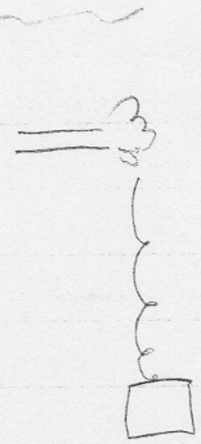
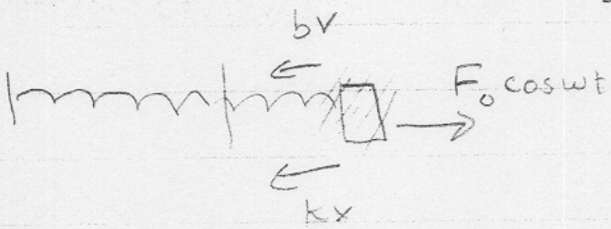
Last Time



- Drive the spring with an external force $F_0 \cos \omega t$

- The motion reaches a steady state

- The amplitude can become large if tunded @ ω_0
  → Remember the bridge



Newtom

$$ma = -kx - bv + F_0 \cos \omega t$$

$$\underset{①}{\frac{d^2x}{dt^2}} + \underset{②}{\frac{b}{m}\frac{dx}{dt}} + \underset{③}{\frac{k}{m}x} = \frac{F_0}{m}\cos \omega t$$

Solution:

$$x = A_\omega \cos(\omega t - \phi_\omega) \qquad \text{Ansatz}$$

$$A = \frac{F_0/m}{\left[(\omega^2 - \omega_0^2)^2 + \left(\frac{b}{m}\omega\right)^2\right]^{1/2}} \qquad \phi = \tan^{-1}\left[\frac{b/m\omega}{(\omega_0^2 - \omega^2)}\right]$$

Derivation - Skipped in class

$$\ddot{x} = -A\omega^2 \cos(\omega t - \phi)$$

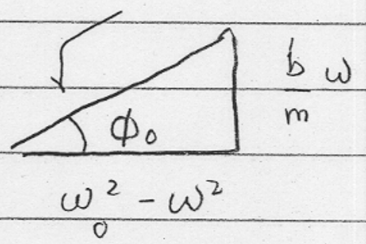
$$\dot{x} = -A\omega(\sin \omega t - \phi)$$

$$x = A\cos(\omega t - \phi)$$

$$\overset{①}{-A\omega^2\cos(\omega t - \phi)} - \overset{②}{\frac{b}{m}\omega A \sin \omega t - \phi} + \overset{③}{\underset{\overset{///}{k/m}}{\omega_0^2}} A\cos(\omega t - \phi) = \frac{F_0}{m}\cos\omega t$$

$$A\left[(\omega_0^2 - \omega^2)\cos(\omega t - \phi) - \frac{b}{m}\omega \sin(\omega t - \phi)\right] = \frac{F_0}{m}\cos\omega t$$

Same trick



$$\sqrt{\ } = \left((\omega_0^2 - \omega^2)^2 + \left(\frac{b}{m}\omega\right)^2\right)^{1/2}$$

$$A\sqrt{\ }\left[\frac{\omega_0^2 - \omega^2}{\sqrt{\ }}\cos(\omega t - \phi) - \frac{(b/m)\omega}{\sqrt{\ }}\sin(\omega t - \phi)\right] = \frac{F_0}{m}\cos\omega t$$

$$A\sqrt{\ }\left[\cos\phi_0 \cos(\omega t - \phi) - \sin\phi_0 \sin(\omega t - \phi)\right] = \frac{F}{m}\cos\omega t$$

$$A\sqrt{\ }\left[\cos(\omega t - \phi + \phi_0)\right] = \frac{F}{m}\cos\omega t$$

Need to choose (to have cos match)

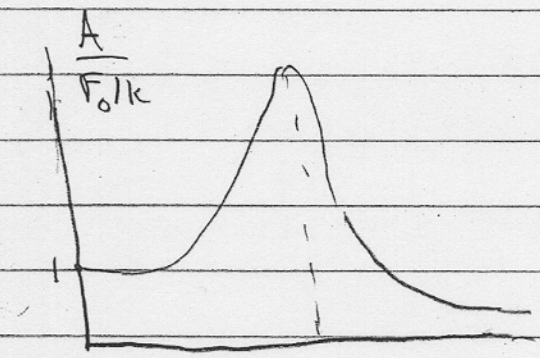
$$\phi = \phi_0 = \boxed{\tan^{-1}\left(\frac{b/m\,\omega}{\omega_0^2 - \omega^2}\right) = \phi}$$

Then

$$A\Gamma = \frac{F_0}{m}$$

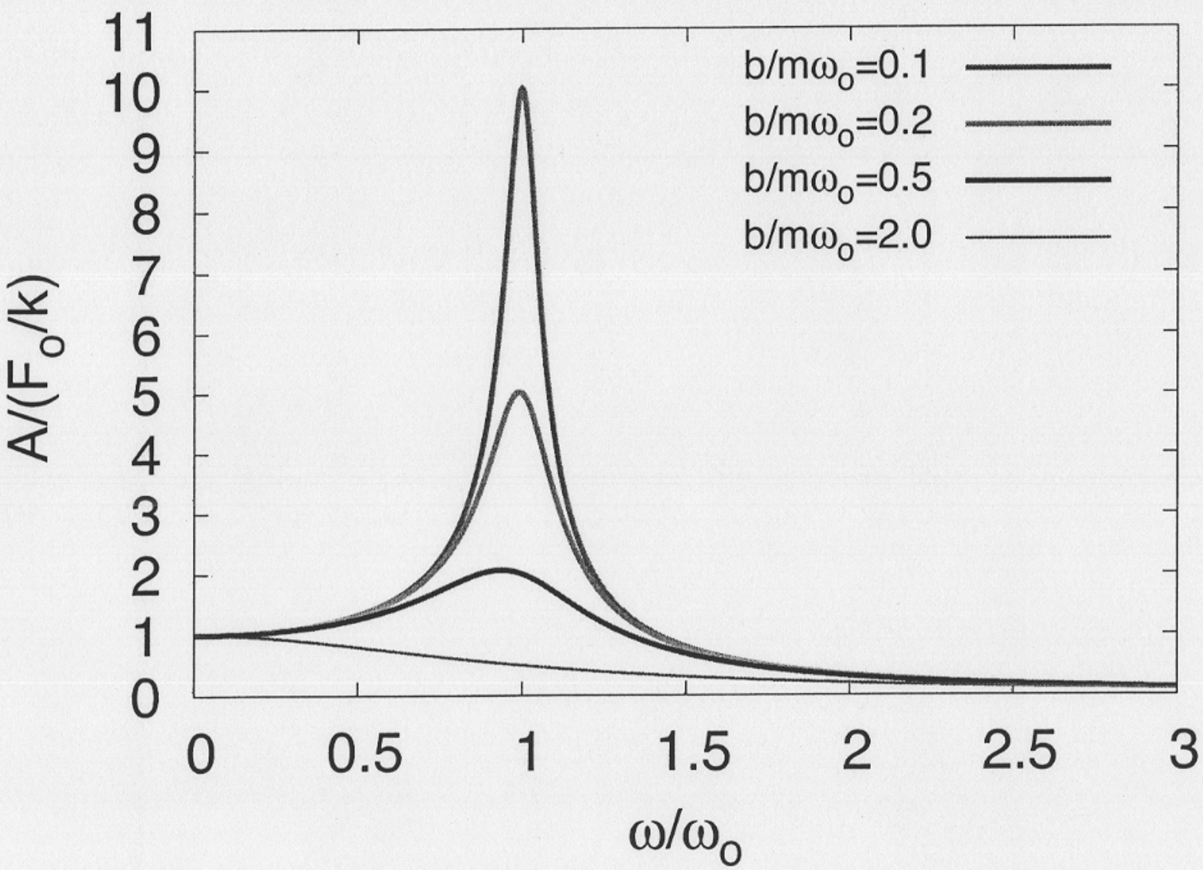
Or
$$\boxed{A = \frac{F_0/m}{\left[(\omega_0^2-\omega^2)^2 + \left(\frac{b}{m}\omega\right)^2\right]^{1/2}}}$$

Usually we are iterested in $\frac{b}{m\omega_0} \ll 1$ (Continue



$\omega \simeq \omega_0$

- Many interesting features
- See slides



A/(F_o/k)
ω/ω_o
b/mω_o=0.1
b/mω_o=0.2
b/mω_o=0.5
b/mω_o=2.0

At Resonance $\omega \simeq \omega_0$

$$A_{max} = \frac{F_0/m}{\left(0^2 + \left(\frac{b}{m}\omega_0\right)^2\right)^{1/2}}$$

$$A_{max} = \frac{F_0/m}{\frac{b}{m}\omega_0} = \frac{F_0}{m\omega_0^2} \cdot \frac{1}{\left(\frac{b}{m\omega_0}\right)} = \boxed{\frac{F_0}{k}\left(\frac{m\omega_0}{b}\right)}$$

Usually we define

$$Q \equiv \frac{m\omega_0}{b} \sim \text{"}\frac{1}{\text{damping in dimensionless units}}\text{"}$$

So

$$A_{max} = \frac{F_0}{k} \cdot Q$$

For a violin $Q \sim 30-50$

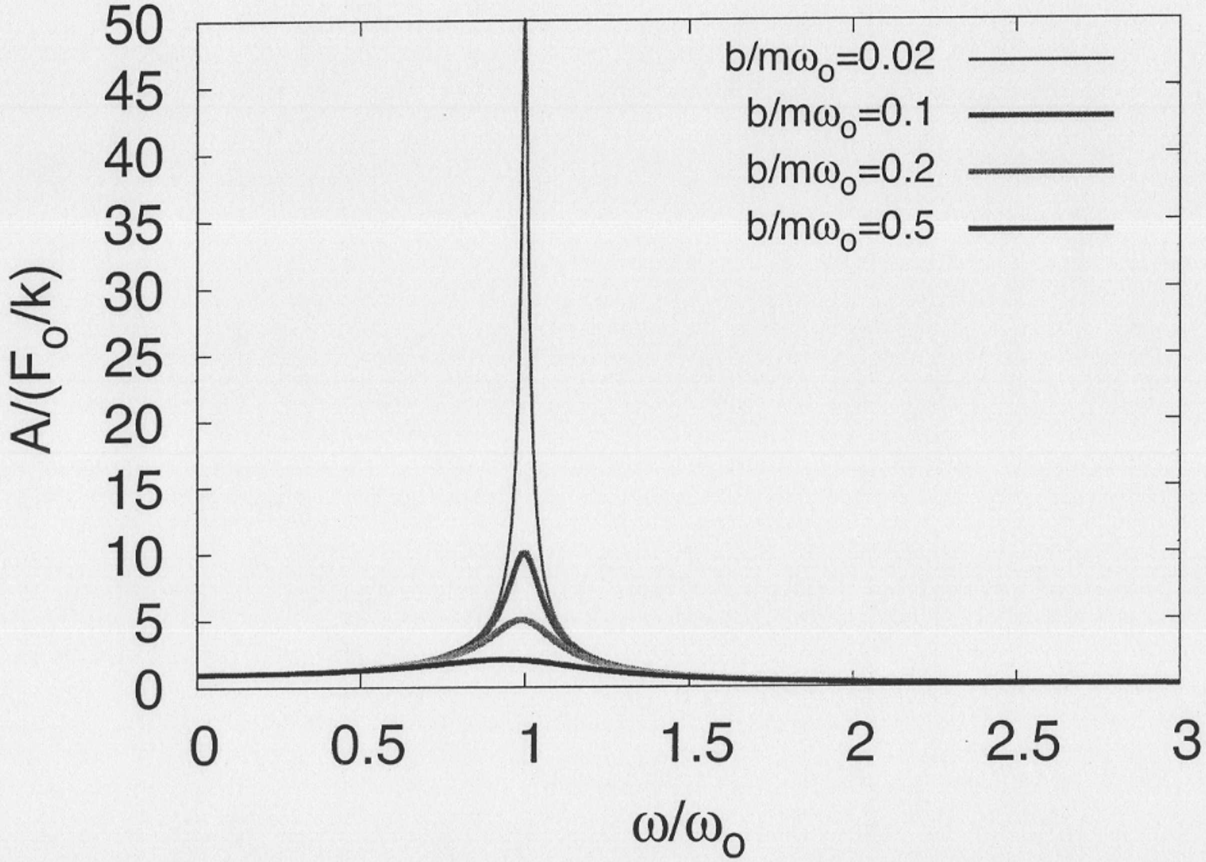
Notice:

→ As the Q becomes Large the "spike" becomes increasingly sharp

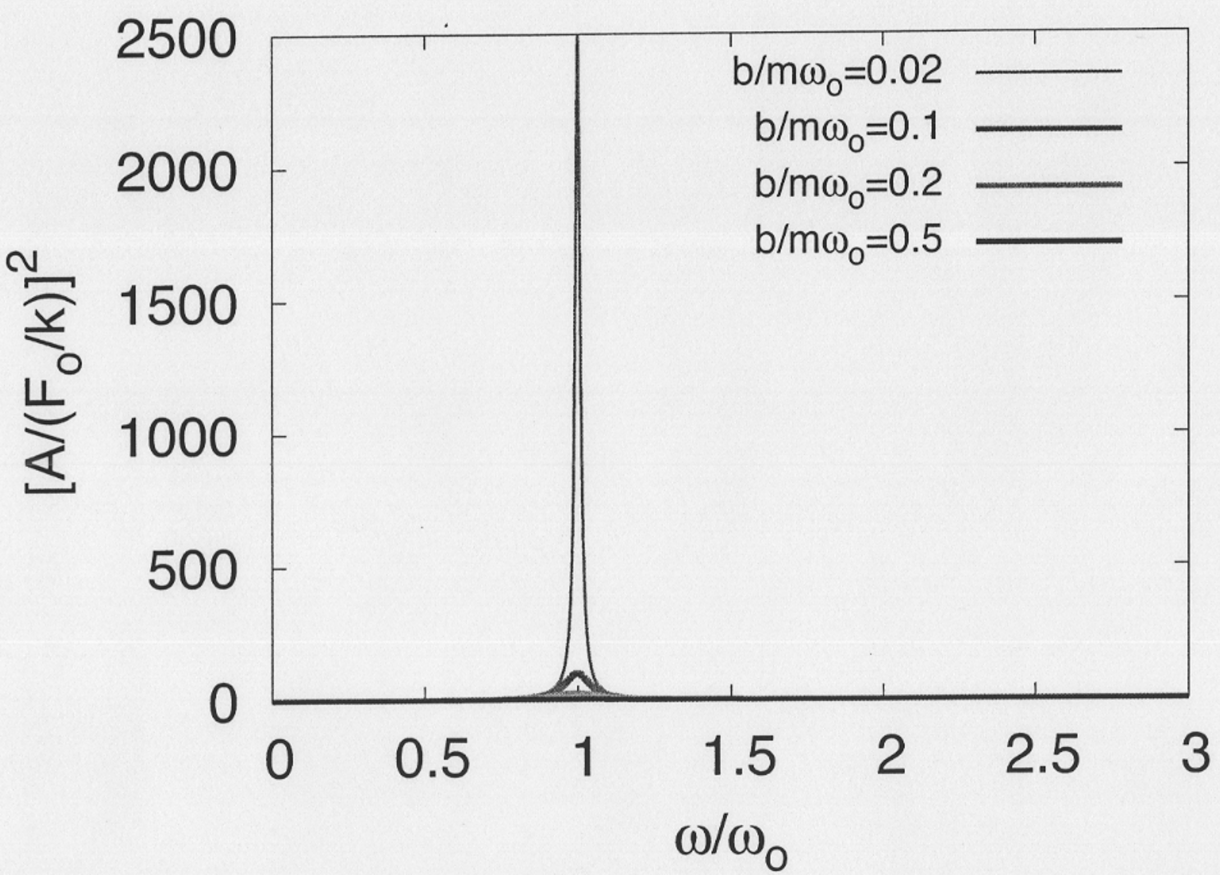
see figure

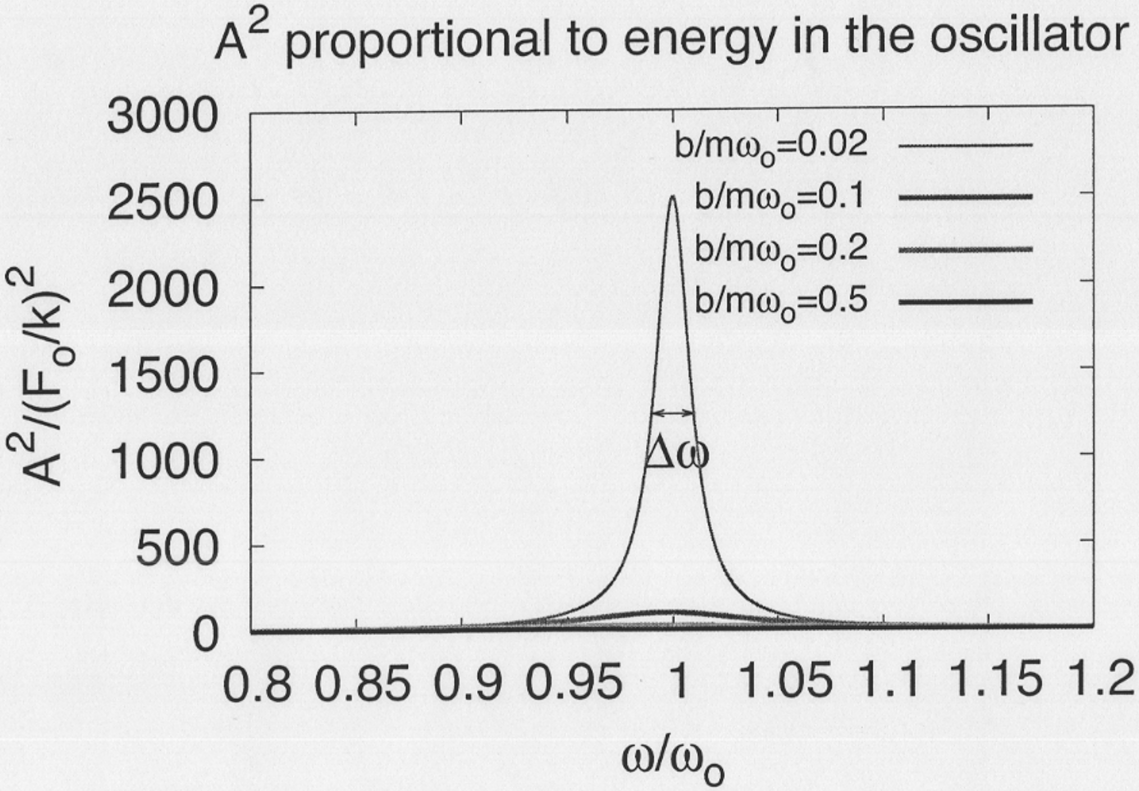
Small → $\frac{1}{Q} \simeq \frac{\Delta\omega}{\omega} = \frac{\text{Full width at half max in } A^2 \text{ plo}}{\omega}$

Realistic $Q \approx 50$ for a violin



Realistic $Q \approx 50$ for a violin, we also care about energy $\propto A^2$

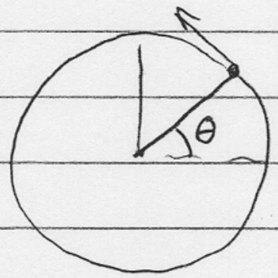




$$\frac{1}{Q} \approx \frac{\Delta\omega}{\omega} = \frac{\text{Full width at half maximum in } A^2 \text{ plot}}{\omega}$$

Rotational Motion

Angular Quantities -- Always use radians internally



$$\theta = \frac{l}{R}$$

$$\omega = \frac{d\theta}{dt} = \frac{dl/dt}{R} = \frac{V}{R}$$

$$\boxed{\omega = \frac{V}{R} \qquad \text{or} \qquad V = \omega R}$$

Units of ω

- rad/s

  or

- rev/s 1 rev = 2π rad , rpm = rotations per minute

Also use the frequency

$$f = \frac{\omega}{2\pi} = \frac{\#\text{ of complete cycles}}{\text{sec}} = \text{Hz}$$

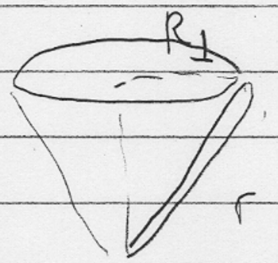
If the motion is uniform define

$$T = 1/f$$

What else needs to be said?

1. Implicit in ω is an agreed upon center point
   - a fixed point
   - Or the CM if the system is moving

2. ω is independent of where you are on the wheel, a property of body & center point

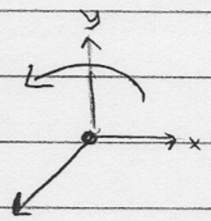
3. 

$$\omega = \frac{d\theta}{dt} = \frac{V}{R_\perp}$$

$$V = \omega R_\perp = \omega r \sin\theta$$

Angular Velocity is actually a vector

1. We choose the angular velocity $\vec{\omega}$ to be the direction of rotating axis according to the "right hand rule"



2. So a counter clockwise rotation in x-y p
we would say is

$$\vec{\omega} = \omega \hat{k} \leftarrow \text{out of the page}$$

magnitude of ω, pos number

While a clockwise rotation is into the page

$$\vec{\omega} = -\omega \hat{k}$$

pos number

3. If we only ever talk about rotation in the x-y plane we can use the sign to indicate the direction of rotation