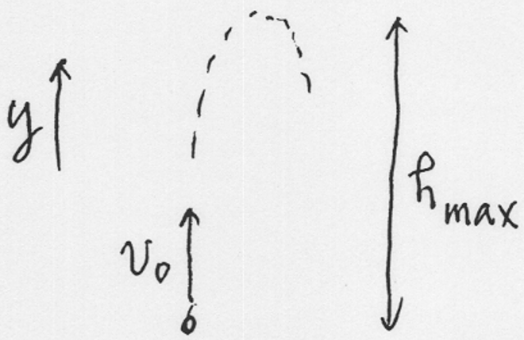
(Lecture)

a throwing ball upward: with friction



if no air friction

$$h_{max} = \frac{v_0^2}{2g}$$

no friction
$V_y = V_0 - gt \Rightarrow t = \frac{V_0}{g}$
$h = V_0 t - \frac{gt^2}{2} = \frac{V_0^2}{2g}$

suppose f = friction force $= \alpha v^2$

α ← cst

find $h_{max} = ?$

Newton: laws of physics are diff. equs!

$$F = ma \qquad -mg - \alpha v^2 = ma = m\frac{dv}{dt}$$

$$k \equiv \frac{\alpha}{m} \qquad -g - kv^2 = \frac{dv}{dt} \quad \text{-----} \quad (1)$$

? how to solve ?

either $dt = \frac{-dv}{+g+kv^2} \rightarrow t = -\int_{v_0}^{v} \frac{dv'}{g+kv'^2}$

but $v = \frac{dy}{dt}$; so $dt = dy/v$

$$v = \frac{dy}{dt} = \frac{dy}{dv}\frac{dv}{dt}$$

$$\rightarrow dy = \frac{v\,dv}{\frac{dv}{dt}} \quad \text{using (1)} \rightarrow \quad dy = \frac{v\,dv}{-g-kv^2} \qquad (2)$$

(2) is easy:

$$\int_0^{h_{max}} dy = \int_{v_0}^{0} \frac{v\,dv}{-g-kv^2} = -\frac{1}{2k}\int_{v_0}^{0} \frac{d(kv^2)}{g+kv^2}$$

$$= -\frac{1}{2k}\ln(g+kv^2)\Big|_{v_0}^{0}$$

$$\therefore h_{max} = -\frac{1}{2k}\left[\ln g - \ln(g + kv_0^2)\right]$$

$$= -\frac{1}{2k}\ln\frac{g+kv_0^2}{g} \qquad (4)$$

naively $\ln\frac{g}{g} = 0$
but $\frac{1}{k}$ so looks $\frac{0}{0}$

when no fric. force $k \to 0$

above should reduce to $\frac{v_0^2}{2g}$?

(4) $\rightarrow$ $h_{max} = -\frac{1}{2k}\ln\left(1 + \frac{kv_0^2}{g}\right)$

(using $\ln(1-x) = -(x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots)$

$$\xrightarrow[k\to 0]{} -\frac{1}{2k}\left(-\frac{kv_0^2}{g}\right) = \frac{v_0^2}{2g}$$ | no, $x = -\frac{kV}{g}$

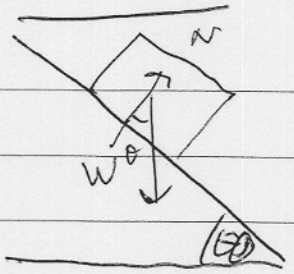
OK ?

let me keep the next term

$$-\frac{1}{2k}\left(-\frac{kV_0^2}{g} - \frac{1}{2}\left(\frac{kV_0^2}{g}\right)^2\right) = +\frac{V_0^2}{g} - \underbrace{\frac{kV_0^4}{4g^2}}_{\text{now we get correcti}} + \ldots$$

Lecture ~~Friction~~ Friction$\boxed{\text{less}}$

motion with Inclined Planes



$$\begin{cases} -W + N\cos\theta = m a_y \\ N\sin\theta = m a_x \end{cases}$$

3 unknown

+ constr.

constraint $y = -x\tan\theta$; $\ddot{y} = -\ddot{x}\tan\theta$

$a_y = -a_x \tan\theta$

$$a_x = N\frac{\sin\theta}{m} \;;\quad a_y = -N\frac{\sin\theta}{m}\frac{\sin\theta}{\cos\theta} = -N\frac{\sin^2\theta}{\cos\theta}$$

$$-mg + N\cos\theta = -N\frac{\sin^2\theta}{\cos\theta}$$

$$N\left(\frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta}\right) = mg$$

$N = mg\cos\theta$ | less than weight

zero if $\theta = \pi/2$

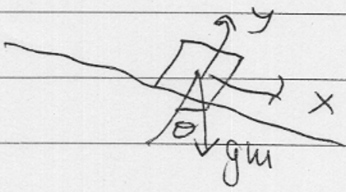
$a_x = g\sin\theta\cos\theta$ zero if $\theta = 0$ or 90°

$a_y = -g\sin^2\theta$ at $\theta = 0$ zero

at $\theta = 90^\circ$ it is $-g$

yes

The "turning head" solution



$gm\sin\theta = m a_x$

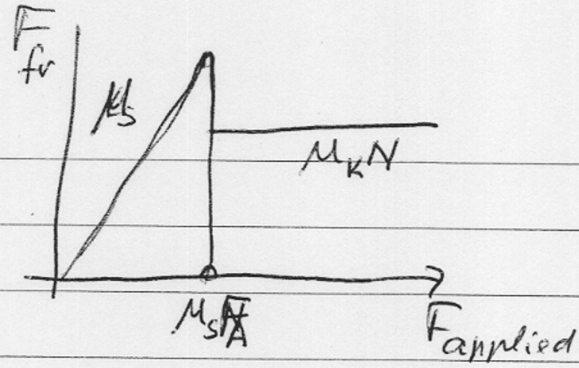
$gm\cos\theta - N = \cancel{m a_y}$ now $y =$ const

$\ddot{y} = 0$

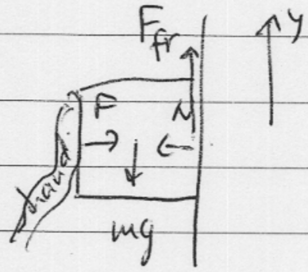
thus they decouple $N = mg\cos\theta$

$a_x = g\sin\theta$

Now with friction



A box against a wall



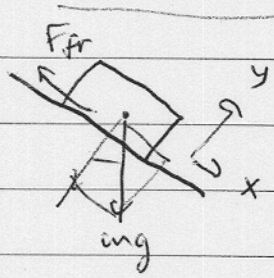
$\mu_s F = F_{Fr} > mg$ condition of no slipping

if it is less, acceleration

$$\mu_k F - mg = ma$$

$$y = \frac{at^2}{2} = \left(-g + \frac{\mu_k F}{m}\right)\frac{t^2}{2} < 0$$

↑ pushing makes falling slower



$$N = mg\cos\theta$$

$$-\mu_k N + mg\sin\theta = ma_x$$

$$-\mu_k \cos\theta + \sin\theta = \frac{a_x}{g}$$ should be > 0 !

critical angle $\mu_k = tg\,\theta$ if friction is larger, no sliding

more accurately, $\boxed{tg\,\theta = \mu_s}$