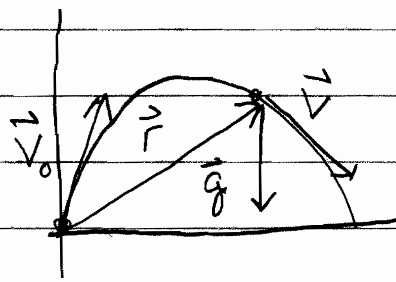
Last Time

Projectile motion



Lots of vectors !

← Illustrate by throwing basketball

$\vec{r}$ = position vector

$\vec{V}$ = velocity vector

$\vec{g}$ = acceleration vector

$\vec{V}_0$ = initial velocity vector

One equation

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{g}t^2 \qquad \vec{g} = \begin{pmatrix} 0 \\ -g \end{pmatrix} \qquad g = 9.8\,m/s$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} v_{0x} \\ v_{0y} \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -g \end{pmatrix} t^2$$

Becomes : Also had

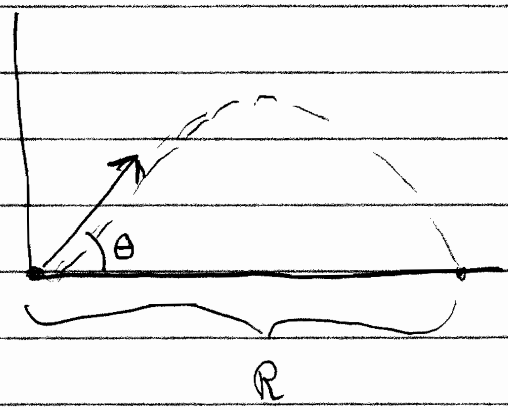
$$x = x_0 + v_{0x} t \qquad V_x = \text{const} = V_{0x}$$

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2 \qquad V_y = V_{0y} - gt$$

- In the x-direction, the x-velocity is constant

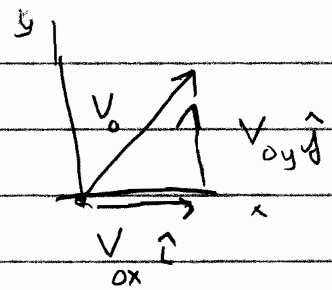
Example:

Level Horizontal Range:



You fire an rocket with speed of mach 1.5
what is the maximum dist it travels? How var will it travel (R) as function of V_0, θ, + g

Step 1: Choose a coordinate system and resolve vectors into components



V_0 = mach 1.5 = 510 m/s

$V_{ox} = V_0 \cos\theta$

$V_{oy} = V_0 \sin\theta$

θ will be determined by maximizing R

The equations of motion:

$$\vec{r} = \cancelto{0}{\vec{r}_0} + \vec{v}_0 t + \frac{1}{2}\vec{g}t^2$$

we start at origin

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} v_0 \cos\theta \\ v_0 \sin\theta \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -g \end{pmatrix} t^2$$

$$x = v_0 \cos\theta \, t$$

$$y = v_0 \sin\theta \, t - \frac{1}{2} g t^2$$

When does it hit the ground?

$$y(t) = v_0 \sin\theta \, t - \frac{1}{2} g t^2 = 0$$

$$y(t) = t \left(v_0 \sin\theta - \frac{1}{2} g t \right) = 0$$

$$t = \frac{2 v_0 \sin\theta}{g}$$

How far does it travel

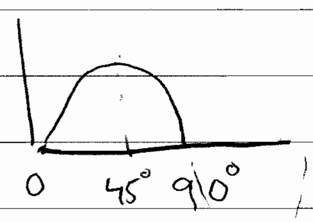
$$R = x = v_0 \cos\theta \, t$$

$$R = x = v_0 \cos\theta \left(\frac{2 v_0 \sin\theta}{g} \right) = \frac{2 v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin 2\theta}{g} = R$$

Note:

$$\sin(2\theta) = 2 \sin\theta \cos\theta$$

$\frac{R}{v_0^2/g}$



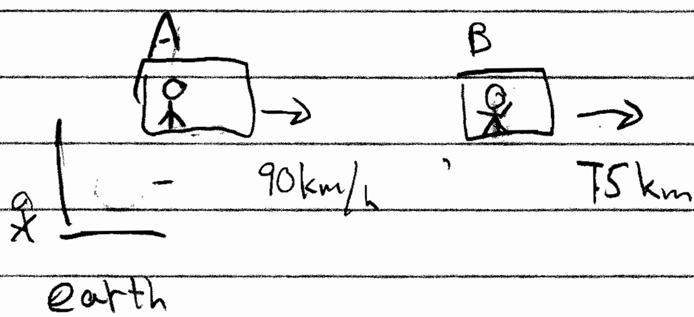
So distance is maximized when $\theta = 45^\circ$

$$R_{max} = v_0^2/g = (510\,m/s)^2 / 9.8\,m/s^2 \approx 27\,km \simeq$$

About the dist. from Stony brook to Port Jeff

Addition of velocity or Relative Velocity

→ Everything around us is in constant motion.



$V_{A|E} = 90\,km/h$ ← velocity of A relative to earth observer

$V_{B|E} = 75\,km/h$ ← velocity of B relative to earth observer

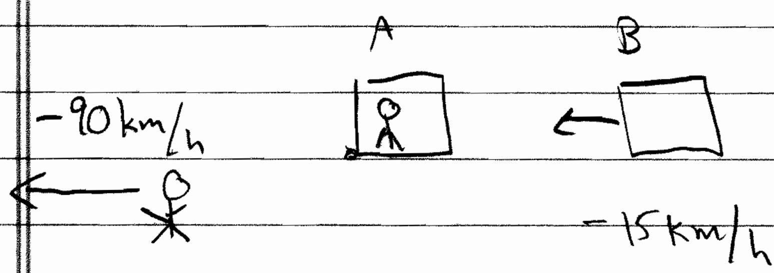
→ Normally leave off the "/E" business but its alway there

Then

$$V_{B|A} = \underbrace{75\,km/h}_{V_{B|E}} - \underbrace{90\,km/h}_{V_{A|E}} = -15\,km/h$$

- A sees B moving to left
- $V_{E|A} = -90\,km/h$

So Picture "A" sees:



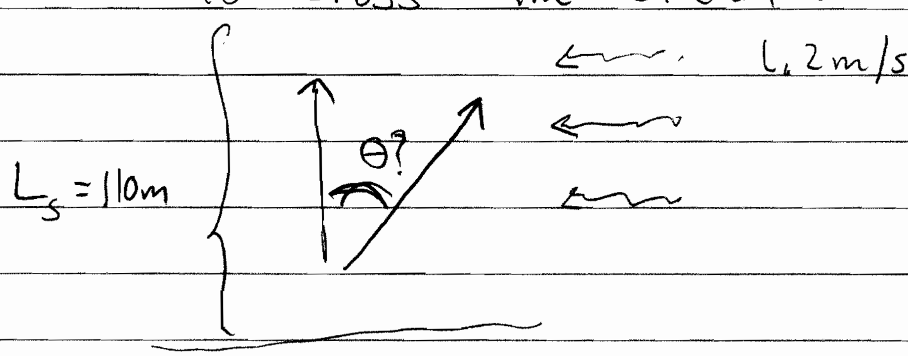
General:

$$\vec{v}_{A|E} = -\vec{v}_{E|A}$$

$$\vec{v}_{A|B} = \vec{v}_{A|E} - \vec{v}_{B|E}$$

Ex 1. If a boat is in still water it travells at a speed of $v_0 = 1.85\, m/s$.

(a) If he now wants to travel directly across the stream what angle should he point. The stream speed is $v_s = 1.2\, m/s$. (b) How long will it take to cross the stream?



Solution

- $\vec{V}_{Bt|W}$ = Velocity of boat relative to water

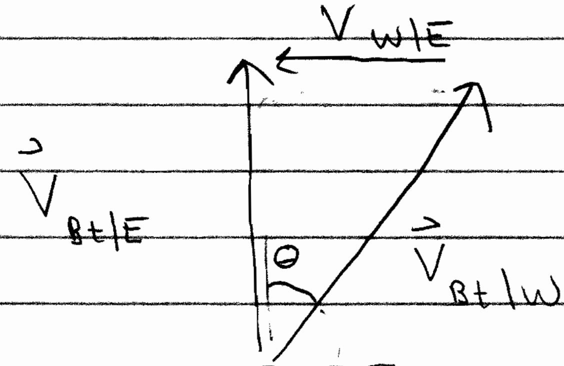
- $\vec{V}_{W|E}$ = Velocity of water relative to earth

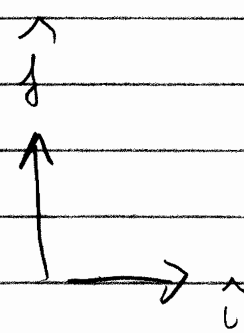
- $\vec{V}_{Bt|E}$ = velocity of Boat relative to earth

$$\vec{V}_{Bt|W} = \vec{V}_{Bt|E} - \vec{V}_{W|E}$$

$$\vec{U}_{Bt|W} + \vec{V}_{W|E} = \vec{V}_{Bt|E}$$

- Picture

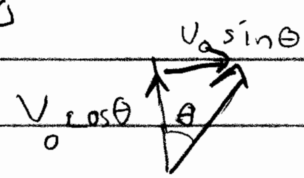




Math

$$\vec{V}_{W|E} = -V_s\,\hat{i} \qquad V_s = 1.2m/s$$

$$\vec{V}_{Bt|W} = V_0 \sin\theta\,\hat{i} + V_0 \cos\theta\,\hat{j} \qquad V_0 = 1.2m/s$$



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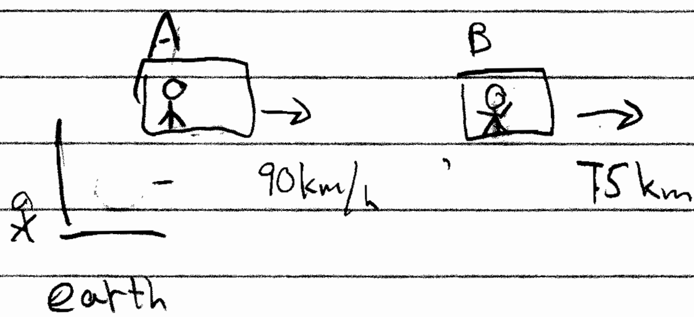

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- A sees B moving to left
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Since

$$\vec{V}_{Bt|E} = \vec{V}_{W|E} + \vec{V}_{Bt|E}$$

$$= (-v_s \hat{\imath}) + (v_0 \sin\theta \,\hat{\imath} + v_0 \cos\theta \,\hat{\jmath})$$

$$\vec{V}_{Bt|E} = (-v_s + v_0 \sin\theta)\,\hat{\imath} + v_0 \cos\theta \,\hat{\jmath}$$

So if we want to go directly across the stream we need the $\hat{\imath}$ component of $\vec{V}_{Bt|E}$ to be zero

$$-v_s + v_0 \sin\theta = 0$$

$$\sin\theta = v_s / v_0 = 1.2\,m/s \,/\, 1.8\,m/s$$

$$\theta \approx 42^\circ$$

Now

$$\vec{V}_{Bt|E} = v_0 \cos\theta \,\hat{\jmath}$$

$$= (1.8\,m/s)\cos(42^\circ)\,\hat{\jmath}$$

$$= 1.34\,m/s\,\hat{\jmath}$$

So the time it takes to cross is:

$$d = v_{Bt|E}\, t \Rightarrow t = \frac{d}{V_{Bt|E}} = \frac{110m}{1.34m/s} = 82\,s \approx 1\tfrac{1}{2}\,min$$