Last Time

- Heat is the transfer of microscopic forms of energy from one system to another.

- It can be used to raise the temperature or change the phase.

Dilute Gasses - Ideal Gasses

- Molecules are sufficiently far apart that they don't interact with each other. Energy depends only on temperature, not on volume or $V/N$.

- All Energy is kinetic. For a mono-atomic gas:

$$U = N \frac{1}{2} m v^2$$

- Want to determine $\frac{1}{2} m v^2$. 
To this end let's estimate the force on the wall.

\[ \Delta p^x = -2m v_{ix} \]

The time it takes to traverse the box is

\[ \Delta t = \frac{2L}{v_{ix}} \]

So

\[ F_{\text{on wall}} = \frac{\Delta p^x}{\Delta t} = \frac{2m v_{ix}^2}{L} \]

due to the \( i \)-th molecule

\[ F_{\text{tot}} = \frac{1}{L} \sum m v_{ix}^2 = \frac{Nm}{L} \left( \frac{\sum v_{ix}^2}{N} \right) \]

\[ F_{\text{tot}} = \frac{Nm v_{ix}^2}{L} \]

\[ P = \frac{F_{\text{tot}}}{L^2} = \frac{Nm v_{ix}^2}{L^3} \]

\( \triangle \) area of wall
Volume of box
\[
V = L^2
\]

Further since all direction are equal
\[
\overline{V_x} = \overline{V_y} = \overline{V_z} = \frac{1}{3} \overline{V^2}
\]

Where
\[
V^2 = V_x^2 + V_y^2 + V_z^2
\]

So
\[
\rho = \frac{N m V^2}{3V} \quad \text{or} \quad \rho V = \frac{2N}{3} \left( \frac{1}{2} m v^2 \right)
\]

Now
\[
\rho V = N k_B T
\]

So
\[
\frac{2}{3} \left( \frac{1}{2} m v^2 \right) = k_B T \quad \text{or} \quad \frac{1}{2} m v^2 = \frac{3}{2} k_B T
\]

Then
\[
U = N \frac{1}{2} m v^2 = N \frac{3}{2} k_B T \quad \text{for ideal mono-atomic gas}
\]
Note:

\[ u = \frac{3}{2} N k_B T = \frac{3}{2} \pi n N_A k_B T \]

\[ u = \frac{3}{2} \pi n RT \]
Problem:

Determine the root mean-square velocity of O$_2$ gas at room temperature 300°K.

\[
V_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}
\]

\[
\frac{1}{2} m v^2 = 3 k_B T
\]

\[
\overline{v^2} = 3k_B T
\]

\[
m_{O_2} = \frac{32 g}{N_A}
\]

O has 8 protons and 8 neutrons and its O$_2$

\[
M_{O_2} = 5.3 \times 10^{-26} \text{ kg}
\]

\[
V_{\text{rms}} = \sqrt{\frac{3k_B T}{M_{O_2}}}
\]

\[
k_B = 1.38 \times 10^{-23} \text{ J/K}
\]

\[
T = 300°K
\]

\[
V_{\text{rms}} = 483 \text{ m/s}
\]

i.e. of order the sound speed in air \( V_s \approx 340 \text{ m/s} \)
Temperature and Equipartition

\[ \frac{1}{2} m v^2 = \frac{3}{2} k_B T \]

Notice each molecule has "3 degrees of freedom" in motion \( x, y, z \):

\[ \frac{1}{2} m v_x^2 = \frac{1}{2} m v_y^2 = \frac{1}{2} m v_z^2 = \frac{1}{2} k_B T \]

There is then a rule: "Equipartition"

Each "degree of freedom" has an average energy \( \sim \frac{1}{2} k_B T \)

Gives us our working definition of temperature:

\( k_B T \) is "Energy per degree of freedom"

So two things equilibrate when the energy per degree of freedom is equal or temperature

1. A more precise meaning of "degree of freedom" can be given
2. Equipartition is violated in a quantum mech setting
Now consider a diatomic molecule $O_2, N_2, H_2$.

Each molecule has two rotational degrees of freedom and 3 translational $(x, y, z)$ degrees of freedom.

$$U = N \left( \frac{3 \cdot 1}{2} k_B T + \frac{2 \cdot 1}{2} k_B T \right)$$

$$U = \frac{5 N k_B T}{2}$$

← energy of a diatomic gas

Can also be written

$$U = \frac{3 n R T}{2}$$