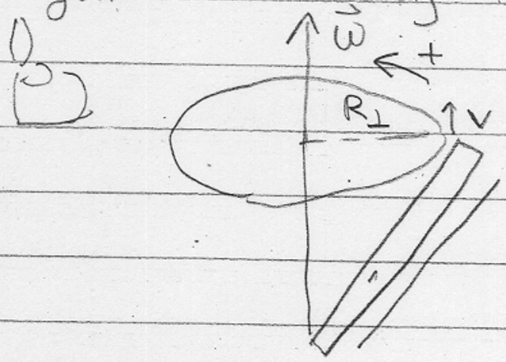
Last Time

Angular Velocity:



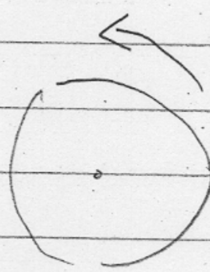
$\vec{\omega}$

① magnitude

$\omega = \dfrac{d\theta}{dt} \Rightarrow v = \omega R_\perp$

$v = \omega r \sin\theta$

② Direction: Right hand Rule, means you curl fingers and thumb points in direction of ω



\+ ③ For "normal" xy rotation

\+ / counter-clock / out of page

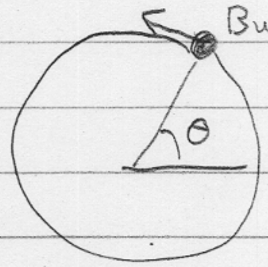
− / clockwise / into page

④ Every point on the solid body has the same ω but different velocities

"ω is a constant of the body"

Angular Acceleration:

- Suppose the wheel spins faster and faster all the time

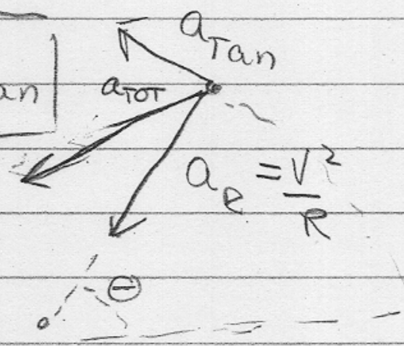


$$\alpha = \frac{d\omega}{dt} = \text{"change in angular velocity per time"}$$

$$\alpha = \frac{d}{dt}\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2}$$

- Suppose I'm a bug on the wheel
  - → my speed is getting faster
  - → my direction is changing

Then:

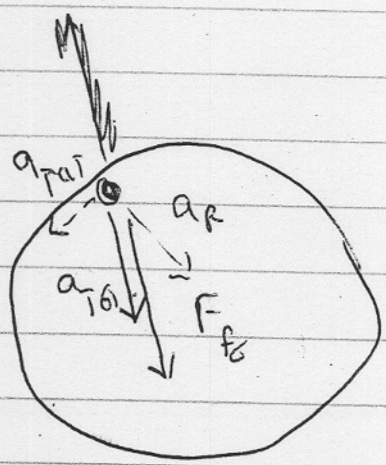


$$a_{Tan} = \frac{dV}{dt} = \frac{d\,R\omega}{dt} = R\frac{d\omega}{dt} = \boxed{R\alpha = a_{Tan}}$$

$$a_R = \frac{V^2}{R} = \frac{(R\omega)^2}{R} = \omega^2 R$$

The total acceleration

$$a_{TOT} = \sqrt{a_{tan}^2 + a_R^2}$$

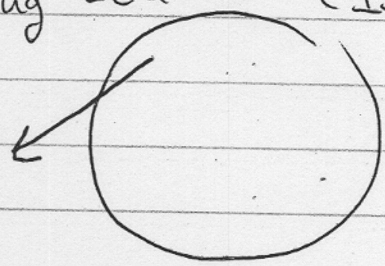


$$F_{fr}^{max} = m\, a_{TOT}$$

$$\mu_s \cancel{m} g = \cancel{m}\, a_{TOT}$$

$$\mu_s = \frac{a_{TOT}}{g}$$

At this point
Bug Leaves (Is thrown off) the wheel

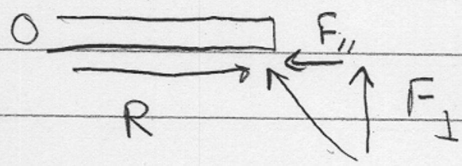


Torque - What makes spin?

1. Force: F

2. Moment Arm: R

3. Only the component of the force perpendicular to the moment arm causes things to spin



$\leftarrow$ force $\perp$ to moment arm

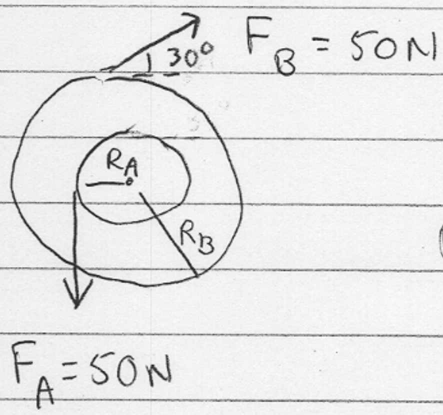
These observations motivate a definition:

$$|\tau| = R F_{\perp} = R F \sin\theta$$

1. Torque is actually a vector (much more on this later!) which will point out of the page for counter clockwise and into the page for clockwise
2. For the moment we just will say that the torque is + if it causes rotation in the counter clockwise direction, − if it causes clockwise rotation.

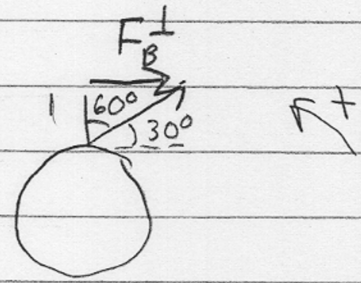
So, $\tau = R F \sin\theta \cdot \{(\pm)\} \leftarrow$

Example 10-7: Torque on a compound wheel



Which way does it turn

$R_A = 30cm$ $R_B = 50cm$



① $\tau_A = R_A F_{A\perp} = R_A F_A$

② $\tau_B = R_B F_{B\perp} = R_B F_B \sin 60^\circ \cdot (-)$

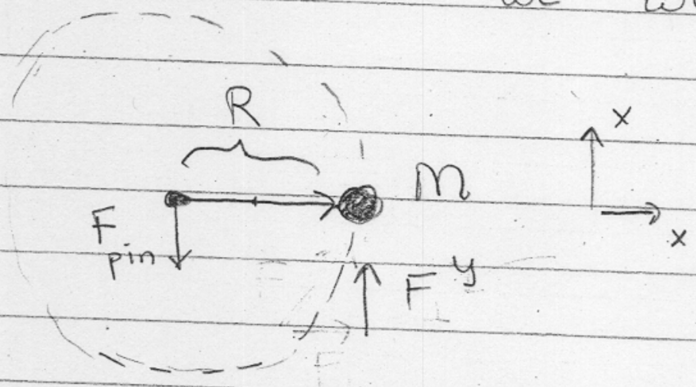
So

$$\tau_A + \tau_B = R_A F_A - R_B F_B \sin 60^\circ = -6.7 N$$

So the net torque is negative indicating it causes a clockwise rotation

Torque causes angular acceleration	Like force causes acceleration
$\tau = I \alpha$	$F = ma$

"moment of intertian"
— we will talk about it now



$$F^y = ma^y$$

$$F^y = m R \alpha$$

$$\underbrace{F\ R} = \underbrace{mR^2}\ \alpha$$

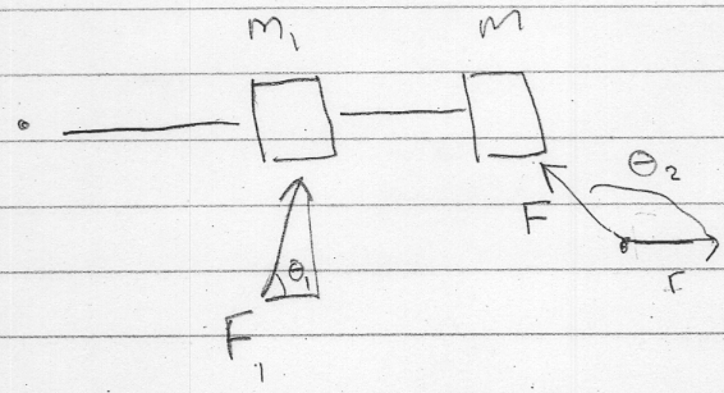
$$\tau = \underbrace{mR^2}\ \alpha$$

So:

$mR^2 \equiv I \rightarrow$ in this case

$$\tau = I \alpha$$

Generalize



Can be shown (a litte later) - Chapter 11)

$$\sum \tau_{ext} = F_1^{\perp} R_1 + F_2^{\perp} R_2 = I\alpha$$

$$= F_1 R_1 \sin\theta_1 + F_2 R_2 \sin\theta = I\alpha$$

$$I = \sum_i m_i r_i^2 \Rightarrow m_1 r_1^2 + m_2 r_2^2$$

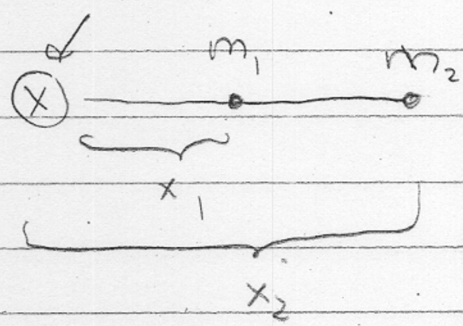
So $\boxed{\sum \tau_{ext} = I\alpha}$ $\quad I = m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots = \int dm\, r^2$

What is I Really ?

$$I = m_{TOT} \langle r^2 \rangle_m \qquad \langle r^2 \rangle_m = \frac{\sum_i m_1 r_1^2 + m_2 r_2^2 + \ldots}{m_1 + m_2}$$

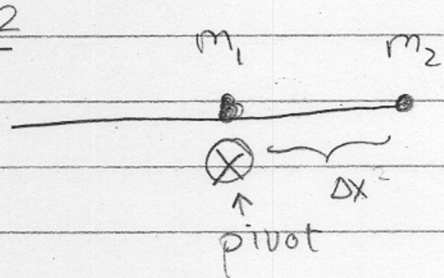
Its kind of a mass weighted position squared

Ex1



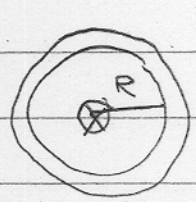
$$I = m_1 x_1^2 + m_2 x_2^2$$

Ex2



$$I = m_1 0^2 + m_2 \Delta x^2$$

Ex3

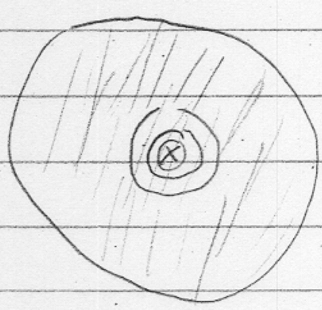


$$I = M_{TOT} \langle r^2 \rangle = M_{TOT} R^2$$

$$I = \int dm\, r^2 = \int dm\, R^2 = MR^2$$

Ex4

Solid Disk, density $\sigma = \frac{M}{\pi R^2}$



$$I = \int dm\, r^2$$

$$I = \int \sigma\, dA\, r^2 = \int_0^R \frac{M}{\pi R^2} 2\pi r\, dr\, r^2$$

$$I = \frac{M}{\cancel{\pi} R^2} \cancel{2\pi} \frac{1}{\cancel{4}\,2} r^4 \Big|_0^R = \frac{1}{2} MR^2$$