Temperature

\[ t_c \quad t_f \quad T \quad \text{where did it come from?} \]

- Celsius, Fahrenheit \( \approx 1724 \), Kelvin

\( T \)

Gas

1. Characterized by constants of motion:
   - \( E \) the total energy in the box
   - \( N \) the total # of part in box

2. Then the other state variables \( P, T, \rho \) are determined by these constants and the properties of the gas.

- In the study of dilute gases find at fixed temperature

(Boyle) \( PV = \text{constant at fixed temperature} \)

Now keep \( V \) fixed and decrease the temperature.

1. \( P \) decreases,

\[
\begin{array}{c}
P \\
\hline
-273^\circ C \quad 0^\circ C \quad T
\end{array}
\]

the special temperature \(-273^\circ C\) where \( P = 0 \) is independent of gas and volume.
So one finds a special temperature $T_c = -273^\circ C$ where pressure becomes zero.

- Define this temperature to be zero

$$T = t_c + 273^\circ C \Rightarrow \text{ie } T=0 \text{ is } -273^\circ C$$

- $T$ temp
- Celsius
- $t_c$ temp in Kelvin

Generally want to use Kelvin.
Last Time

- **Temperature Scales**
  
  \[ t_C \leftarrow \text{Celsius} \]
  
  \[ t_F \leftarrow \text{Fahrenheit} \quad t_F = \frac{9}{5} t_C + 32^\circ F \]
  
  \[ T \leftarrow \text{Kelvin} \quad T = t_C + 273^\circ C \]

  - always use it when doing calcs
  - Defined to be zero when pressure = 0

- **Dilute Gases (Ideal Gasses)**
  
  \[ PV = \text{const} \quad \text{fixed} \ T \quad \text{(Boyle law)} \]
  
  \[ P \propto T \quad \text{fixed} \ V \quad \text{(Gay-Lussac)} \]
  
  \[ V \propto T \quad \text{fixed} \ P \quad \text{(Gay-Lussac)} \]
What is $V$?

- What we really mean is the volume for a fixed number of particles, $V/N$.

- Can double the volume and double the number of particles at the same temp and get the same pressure.

So

$$\frac{PV}{N} = \text{const}$$

All these are combined to give

$$PV = Nk_B T$$

Where

$$k_B = 1.38 \times 10^{-23} \text{ J/K}$$

Note: $k_B \approx 10^{-23}$, $N \approx 10^{23}$, $Nk_B \approx 1$
Usually work with moles

1 mol = 1 \( N_A \) of particles \( N_A = 6.02 \times 10^{23} \)

So number of moles

\[ N = n \cdot N_A \]

The

\[ PV = n \cdot N_A \cdot k_B \cdot T \quad \text{or} \quad PV = nRT \]

\[ R \equiv \text{the ideal gas constant} = 8.31 \frac{J}{\text{K mol}} = N_A \cdot k_B \]

Similarly:

1 mol weight = \( M \equiv \) molar mass

Example: \( M = 55.8 \text{ g} \) for iron

\[ m = nM \]

mass of substance
Trivia

1. How much does $N_A$ of protons weigh?
   Approx. 1g

2. The mass of proton $\approx$ mass neutron $m_p = m_n$
   Electron mass is negligible $\frac{m_e}{m_p} \approx \frac{1}{2000}$

3. Iron has 26 protons + 30 neutrons $\approx$ 56 nucleons
   1 mol of Fe weighs $\approx$ 56 g
Problem: Volume of 1 mole at 0°C and 1 atm?

\[ PV = nRT \]

\[ \text{latm} = 1 \text{ bar} = \frac{10^5 N}{m^2} \]

\[ V = (1 \text{ mol}) \left( 8.31 \text{ J} \right) \left( 273^\circ \text{ K} \right) \]

\[ \frac{10^5 N}{m^2} \]

\[ = 22.4 \text{ L} \]

Temp and Expansion:

\[ \Delta L = \alpha \Delta T \]

\[ \text{AT at constant Pressure:} \]

Coefficient of expansion

\[ \frac{\Delta L}{L} \sim 10^{-5} \text{ per degree} \]

\[ L \sim 30 \text{ m} \quad \Delta T \sim 20^\circ \quad \Delta L \sim 0.5 \text{ cm} \]
For volume expansion

\[ \Delta V = \beta \Delta T \]

Now:

\[ V = L^3 \]

\[ \Delta V = 3 \cdot L^2 \Delta L \]

\[ \frac{\Delta V}{V} = \frac{3 \cdot \Delta L}{L} = 3 \alpha \Delta T \]

So

\[ \beta \approx 3 \alpha \]
Heat

To raise the temperature of a body, we have to add energy. Thermal energy is known as heat transferred from one system to another, i.e., the kinetic and potential energies of all the molecules and light.

\[ \Delta Q \propto \Delta T \]

specific heat of a body

If you have a lot of stuff, it takes a lot of heat.

\[ \Delta Q = m \cdot c_p \cdot \Delta T \]

at constant pressure

specific heat per mass

\[ c_{water} = 4190 \, \text{J/kg}^\circ\text{C} \]

1 cal = 4.184 J/kg \^\circ\text{C}

Also write:

\[ \Delta Q = n \cdot c_p \cdot \Delta T \]

specific heat per mol

\[ c_{H_2O} = 75.4 \, \text{J/mol} \cdot \text{°K} \]

number of moles
Comments

1. Now \( \frac{nM}{C_p} \) \[ \Delta Q = mC_p \Delta T = nM C_p \Delta T \]

Compare

\( M C_p = C_p \) Too easy!

2. Since \( \Delta T = \frac{\Delta t}{k} \) \[ Q = mc_p \Delta T \]

\[ Q = mc_p \Delta t \]

\[ \text{we can use celsius in working these probs} \]
Example: determining the specific heat of steel.

A steel ball of mass $80\,g = m_s$ has an initial temp of $200^\circ C$ is placed in a bucket at water of $m = 250\,g$. The final temp is $25.8^\circ C = T_f$ (neglect the cup).

Initial

\[
\begin{array}{c}
\text{200}^\circ C \\
\text{20}\,^\circ C \\
\end{array}
\]

\[
\begin{array}{c}
\text{80}^\circ C \\
\text{80}^\circ C \\
\end{array}
\]

Sol: No energy is added:

\[
\Delta Q_w + \Delta Q_s = 0
\]

\[
\Delta Q_s = -\Delta Q_w \quad \text{heat to}
\]

\[
m_s c_s \Delta T_s = -m_w c_w \Delta T_w
\]

\[
c_s = -m_w c_w \Delta T_w \quad \Delta T_w = 5.8^\circ C
\]

\[
m_s \Delta T_s \quad \Delta T_s = -174.2^\circ C
\]

\[
c_s = \frac{436}{10\,\text{tons}} = 4190\,J
\]

\[
\text{10 tons smaller than } H_2O
\]
Latent Heat

Sometimes adding energy does not change the temperature (\(~ KE \) but rather changes the structure (\(~ PE \)) of the material.

\[
\begin{align*}
\text{T}^\circ C & \quad 100^\circ C & \quad \text{Water vaporizes} \\
\text{Ice melts} & \quad 0^\circ C & \quad \text{Icy} \\
\text{Ice} & \quad 334 J & \quad 2300 J \\
1 \text{ kg of } H_2O
\end{align*}
\]

\[\Delta Q = m \cdot L\]

Latent heat of fusion \(\approx 334 \text{ J/kg}\)

heat required to melt

Latent Heat of Vaporization

\[\Delta Q_{\text{vaporize}} = m \cdot L \approx 2300 \text{ J/kg for water to steam}\]
Example

A 2 kg Chunk of ice at -10°C is added to 5 kg of liquid water at 15°C. What is the final temp?

Possible Outcomes:
- Ice could partially melt after rising to 0°C, leaving ice-water mix at 0°C.
- Ice could completely melt and the T could rise above 0°C.

Sol

- The heat needed to raise ice to 0°C

\[ Q_1 = m_1 \cdot c_i \cdot \Delta T_1 = 4.4 \times 10^4 \text{ J} \]

\[ \frac{c_i}{kg \cdot ^\circ C} = 2200 \frac{\text{J}}{kg \cdot ^\circ C} \text{ specific heat of ice} \]

\[ \Delta T = 10^\circ K \text{ from -10} \to 0 \]

\[ m_1 = 2 kg \]

- The heat required to melt the ice:

\[ Q_2 = m_1 \cdot L_i = (2 kg) \cdot (334 J/kg) = 668 J \]
Finally the heat taken away from the water to lower it to zero 0°C

\[ \Delta Q = m_w c_w \Delta T = -3.14 \times 10^5 \text{J} \]

\[ m_w = 5 \text{kg} \quad c_w = 4.190 \text{J/kg} \cdot ^\circ\text{C} \quad \Delta T = -15^\circ\text{C} \]

So since

\[ (Q_3) > (Q_1 + Q_2) \]

the water will have enough energy to melt the ice and raise its temp. To find the final temp we have

\[ \Delta Q = 0 \quad \Rightarrow \quad \text{No heat inflow} \]

\[ Q_1 + Q_2 + m_i c_i (t_f - 0^\circ\text{C}) + m_w c_w (t_f - 15^\circ\text{C}) = 0 \]

heat into ice

heat into water

\[ (m_i c_i + m_w c_w) t_f = m_i c_i 0^\circ\text{C} + m_w c_w 15^\circ\text{C} - (Q_1 + Q_2) \]

\[ t_f = \frac{0^\circ\text{C} + m_w c_w 15^\circ\text{C} - (Q_1 + Q_2)}{m_i c_i + m_w c_w} \]

\[ t_f = 9.2^\circ\text{C} \]