Last Time

- We talked about pos vs. time (see handout)
- In each bin there is a local slope:

$$v = \frac{\Delta x}{\Delta t} \Rightarrow \frac{dx}{dt}$$

- Now

$$\Delta x_1 = v_1 \Delta t$$

area of the first bin of v vs. t

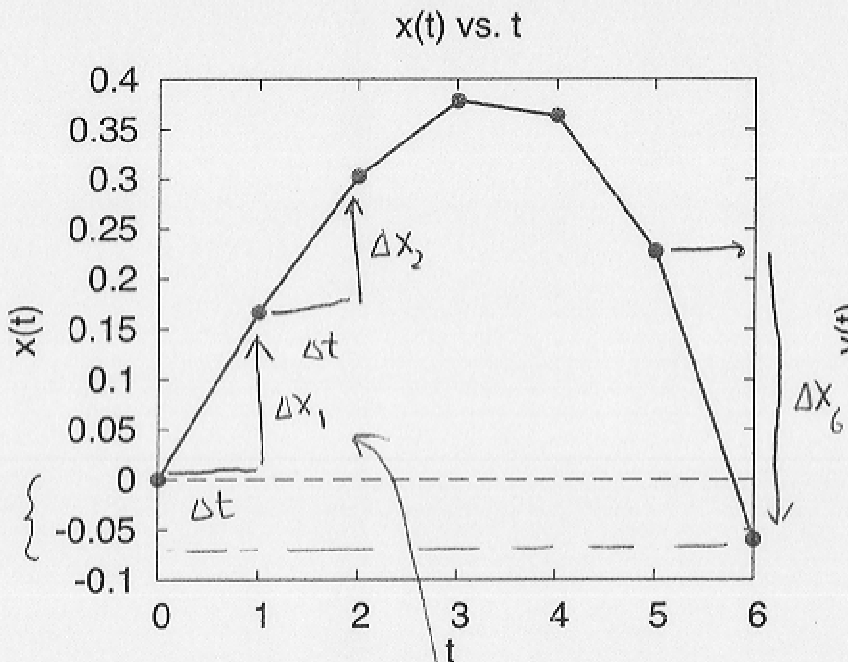
$$\Delta x_2 = v_2 \Delta t$$

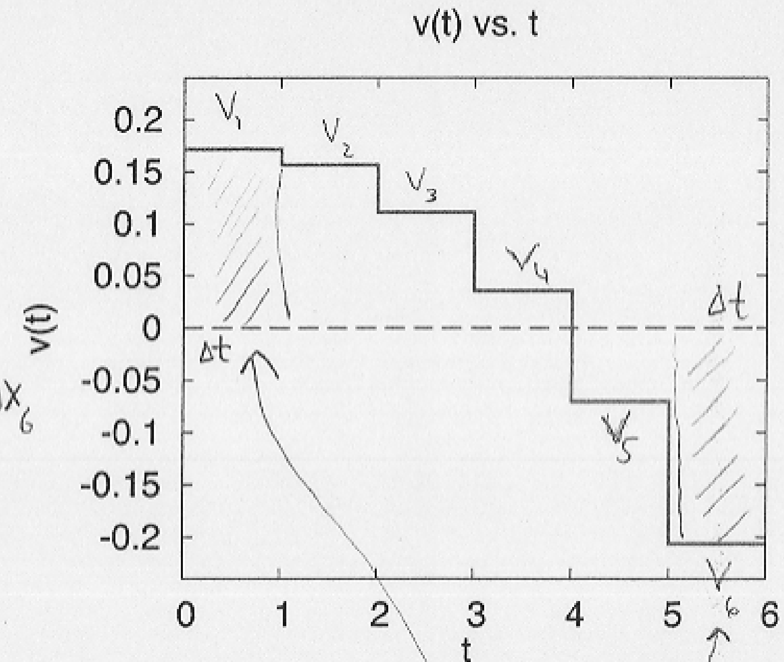
$$\vdots$$

$$\Delta x_6 = v_6 \Delta t$$

$$\Delta x_{Total} = x_f - x_i = \sum_i \Delta x_i = \sum_i v_i \Delta t$$

Position vs. Time





$\Delta X_{TOT} = X_f - X_i$

$\Delta X_1 = V_1 \Delta t$ = area of v vs. t

$\Delta X_6 = V_6 \Delta t$ = signed area of v vs t

Total change in pos $= \sum_i \Delta x_i = \sum v_i \Delta t$

$$\boxed{x_f - x_i = \int_{t_i}^{t_f} v \, dt}$$

Total Displacement = Area under v vs. t curve

Calculus Derivation of All Prior Results

$$v(t) = \frac{dx}{dt}$$

so $\quad v(t)\,dt = dx$

Integrate:

$$\int_i^f v(t)\,dt = \int_i^f dx = x_f - x_i \checkmark$$

If $\quad v(t) = \text{const} = v$

$$\int_{t_i}^{t_f} v\,dt = x_f - x_i$$

$$x_f - x_i = V\,(t_f - t_i)$$

$$x_f = x_i + V\,(t_f - t_i) \checkmark$$

Finally

from above

$$\overline{V} = \frac{\int_{t_f}^{t_i} v(t)}{t_f - t_i} = \frac{x_f - x_i}{t_f - t_i} \checkmark$$

remember

$$\overline{f(x)} = \frac{\int_a^b f(x)}{b-a}$$

Acceleration

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{\text{change in velocity}}{\text{elapsed time}}$$

$$a = \frac{\Delta v}{\Delta t}$$

for constant acceleration

In general:

$$\boxed{a(t) = \frac{dv}{dt}}$$

The acceleration is the second derivative: of position

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$\boxed{a = \frac{d^2x}{dt^2}}$$

General

$$\int_{v_i}^{v_f} dv = \int_{t_i}^{t_f} a(t)\, dt$$

$$v_f - v_i = \int_{t_i}^{t_f} a(t)\, dt$$

Signs of Accel

Q: A biker is moving to the left at 10 m/s and decides to go faster to 30 m/s increasing his speed over 5s. Is his acceleration positive or negative

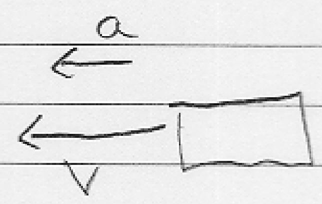
A: Negative

$$a = \frac{\Delta V}{\Delta t} = \frac{V_f - V_i}{t_f - t_i}$$

$$a = \frac{(-30\,m/s) - (-10\,m/s)}{5s}$$

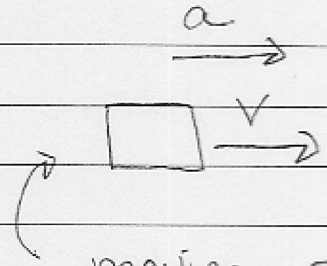
$$a = \frac{-20\,m/s}{5s} = -4\,m/s^2$$

Picture:



His acceleration is to the Left (negative)

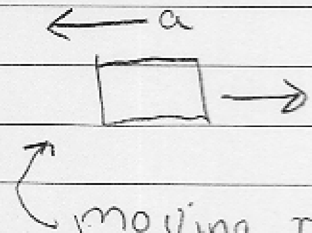
Cases



$a > 0$

$v > 0$

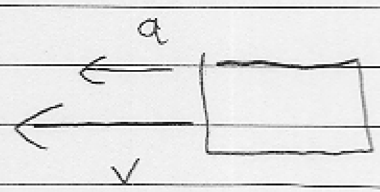
moving right and speeding up



$a < 0$

$v > 0$

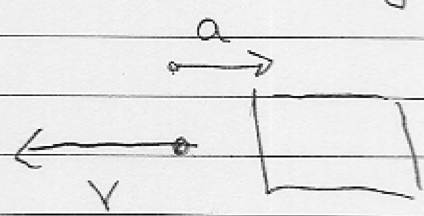
moving right + slowing down



$a < 0$

$v < 0$

moving left and speeding up



$a > 0$

$v < 0$

moving left and slowing down

Constant Acceleration

$$\frac{dv}{dt} = a$$

$$\int_{v_0}^{v} dv = \int_{0}^{t} a \, dt$$

$$v - v_0 = a t$$

(1) $$\boxed{v = v_0 + a t}$$ (v vs. t)

Now

$$\frac{dx}{dt} = v$$

$$\frac{dx}{dt} = v_0 + at$$

$$\int_{x_0}^{x} dx = \int_{0}^{t} (v_0 + at) \, dt$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

(2) $$\boxed{x = x_0 + v_0 t + \frac{1}{2} a t^2}$$ (x vs. t)

One more! it doesn't involve time

$$a = \frac{dv}{dt}$$

The energy trick

$$a = \frac{dv}{dx}\overbrace{\frac{dx}{dt}}^{v}$$

$$a = \frac{dv}{dx}\,v$$

$$\int_{x_0}^{x} a\,dx = \int_{v_0}^{v} v\,dv$$

$$a(x - x_0) = \frac{1}{2}v^2\Big|_{v_0}^{v}$$

$$a(x - x_0) = \frac{1}{2}v^2 - \frac{1}{2}v_0^2$$

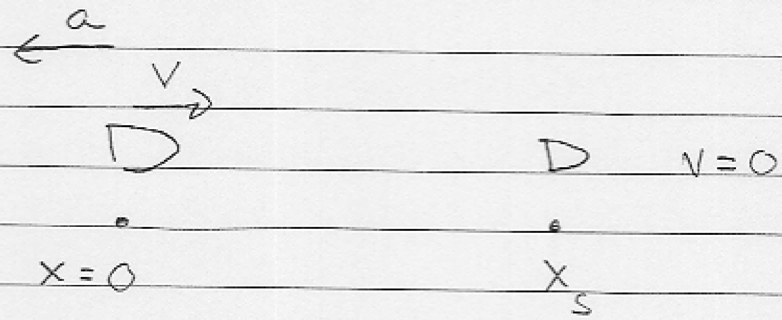
3 $$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$ (v vs. x)

Summary

Eqs 1, 2, 3 are for constant accel

Prob: Estimate the minimum length required for a normal comercial runway

Sol:



Requirements: ① plane comes in @ speed ~50m/s

② The acceleration cant be greater than $\sim \frac{1}{5} g \simeq 2m/s^2$

Notice the information does not involve time So the (v vs x) equation is a good choice

$$\cancel{v}^{2} = v_0^2 + 2\vec{a}(x - \cancel{x_0})$$

$$0 = v_0 - 2|a|\, x_s \qquad |a| = 2m/s^2$$

$$0 = v_0^2 - 2|a|x_s$$

$$\frac{v_0^2}{2|a|} = x_s$$

$$\frac{(50m/s)^2}{2(2m/s^2)} = x_s = 625m \sim 6 \text{ football fields}$$

Gravity

- Things fall the velocity changes there is acceleration

- Detailed measurement show (Galileo)
  - The acceleration is constant $g \simeq 9.8 m/s^2$ and downward
  - Independent of velocity
  - Indep of mass

Q: A persons stands on top of a 50m cliff and throws a ball straight up with a speed of 15m/s. When is the ball 18m high.

Sol. This problem involves how high (8m) and time, so use the (x vs. t) equation

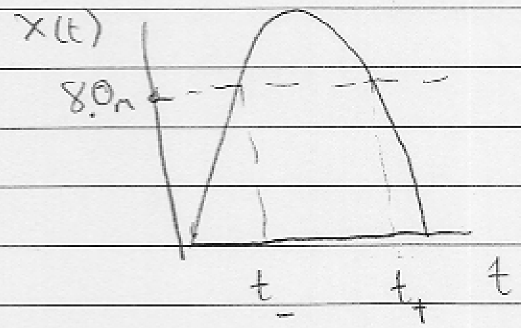
$$x(t) = \cancel{x_0}^{0} + v_0 t + \frac{1}{2} \vec{a}\, t^2 \qquad \vec{a} = -g$$

$v_0 = 15 m/s >$

$=$

$$x(t) = (15\,m/s)\,t - \frac{1}{2}(9.8\,m/s^2)\,t^2$$

Graph



Solving

$$8\,m = (15\,m/s)t - (4.9\,m/s^2)\,t^2$$

$$0 = (4.9\,m/s^2)t^2 - (15\,m/s)t + 8\,m$$

$$t = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \left(\frac{c}{a}\right)}$$

$a = 4.9\,m/s^2$
$b = -15\,m/s$
$c = 8\,m$

$t = 0.69\,s$ and $t = 2.37\,s$