \qquad \text{note } v^2 = 2gh_o$$

$$\boxed{h = 9h_o}$$