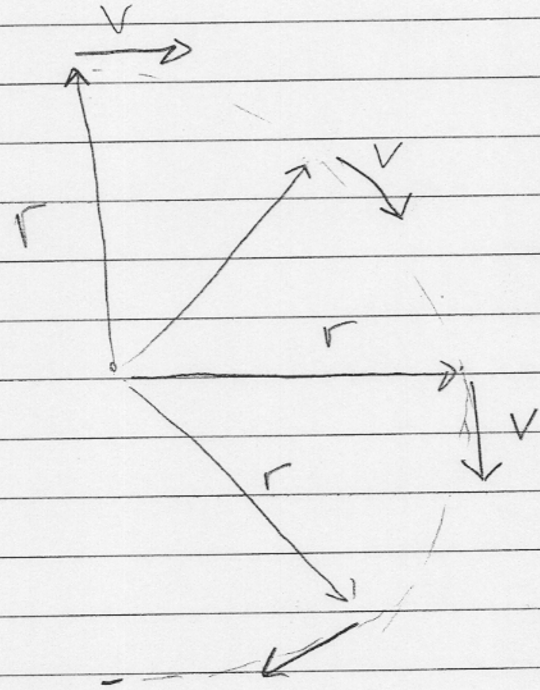
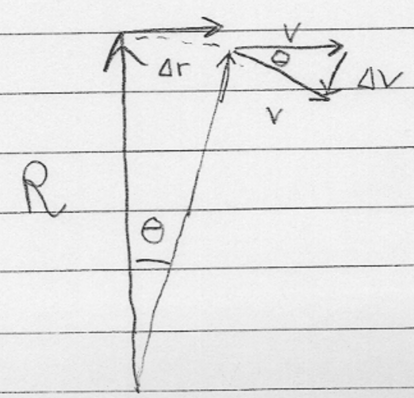
Uniform Circular motion (constant velocity)

- Speed, $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$, is constont
- The radius $|\vec{R}| = R$ is constanb
- but $\vec{v}$, and $\vec{R}$ are changing directs all the time



$$\vec{a} = \frac{d\vec{v}}{dt} \neq 0$$

Phsical Analysis



$$\frac{\Delta V}{V} = \frac{\Delta r}{R}$$

$$\Delta V = \frac{V}{R} \Delta r$$

The magnitude of accel.

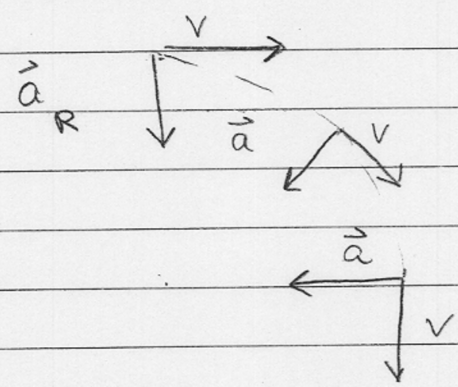
$$a = \frac{\Delta V}{\Delta t} = \frac{V}{R} \overbrace{\frac{\Delta r}{\Delta t}}^{V} = \boxed{\frac{V^2}{R} = a}$$

The direction of the radial acceleration is in

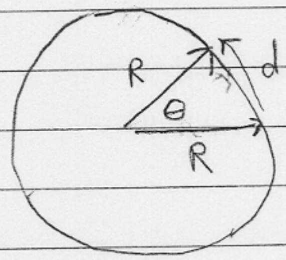
$$\vec{a}_R = \frac{v^2}{r} \times (\text{unit vector pointing toward center of circ}$$

$$\vec{a}_R = \frac{v^2}{r} \cdot (-\hat{R})$$

Picture



Mathematical Derivation:



The ball is going around in a circle at a constant rate

$$V = \frac{2\pi r}{T}$$

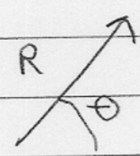
T = time to make one revolution
period

f = frequency = $\frac{1}{T}$ = # of revs per sec

$$\theta = \frac{d}{R} = \frac{V \cdot t}{R} = \frac{2\pi \cancel{R} \cdot t}{\cancel{R}\, T}$$

$\theta = \frac{2\pi t}{T}$ note: for $t = T$ $\theta = 2\pi$ ✓

So

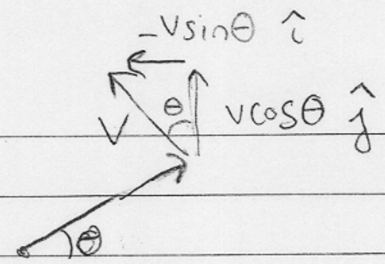


$$\vec{R} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$

$$\vec{R} = R\cos\frac{2\pi t}{T}\,\hat{i} + R\sin\frac{2\pi t}{T}\,\hat{j}$$

$$\frac{d\vec{R}}{dt} = -R\sin\frac{2\pi t}{T}\cdot\frac{2\pi}{T}\,\hat{i} + R\cos\frac{2\pi t}{T}\cdot\frac{2\pi}{T}\,\hat{j}$$

$$= -\frac{2\pi R}{T}\sin\theta\,\hat{i} + \frac{R\,2\pi}{T}\cos\theta\,\hat{j}$$



$$\vec{v} = \frac{d\vec{R}}{dt} = -v\sin\theta\ \hat{i} + v\cos\theta\ \hat{j}$$

$$|\vec{v}| = \left(v^2\sin^2\theta + v^2\cos^2\theta\right)^{1/2} = v \quad ✓$$

Now

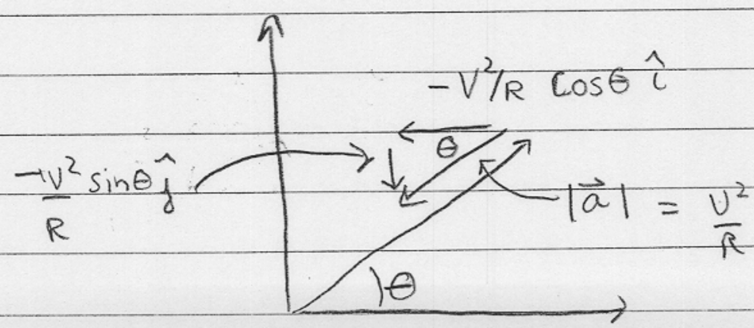
$$\vec{v} = v\left(-\sin\frac{2\pi t}{T}\right)\hat{i} + v\left(\cos\frac{2\pi t}{T}\right)\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = v\left(-\cos\frac{2\pi t}{T}\right)\cdot\frac{2\pi}{T}\ \hat{i} + \frac{v}{R}\left(-\sin\frac{2\pi t}{T}\right)\cdot\frac{2\pi}{T}\ \hat{j}$$

Multiply & Divide By R

$$\vec{a} = -\frac{v}{R}\overbrace{\frac{2\pi R}{T}}^{v}\left(+\cos\theta\ \hat{i} + (+\sin\theta\ \hat{j})\right)$$

$$\vec{a} = -\frac{v^2}{R}\left(\cos\theta\ \hat{i} + \sin\theta\ \hat{j}\right)$$

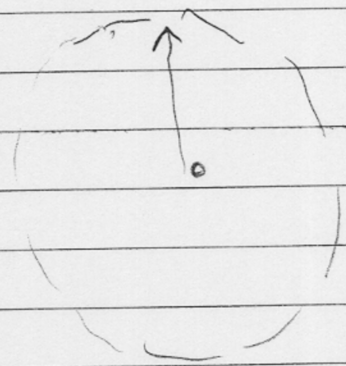


Newtons Laws and Circular Motion

- Whenever there is acceleration there must be force
- In the absence of forces objects move in a straight line (demonstrate)

Prob

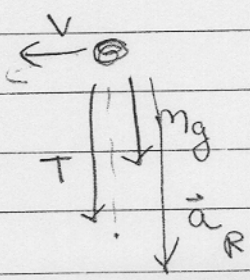
A ball is connected to a string and whirled in a vert. circ. Estimate the tension in the rope at the top



Solution:

$$v \sim \frac{2\pi R}{T} \sim \frac{2\pi \, 1/4\,m}{0.33\,s} \sim 5\,m/s \qquad m \sim 20\,g$$

Analyze Newtons laws at top



$$-T - mg = -m\frac{v^2}{R}$$

$$T = \frac{mv^2}{R} - mg \sim 18.4\,N$$

As the velocity becomes less and less, the tension becomes less and less

The minimum velocity before the ball falls can be found by setting $T=0$

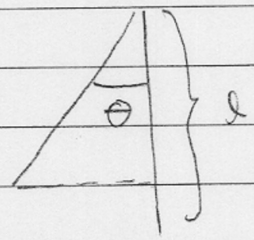
$$0 = mg - \frac{mv^2_{min}}{R}$$

$$v_{min} = \sqrt{gR}$$

$$v_{min} = (9.8\,m/s^2 \; 0.25\,m)^{1/2} \approx 1.5\,m/s$$

Problem

A ball is circulating in a conical Pendulum



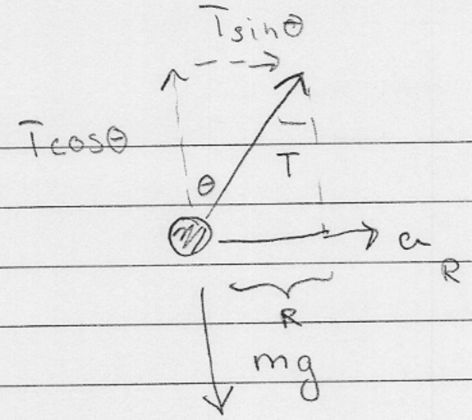
- Determine the frequency of rotation as a function of l and θ

Observations based on class: → larger the angle the larger the frequenc
→ smaller l faster frequenc

Solution ① Draw w/ free body diagram w/ all possible forces in all possible direction

② analyze constraints

③ work /enthusiasm



① $x\rfloor$

$x\rfloor$ $\quad T\sin\theta = m a_R = m v_0^2/R_\perp$

$y\rfloor$ $\quad +T\cos\theta - mg = m\cancel{a_y}$

② We have analyzed constraints a_y, $a_R = \frac{v^2}{R}$

③ Work

$$T = \frac{mg}{\cos\theta} \Rightarrow \cancel{m}g\frac{\sin\theta}{\cos\theta} = \frac{\cancel{m}v^2}{R_\perp}$$

$$g\frac{\sin\theta}{\cos\theta} = \frac{v^2}{l\sin\theta}$$

$$\sqrt{\frac{gl}{\cos\theta}}\sin\theta = v$$

Now $\quad f = \frac{1}{T} = \frac{1}{\frac{2\pi R_\perp}{v}} = \frac{v}{2\pi R_\perp} = \sqrt{\frac{gl}{\cos\theta}}\frac{\sin\theta}{l\sin\theta}$

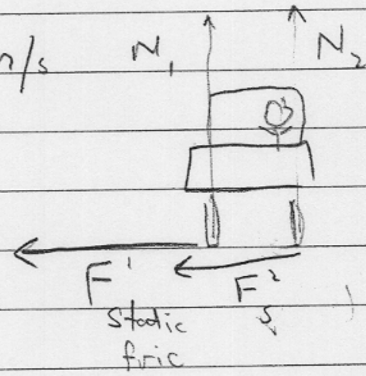
$f \simeq \sqrt{\frac{g}{l\cos\theta}} \Rightarrow$ Note; ① when l smaller higher freq.
② when θ larger, $\cos\theta$ smaller, and frequency higher

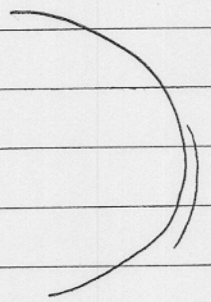
Banked Roads and highways.

- You wish to divert I95 by 90°, estimate the minimum Radius of curvature needed to accomplish this

Sol; $m_{car} \sim 2000\,kg$

$U_{car} \sim 60\,mph \sim 25\,m/s$





Typical $\mu_s \sim 0.4$

Normal force from each tire

$$4N - mg = m\cancel{a_y}$$

$$N = mg/4$$

Four tires

$$F_s^{max} = m\frac{U^2}{R}$$

$$4 \cdot \mu_s N = \frac{mU^2}{R}$$

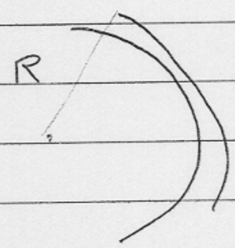
$$\cancel{4}\,\mu_s \frac{\cancel{m}g}{\cancel{4}} = \frac{\cancel{m}U^2}{R}$$

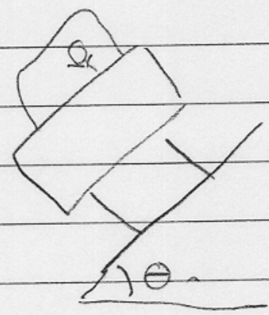
$$R = \frac{U^2}{g}\frac{1}{\mu_s} \simeq 159\,m$$

A typical exit ramp:

1. Estimate the recommended speed $\sim 30\text{mph} \sim 13\text{m/s}$
2. Estimate the typical radius of curvature ~ 30,

Now typically the road is banked



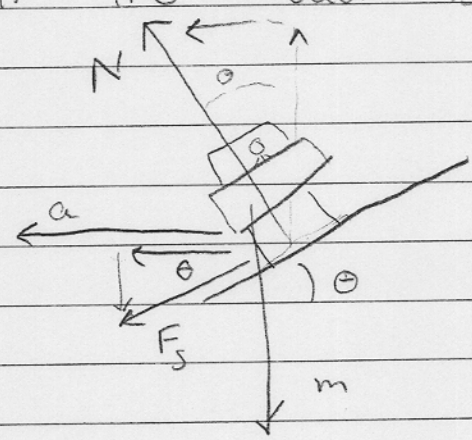


- Estimate the minimum μ_s which will hold the car on the road in the absence of banking.

$$\mu_s^0 = \frac{v^2}{gR} \simeq 0.58 = \frac{(13\text{m/s})^2}{(9.8\text{m/s}^2)(30\text{m})}$$

- Determine the μ_s which will hold the car on the road for a banking angle of 15°

Now if the road is banked



① x|

$$-N\sin\theta - \mu_s N\cos\theta = -m\overbrace{\frac{v^2}{R}}^{a_x}$$

y|

$$N\cos\theta - mg - \mu_s N\sin\theta = m\cancelto{0}{a_y}$$

② Constraints $a_y = 0$ $a_x = v^2/R$

③ Count:

Unknowns : N, μ_s ✓

Equations: 2

Work

$$N(\cos\theta - \mu_s\sin\theta) = mg$$

$$N = mg/(\cos\theta - \mu_s\sin\theta)$$

$$N\,(\sin\theta + \mu_s\cos\theta) = m\frac{v^2}{R}$$

$$\cancel{m}g\,\frac{(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \cancel{m}\frac{v^2}{R}$$

$$\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = \frac{v^2}{Rg}$$

Note

$$\left\{\begin{array}{l} c \equiv \cos\theta \\ s \equiv \sin\theta \end{array}\right.$$

$$s + \mu_s c = \frac{v^2}{Rg}(c - \mu_s s)$$

$$-s + \frac{v^2}{Rg}c = +\mu_s c + \mu_s s\frac{v^2}{Rg}$$

$$\frac{\frac{v^2}{Rg}c - s}{c + s\frac{v^2}{Rg}} = \mu_s$$

$$\boxed{\frac{\frac{v^2}{Rg} - \tan\theta}{1 + v^2/Rg\tan\theta} = \mu_s}$$

$v = 13\,m/s$ ~ 30mph

$\theta \simeq 15^\circ$

$R = 30m$

$v^2/gR = 0.58$

$$0.27 \simeq \mu_s$$

wet conditions