Lost Time

Angular Velocity:

1. \( \omega = \frac{d\theta}{dt} \) \( \Rightarrow \) \( \mathbf{v} = \omega \mathbf{R} \)

2. Direction: Right hand Rule means you curl fingers and thumb points in direction of \( \omega \)

3. For "normal" x y rotation:
   - + counter-clock out of page
   - - clockwise into page

4. Every point on the solid body has the same \( \omega \) but different velocities
   "\( \omega \) is a constant of the body"
Angular Acceleration:

- Suppose the wheel spins faster and faster all the time.

\[ \alpha = \frac{dw}{dt} = \text{"change in angular velocity per time"} \]

\[ \alpha = \frac{d}{dt} \left( \frac{d\theta}{dt} \right) = \frac{d^2\theta}{dt^2} \]

- Suppose I’m a bug on the wheel.
  → my speed is getting faster.
  → my direction is changing.

Then:

\[ a_{\text{tan}} = \frac{dv}{dt} = \frac{d}{dt} \left( \frac{Rw}{dt} \right) = \frac{R \alpha}{dt} = a_{\text{tan}} \]

\[ a_R = \frac{V^2}{R} = \left( \frac{Rw}{R} \right)^2 = \omega^2 R \]

The total acceleration:

\[ a_{\text{tor}} = \sqrt{a_{\text{tan}}^2 + a_R^2} \]
At this point, the bug leaves (is thrown off) the wheel.

\[ F_{fr} = ma_{tor} \]
\[ \mu_s m'g = ma_{tor} \]
\[ \mu_s = \frac{a_{tor}}{g} \]
Torque – What makes spin?

1. Force: \( F \)
2. Moment Arm: \( R \)
3. Only the component of the force perpendicular to the moment arm causes things to spin.

\[ \text{These observations motivate a definition:} \]

\[ |\mathbf{T}| = RF_\perp = RF \sin \theta \]

1. Torque is actually a vector (much more on this later!) which will point out of the page for counter-clockwise and into the page for clockwise.
2. For the moment we just will say that the torque is + if it causes rotation in the counter-clockwise direction, – if it causes clockwise rotation.

So, \( \mathbf{T} = RF \sin \theta \cdot \{+\} \)
Example 10-7: Torque on a compound wheel

$F_B = 50\, \text{N}$, which way does it turn?

$R_A = 30\, \text{cm}$, $R_B = 50\, \text{cm}$

$F_A = 50\, \text{N}$

1. $\tau_A = R_A F_{A_1} = R_A F_A$

2. $\tau_B = R_B F_{B_1} = R_B F_B \sin 60^\circ \cdot (-)$

So

$\tau_A + \tau_B = R_A F_A - R_B F_B \sin 60^\circ = -6.7\, \text{N}$

So the net torque is negative indicating it causes a clockwise rotation.
Torque causes angular acceleration. 
Like force causes acceleration.

\[ I = I \alpha \]
\[ F = ma \]

"moment of inertia" - we will talk about it now.

\[ F_y = m a_y \]
\[ F \cdot R = m R^2 \alpha \]
\[ I = m R^2 \alpha \]

So:
\[ m R^2 \equiv I \rightarrow \text{in this case} \]
\[ I = I \alpha \]
Generalize

\[ \sum \tau_{\text{ext}} = F_1 R_1 + F_2 R_2 = I \alpha \]

\[ = F_1 R_1 \sin \theta_1 + F_2 R_2 \sin \theta_2 = I \alpha \]

\[ I = \sum m_i r_i^2 \Rightarrow m_1 r_1^2 + m_2 r_2^2 \]

So

\[ \sum \tau_{\text{ext}} = I \alpha \]

\[ I = m_1 r_1^2 + m_2 r_2^2 + \cdots = \int dm \ r^2 \]

What is \( I \) Really?

\[ I = M_{\text{tot}} \langle r^2 \rangle_m \]

\[ \langle r^2 \rangle_m = \sum m_i r_i^2 \]

\[ \frac{m_1 + m_2}{m_1} \]

It's kind of a mass weighted position squared
Ex 1
\[ I = m_1 x_1^2 + m_2 x_2^2 \]

Ex 2
\[ I = m_1 \alpha^2 + m_2 \Delta x^2 \]

Ex 3
\[ I = M_{\text{tot}} \langle r^2 \rangle = M_{\text{tot}} R^2 \]
\[ I = \int dm \, r^2 = \int dm \, R^2 = MR^2 \]

Ex 4
Solid Disk, density \( \sigma = \frac{M}{\pi R^2} \)
\[ I = \int dm \, r^2 \]
\[ I = \int \sigma \, dA \, r^2 = \int_0^R \frac{M}{\pi R^2} \, 2\pi r \, dr \, r^2 \]
\[ I = \frac{M}{\pi R^2} \int_0^R r^4 \, dr = \frac{1}{2} MR^2 \]