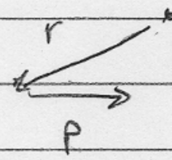
Last Time:

① Defined angular momentum of a particle

$$\vec{l} = \vec{r} \times \vec{p} \qquad l$$



$r_\perp$ $|\vec{l}| = r_\perp p$ out of page

Can easily show: the

$$\frac{d\vec{l}}{dt} = \sum \vec{r} \times \vec{F}_{ext} = \sum_i \vec{\tau}_{ext} \leftarrow \text{analogous to } \vec{F} = \frac{d\vec{p}}{dt}$$

" the sum of $\vec{\text{Torque}}$ is equal to the rate of change of $\vec{l}$ "

Proof:

$$\frac{d(\vec{r} \times \vec{p})}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} \qquad \vec{F} = \frac{d\vec{p}}{dt}$$

$$= \cancel{\vec{v} \times m\vec{v}}^{\,0} + \vec{r} \times \vec{F}$$

in a frame which moves @ costant velc

② For many partice

$$\vec{L} = \sum_i \vec{l}_i$$

One shows

$$\frac{d\vec{L}}{dt} = \sum_i \vec{\tau}_{ext} \leftarrow \text{holds in an inertial frame}$$

As a benefi

$$\frac{d\vec{L}_{cm}}{dt} = \sum \vec{\tau}^{\,cm}_{ext} \quad \leftarrow \text{always true}$$

(3) For a rigid body spinning around an axis

$$\vec{L} = I\vec{\omega}$$

$$\frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt} = I\vec{\alpha}$$

(4) If no external Torques

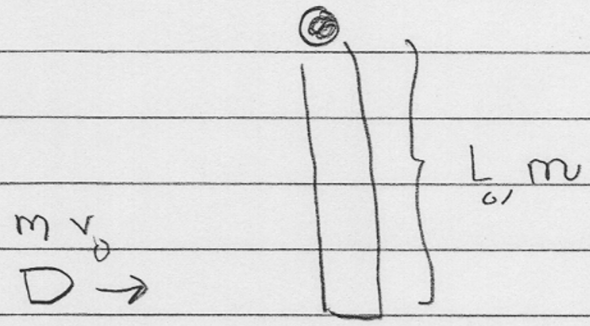
$$\frac{d\vec{L}}{dt} = 0 \qquad \vec{L} = \text{constant}$$

= Angular momentum constant

For simple cases (skater, cat)

$$I_i\omega_i = I_f\omega_f$$

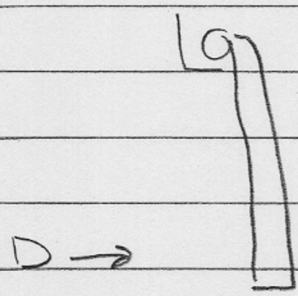
Problem:



Determine the speed v_0 required so that the rod reaches the top of the arc

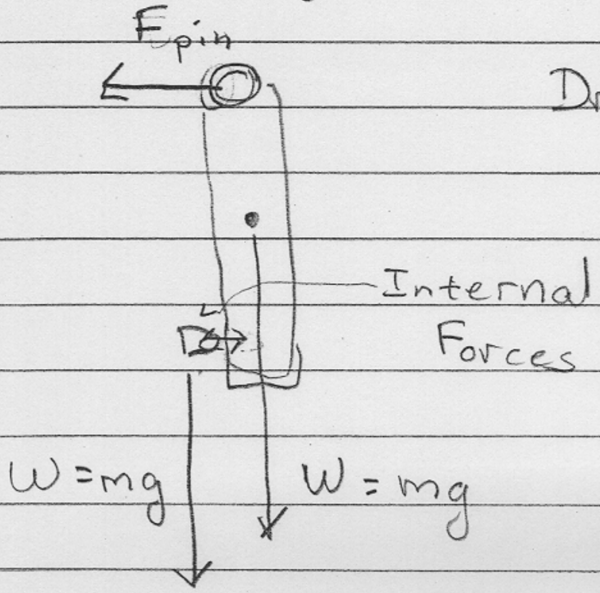
L_0 = Length $\neq$ angular momentum

Q: Is momentum conserved? No!



Compare to a rod just placed on a ledge. Then the rod would go flying

Q: Is angular momentum conserved? Yes!



Draw a FBD

$$\frac{d\vec{L}}{dt} = \sum \vec{\tau}_{ext}$$

$$= \overset{0\cdot R}{\cancel{\vec{\tau}_{pin}}} + \overset{F_\perp = 0}{\cancel{\vec{\tau}^{\,rod}_{g}}} + \overset{F_\perp = 0}{\cancel{\vec{\tau}^{\,bullet}_{g}}}$$

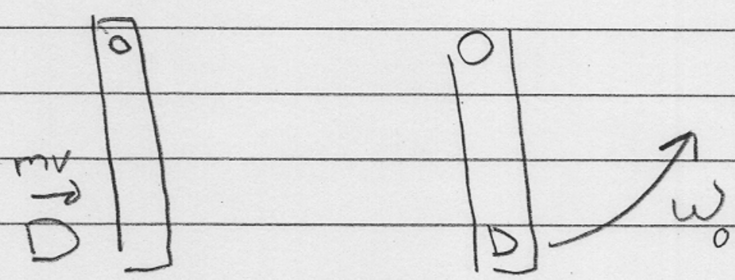
$$\vec{L} = \text{constant}$$

Since

Force $\perp$ to R

$$|\vec{\tau}| = |\vec{R} \times \vec{F}| = R F_\perp$$

So



before after

$$L_{before} = L_{bullet} + \cancel{L_{rod}} = L_{after}$$

$$= r_\perp p = I_f \omega_0$$

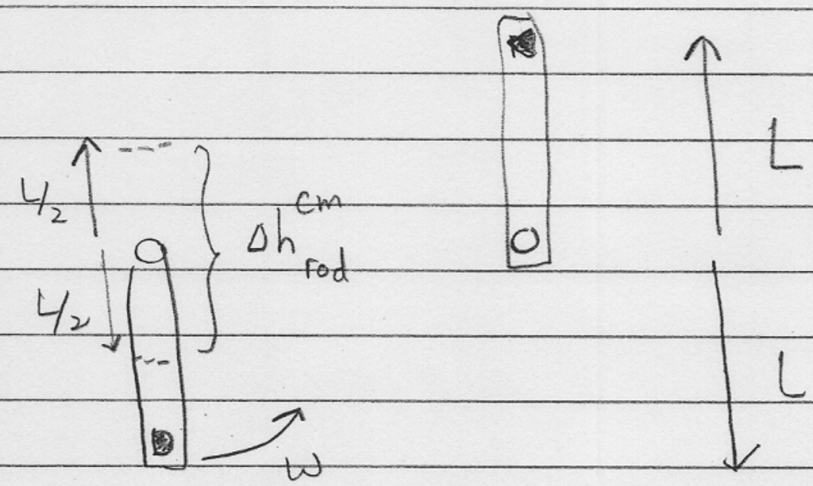
$$= L_0 m v_B = I_f \omega_0$$

$$I_f = I_{bullet} + I_{rod} = ML_0^2 + \underbrace{\frac{1}{3} ML_0^2}_{\text{from table}} = \frac{4}{3} mL^2$$

So

$$L_0 m v_B = \frac{4}{3} ML_0^2 \omega \Rightarrow \boxed{\omega_0 = \frac{3}{4} \frac{v_B}{L_0} \quad \text{units } \frac{1}{s}}$$

Now if the bullet/rod just reaches top



$$KE_{bottom} = \Delta PE$$

$$\frac{1}{2} I \omega^2 = \Delta PE_{rod} + \Delta PE_{bullet}$$

$$\frac{1}{2}\left(\frac{4}{3} m L_0^2\right)\left(\frac{3}{4}\frac{V_B}{L_0}\right)^2 = mg L_0 + mg 2L_0$$

$$\frac{1}{2} \cancel{m} v_B^2 \cdot \frac{3}{4} = 3 \cancel{m} g L_0$$

$$v_B^2 = 8gL \Rightarrow v_B = \sqrt{8gL}$$

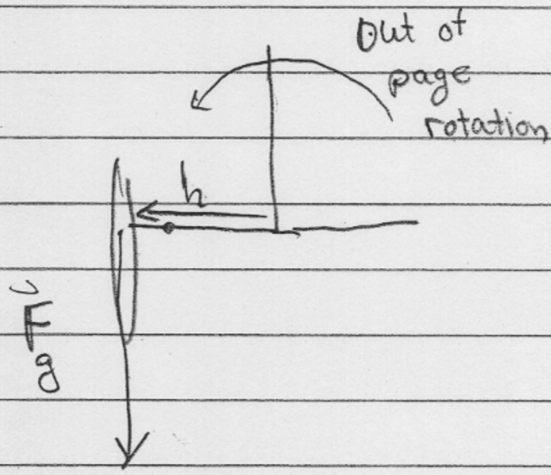
Precession of Gyro Scopes / Tops

→ Unbelievably Rich Subject

→ Studied in Amazing Detail, Klein + Sommerfeld

"Theorie des Kreisels" in four volumes

Example One - Demonstrated (No Spinning)

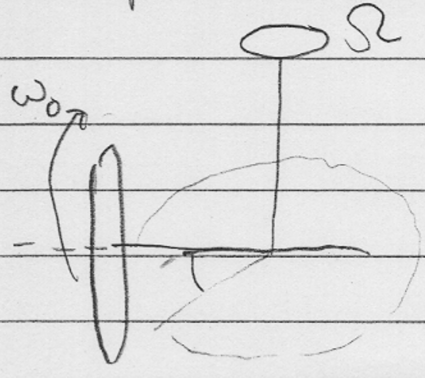


$$\vec{\tau} = \vec{R} \times \vec{F}_g \quad \text{out of page}$$

$$\frac{d\vec{L}}{dt} = hF\,\hat{k}$$

↖ The torque causes out of page rotation

Example 2 - Demonstrated (with spinning)



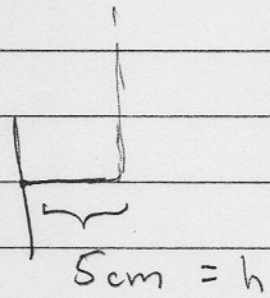
- first observation is that the wheel doesn't topple
- Next observation the wheel precesses with angular velocity Ω

Estimates:

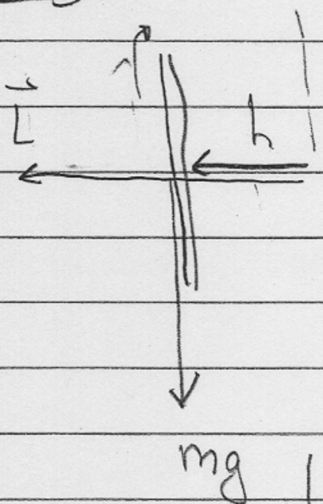
- $V \sim 2\,m/s$
  $R \sim 40\,cm$

  $$\omega_o \sim \frac{V}{R} \sim 5\ rad/s \sim 0.8\ rev/s$$

- $M \sim 20\,lbs \sim 9\,kg$

- 

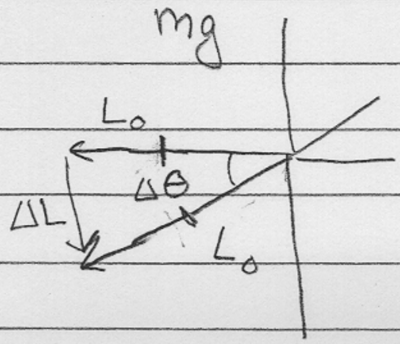
Analysis:



$$\vec{L}_o = I\vec{\omega}_o$$

$$\vec{\tau} = \vec{R} \times \vec{F} = hmg\,\hat{k}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$



$$hmg = \frac{\Delta L}{\Delta t}$$

$$\Delta t\,\frac{hmg}{L_o} = \frac{\Delta L}{L_o}$$

Conclude

$$\Delta t \frac{hmg}{I\omega_0} = \Delta\theta$$

precession frequency

$$\frac{hmg}{mR^2\omega_0} = \frac{\Delta\theta}{\Delta t} = \Omega$$

$$\boxed{\frac{hg}{R^2\omega_0} = \Omega}$$

Lets estimate :

$$\Omega = \frac{(0.05m)(9.8m/s^2)}{(0.4m)^2(5\frac{1}{s})} = 0.6125 \frac{1}{s}$$ ← units radians

$$f = 0.1 \text{ rev/s} = \frac{\Omega}{2\pi}$$

$$\boxed{\text{Period} = \frac{1}{f} \simeq 10s}$$