Last Time

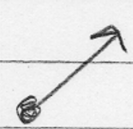
- Heat is the transfer of microscopic forms of energy from one system to another
    - It can be used to raise the temperature or change the phase

Dilute Gasses - Ideal Gasses

- molecules are sufficiently far apart that they dont interact with each other. Energy depends only on temperature not on Volume or V/N
- All Energy is kinetic. For a mono-atomic gas

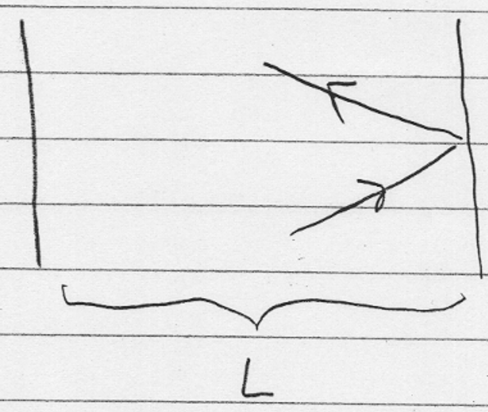
$$U = N \overline{\frac{1}{2} m v^2}$$

mass of atom



- Want to determine $\overline{\frac{1}{2} m v^2}$

To this end lets estimate the force on the wall



$$\Delta p^x = -2 m v_{ix}$$

The time it takes to traverse the box is

$$\Delta t = \frac{2L}{v_{ix}}$$

So

F on wall due to the i-th molecule $= -\frac{\Delta p_x}{\Delta t} = \frac{2 m v_{ix}}{\frac{2L}{v_{ix}}} = \frac{m v_{ix}^2}{L}$

$$F_{TOT} = \frac{1}{L} \sum m v_{ix}^2 = \frac{N m}{L}\left(\frac{\sum_i v_{ix}^2}{N}\right)$$

$$F_{TOT} = \frac{N m \overline{v_x^2}}{L}$$

press

$$P = \frac{F_{TOT}}{L^2} = \frac{N m}{L^3} \overline{v_x^2}$$

area of wall

Volume of box

$V = L^3$

Further since all direction are equal

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{1}{3}\overline{v^2}$$

Where

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

So

$$p = \frac{N}{3V} m \overline{v^2} \quad \text{or} \quad \boxed{pV = \frac{2}{3} N \left(\frac{1}{2} m \overline{v^2}\right)}$$

Now

$$pV = N k_B T$$

So

$$\frac{2}{3}\left(\frac{1}{2} m \overline{v^2}\right) = k_B T \quad \text{or} \quad \boxed{\frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T}$$

Then

$$\boxed{U = N \frac{1}{2} m \overline{v^2} = N \frac{3}{2} k_B T}$$

for ideal mono-atomic gas

Note:

$$U = \frac{3}{2} N k_B T = \frac{3}{2} n \underbrace{N_A k_B}_{\equiv R} T$$

$$\boxed{U = \frac{3}{2} n R T}$$

Problem:

Determine the root mean-square velocity of O_2 gas at room temp $300^\circ K$

$$V_{rms} \equiv \sqrt{\overline{V^2}} = \sqrt{\frac{3k_B T}{m}} \qquad \frac{1}{2}\overline{mV^2} = \frac{3}{2}k_B T$$

$$\overline{V^2} = \frac{3k_B T}{m}$$

$m_{O_2} = \frac{32\,g}{N_A}$ ← O has 8 protons and 8 neutrons and its O_2

$m_{O_2} = 5.3\times10^{-26}\,kg$

$$V_{rms} = \sqrt{\frac{3k_B T}{m_{O_2}}} \qquad k_B = 1.38\times10^{-23}\,J/K \quad T = 300^\circ k$$

$V_{rms} = 483\,m/s$

i.e. of order the sound speed in air $V_s \sim 340\,m/s$

Temperature and Equipartion

$$\overline{\frac{1}{2} m v^2} = \frac{3}{2} k_B T$$

Notice each molecule has "3 degrees of freedom" motion in x, y, z

$$\overline{\frac{1}{2} m v_x^2} = \overline{\frac{1}{2} m v_y^2} = \overline{\frac{1}{2} m v_z^2} = \frac{1}{2} k_B T$$

There is then a rule: -- "Equipartion"

Each "degree of freedom" has an average energy $\sim \frac{1}{2} k_B T$

Gives us our working definition of temperature

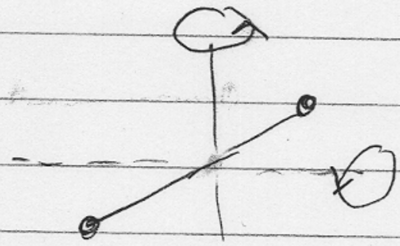
$k_B T$ is the Emergy per degree of freedom

So two things equilibrate when the $\underbrace{\text{energy per degree of freedom}}_{\text{or temperature}}$ is equal

(1) A more precise meaning of "degree of freedom" can be given

(2) Equipartition is violated in a quantum mech setting

Now Consider a diatomic molecule O_2, N_2, H_2



Each molecule has
Two rotational degrees of freedom
and 3 translational (x,y,z) degrees of freedom

$$U = N\left(3 \cdot \frac{1}{2} k_B T + 2 \cdot \frac{1}{2} k_B T\right)$$

$$\boxed{U = \frac{5}{2} N k_B T} \longleftarrow$$ energy of a diatomic gas

Can also be written

$$\boxed{U = \frac{5}{2} n R T}$$