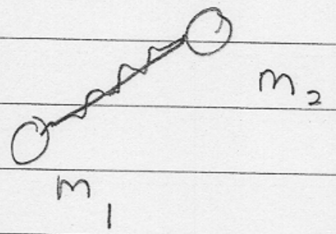
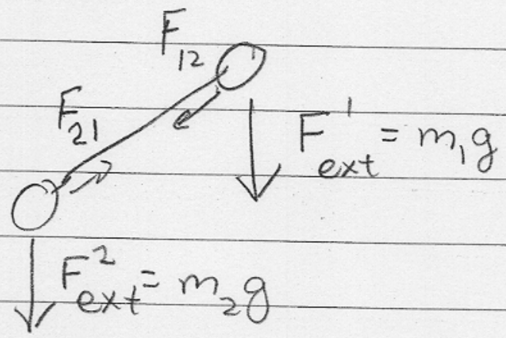
Last Time

- Example Professor throws book in air

→ Lots of internal tugging and Pulling



→ Isn't there something we can say abot the body as a whole



Newtons Laws

$$\vec{F}^1_{ext} + \vec{F}_{12} = m_1 \vec{a}_1$$

$$\vec{F}_{21} = -\vec{F}_{12}$$

$$+ \quad \vec{F}^2_{ext} + \vec{F}_{21} = m_2 \vec{a}_2$$

$$\vec{F}^1_{ext} + \vec{F}^2_{ext} = m_1 \vec{a}_1 + m_2 a_2$$

So

$$\vec{F}^{\,TOT}_{ext} = M_{TOT} \underbrace{\frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1+m_2}}_{\equiv\, a_{cm}} \qquad M_{TOT} = m_1+m_2$$

words

So

$$a_{cm} \equiv \text{mass weighted average of accel} = \langle \vec{a}_{cm} \rangle_m$$

This motivates

$$x_{cm} \equiv \frac{\vec{x}_1 m_1 + \vec{x}_2 m_2}{m_1+m_2}$$

mass weighted position

"Center of gravity"

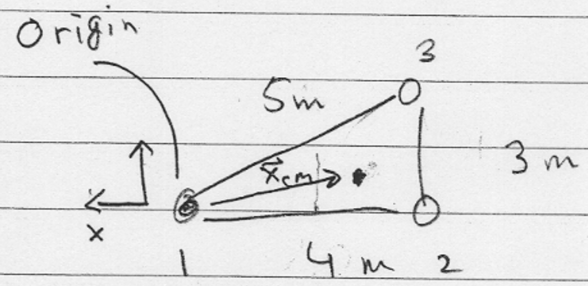
Which becomes

$$V_{cm} = \dot{x}_{cm} = \frac{\dot{x}_1 m_1 + \dot{x}_2 m_2}{m_1+m_2} = \frac{v_1 m_1 + v_2 m_2}{m_1+m_2}$$

mass weighted velocity

$$a_{cm} = \ddot{V}_{cm} = \ddot{x}_{cm}$$

One computation of cm



equal mass m_B

$$\vec{x}_1 = \vec{0} \qquad \vec{x}_2 = 4\,\hat{i} \qquad \vec{x}_3 = 4\,\hat{i} + 3\,\hat{j}$$

So

$$\vec{x}_{cm} = \langle x \rangle_m = \frac{\cancel{m\vec{x}_1}^{\,0} + m\,\vec{x}_2 + m\,\vec{x}_3}{m + m + m}$$

$$= \frac{m}{3m}(\vec{x}_2) + \frac{m}{3m}\,\vec{x}_3$$

$$= \frac{1}{3}(4\hat{i}) + \frac{1}{3}(4\hat{i} + 3\hat{j})$$

$$= \frac{8}{3}\hat{i} + \hat{j}$$

Roughly agrees with intuition

The importance of Center of mass

$$\sum \vec{F}_{ext} = M_{TOT} \vec{a}_{cm} = \frac{dP_{cm}}{dt} = \frac{dP_{TOT}}{dt}$$

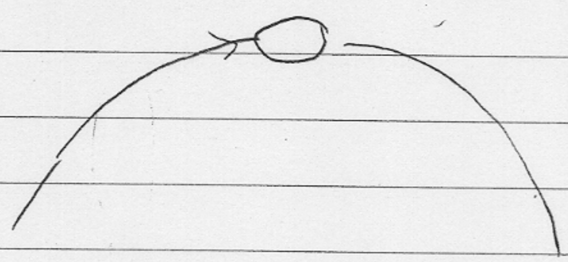
① The sum of all forces is equal to the total mass of the system x acceleration of CM

② The center of mass of a set of objects, moves like a single particle with mass M acted upon by the same external force

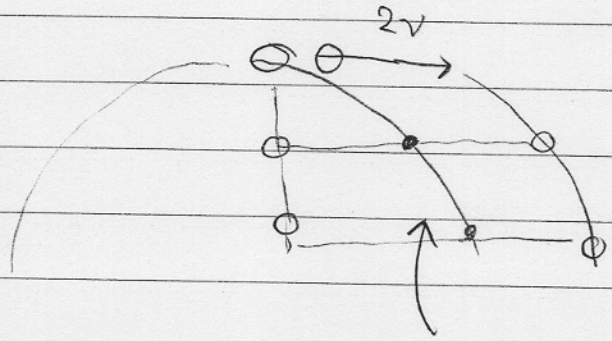
- Back to the book example: ⇝ Complicated internal motion

- Rocket Ship

- Arc of a ship



- Suppose it explodes in mid flight

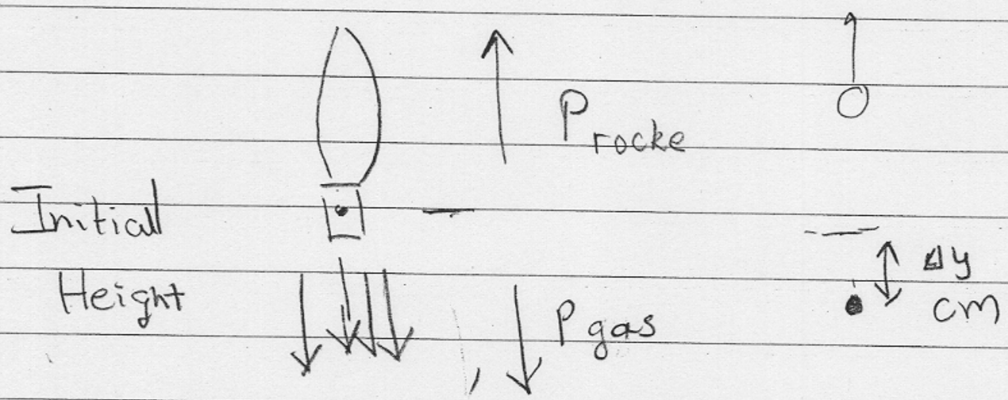


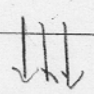
CM obeys the same free fall or bit

* Which Begs the question How does a rocket or jet plane ever fly?

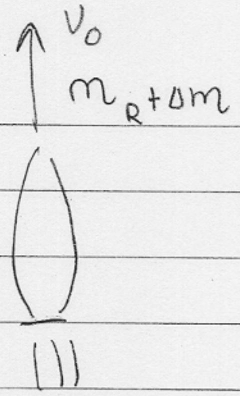
<u>Variable mass</u>

Take the rocket first:



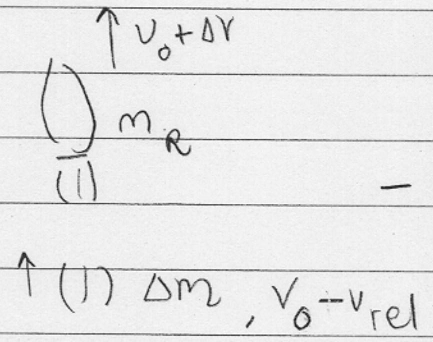


t



$$F_{ext}\,\Delta t = \Delta P_{rocket + gas}$$

t+Δt



$$-(m_R + \Delta m)\, g\, \Delta t = \Delta P$$

So with $\Delta m\, \Delta t \simeq 0$

$$-m_R\, g\, \Delta t = m_R (v_0 + \Delta v) + \Delta m (v_0 - v_{rel}) - (m_R + \Delta m)\, v_0$$

$$-m_R\, g\, \Delta t = m_R\, \Delta v + \Delta m\, v_{rel}$$

$$-m_R\, g + \frac{\Delta m}{\Delta t} v_{rel} = m_R \frac{\Delta v}{\Delta t}$$

note $\frac{dm}{dt} = -\frac{dm_R}{dt}$

If the rocket burns 3/4 of its initial mass whats its speed. Assume $v_{rel} = 2800\,m/s$ and $dm/dt = 190\,kg/s$ and $m_R = 20{,}000\,kg$.

$$-g\,\Delta t - \frac{dm_R}{m_R} v_{rel} = dv$$

$$-g\,\Delta t - v_{rel} \ln \frac{m_f}{m_i} = v_f - v_i$$

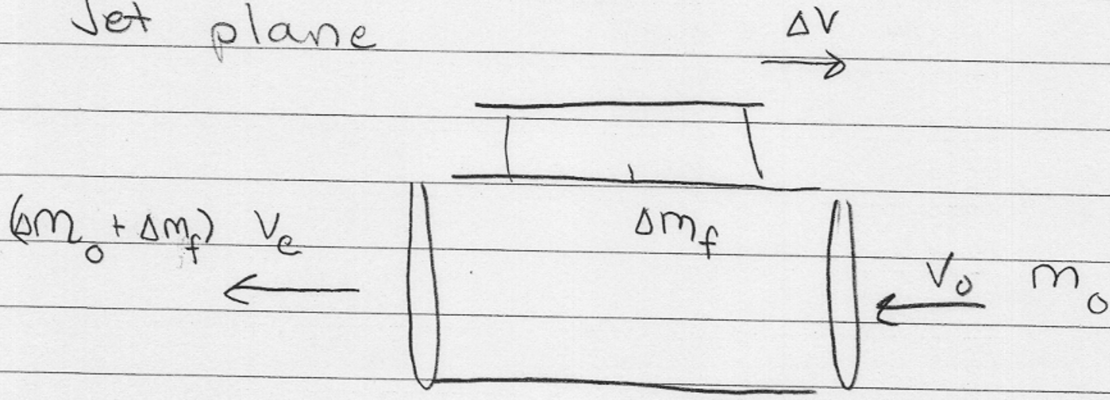
If $dm/dt = 190\,kg/s$

Then $\frac{dm}{dt}\Delta t = \frac{3}{4} m_R \Rightarrow \Delta t = \frac{3/4\, m_R}{dm/dt} = 79s$

$$-g\,\Delta t - v_{rel} \ln\left(\frac{1/4\,\cancel{m}_R}{\cancel{m}_R}\right) = v_f - \cancel{v_i}^{\,0}$$

$$3100\,m/s = v_f$$

Jet plane



$$F^x_{ext}\,\Delta t = 0 = \Delta P_{air + fuel + plane}$$

$$0 = P_f - P_i$$

$$0 = -(\Delta m_o + \Delta m_f) V_e + m_p(\cancel{V_p} + \Delta V)$$

$$-\left[(m_p + \Delta m_f)\cancel{V_p} - \Delta m_o V_o\right]$$

$$0 = -(\Delta m_o + \Delta m_f) V_e + m_p \Delta V + \Delta m_o V_o$$

$$\Delta m_o (V_e - V_o) + \Delta m_f V_e = m_p \Delta V$$

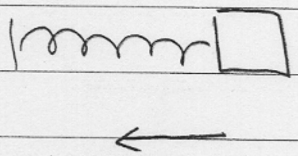
difference in intake and exhaust speeds

$$\frac{\Delta m_o}{\Delta t}(V_e - V_o) + \frac{\Delta m_f}{\Delta t} V_e = m_p \frac{\Delta V}{\Delta t}$$

depends on size of engine intake of aire

burning fuel

Oscillations



Newton Law

$$m\frac{d^2x}{dt^2} = F = -kx$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$ (A differential equation)

Guess a solution

$$x = \sin(\omega_0 t) \Rightarrow \dot{x} = \cos \omega t \; \omega_0 \Rightarrow \ddot{x} = -\omega_0^2 \sin(\omega_0 t)$$

So

$\ddot{x} = -\omega_0^2 x$ so $\sin(\omega_0 t)$ is a solution provided $\omega_0 = \sqrt{k/m}$

But it is not the only solution $\cos(\omega_0 t)$ is also a solution, so is $3\sin(\omega_0 t) + 4\cos(\omega_0 t)$

The most general solution

free constant

$$x(t) = A\cos\omega_0 t + B\sin\omega_0 t$$