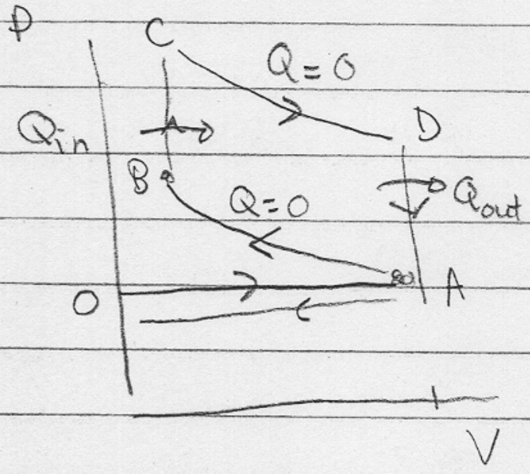
Last Time - Otto Cycle



① Intake OA

② Compress AB $\quad r = \frac{V_A}{V_B} \sim 8$

③ Ignite BC

④ Power CD

⑤ Exhaust DA

⑥ Epell AO

P1: Estimate the volume of air in a typical car, determine the number of moles:

A: 3L = V $\qquad$ B: 22.4 at STP 1mole

so 3L at STP 1/8 mole

P2: Find the temp increase in the compression stroke

$$TV^{\gamma-1} = C \qquad T_B = T_A \left(\frac{V_A}{V_B}\right)^{\gamma-1} = T_A\, r^{\gamma-1}$$

Similarly $T_c > T_D$

$$T_C = T_D \left(\frac{V_D}{V_C}\right)^{\gamma-1} = T_D\, r^{\gamma-1}$$

$$T_A \simeq 300^\circ \qquad T_B = T_A \cdot (8)^{0.4} \simeq 689^\circ K$$

Problem 3

• The efficiency is defined as the work done per unit heat input

$$\varepsilon = \frac{W}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Since: $\cancel{\Delta U}^{0} = Q - W$ $Q = Q_{in} - Q_{out}$

$$Q_{in} - Q_{out} = W$$

Want to determine the efficiency.

① Show that:

$$e = 1 - \left(\frac{V_B}{V_A}\right)^{\gamma - 1}$$

$\frac{V_A}{V_B} \equiv$ the compression ratio $\equiv r \sim 8$

$$e = 1 - \frac{1}{r^{\gamma-1}}$$

Solution:

Analyzé the heat brought in

$$\Delta U = Q - \cancel{W}^{0}$$

$$n\, C_V\, dT = dQ$$

$$Q_H = nC_v(T_C - T_B)$$

Now Q_L

$$\Delta U = Q - \cancel{W}^{\,0}$$

heat flowing into gas

$$\Delta U = nC_v(T_A - T_D) = Q$$

$T_A < T_B$

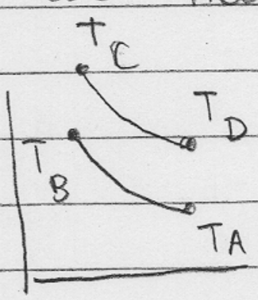
So

$$Q_{out} = -Q = nC_v(T_D - T_A)$$

Then

$$\varepsilon = \frac{W}{Q_{out}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \left[\frac{T_D - T_A}{T_C - T_B}\right]\frac{\cancel{nC_v}}{\cancel{nC_v}}$$

Now we need to relate



T_C and T_D and T_B and T_A

Using

$$T_C = T_D\, r^{\gamma-1} \qquad T_B = T_A\, r^{\gamma-1}$$

So

$$e = 1 - \left[\frac{T_D - T_A}{T_D r^{\gamma-1} - T_A r^{\gamma-1}} \right]$$

$$e = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{8^{0.4}} \simeq 0.56$$

S

P4: Determine the work per stroke if $\Delta T = 600°K$ during the ignition step

$$W = Q_{in} - Q_{out} = Q_{in}\left(1 - \frac{Q_{out}}{Q_{in}}\right)$$

$$W = Q_{in}\, e \qquad e = 0.56$$

From the ignition step

$$dU = dQ - \cancel{dW}^{\,0}$$

$$n C_v\, dT = dQ$$

So $\quad \Delta Q_{in} = n C_v \Delta T$

$$W = n C_v \Delta T\, e$$

Now

$$P = \underbrace{W}_{\text{work per cycle}} \cdot f$$

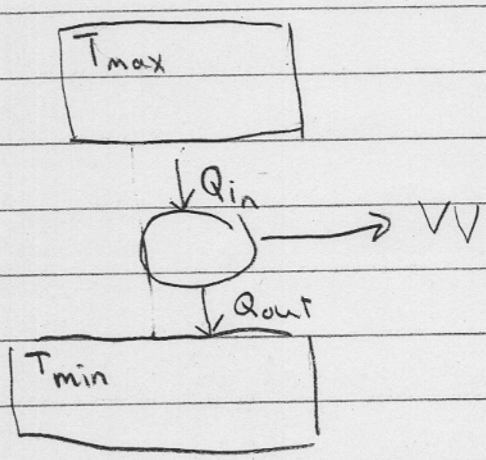
f ← cycles per sec usually quoted in rpm ⇔ rot per min

P ← power usually quoted in hp = 745 Watts

$$P = n C_v \Delta T\, e\, f = 98\,hp \cdot \left(\frac{\Delta T}{600°K}\right)\left(\frac{f}{5000\,RPM}\right)$$

← Toyota corrolla 150 hp at 4000 rpm $\qquad V = 2.4L$

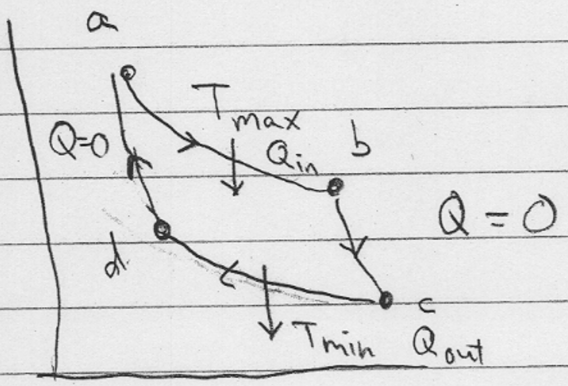
Carnot



① Clausius: not all Q_{in} can be converted to W.

② Need a temp difference
A Lorge temp difference makes for an efficient engine

③ Most efficient enigine:
- all heat in exchanged at T_{max}
- all heat out exchanget at T_{min}



$$e_{carnot} = 1 - \frac{T_{min}}{T_{max}}$$

maximum possible for an engine opperating between T_{min} and T_{max}

Then first Consider CD and consider an ideal gas

why

$$\cancel{\Delta U} = Q - W$$

$$Q_{in} = W = \int_{V_i}^{V_f} p\,dV \qquad p = \frac{nRT}{V}$$

$$Q_{in} = nRT_{min} \log \frac{V_C}{V_D}$$

Similarly

$$Q_{ab} = nRT_{max} \log \frac{V_B}{V_A} = -nRT_{max} \log \frac{V_a}{V_b}$$

$$Q_{out} = -Q_{ab} = nRT_{mx} \log \frac{V_A}{V_B}$$

Then we note in bc

$$T_{max} V_b^{\gamma-1} = T_{min} V_c^{\gamma-1} \Rightarrow \frac{T_{min}}{T_{max}} = \left(\frac{V_C}{V_B}\right)^{\gamma-1}$$

$$T_{max} V_a^{\gamma-1} = T_{min} V_d^{\gamma-1} \Rightarrow \frac{T_{min}}{T_{max}} = \left(\frac{V_D}{V_A}\right)^{\gamma-1}$$

So

$$\frac{V_C}{V_B} = \frac{V_D}{V_A} \Rightarrow \frac{V_A}{V_B} = \frac{V_c}{V_D}$$

So

$$\frac{Q_{in}}{Q_{out}} = \frac{nRT_{max} \log (V_c/V_d)}{nRT_{min} \log (V_a/V_b)} = \frac{T_{max}}{T_{min}}$$

So

$$e_{carnot} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{T_{min}}{T_{max}}$$

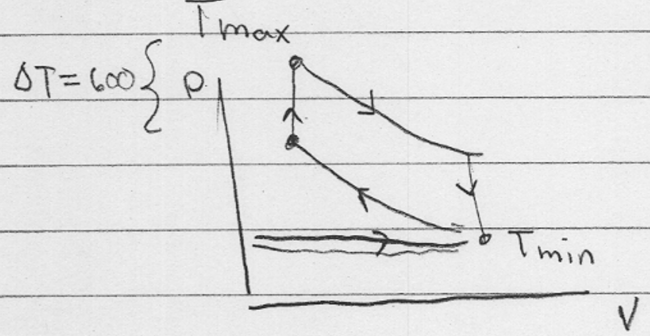
$$\boxed{e_{carnot} = 1 - \frac{T_{min}}{T_{max}}}$$

Carnot Efficiency and the Otto Cycle

$$e_{carnot} = 1 - \frac{T_{min}}{T_{max}}$$

maximum possible efficiency between T_{max} and T_{m}

In Otto-Cycle



We had:

$$T_{max} = 689^\circ K + 600^\circ K$$

$$T_{min} = 300^\circ K$$

$$e_{carnot} = 1 - \frac{300^\circ}{1289^\circ} = 0.76$$

$$e_{actual} = 0.56 < e_{carnot}$$