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A mass  $m$  is dropped from a height  $h$  onto a platform resting on top of a spring at equilibrium. We say that  $t_o$  is when the mass comes into first contact with the platform. After a change in time ( $t$ ) of 0.148s the platform passes the new equilibrium point a distance ( $d$ ) of 19.6cm from its initial position at  $t_o$ . Find a function for the position  $y$  as a function of time, the height ( $h$ ) it was released from above the platform, and its initial velocity ( $v_o$ ) at time  $t_o$ .

$$mg - kx = m \frac{d^2x}{dt^2} \quad (1)$$

$$x = \frac{mg}{k} + y \quad (2)$$

Plugging equation (2) into equation (1) we get:

$$mg - k\left(\frac{mg}{k} + y\right) = m \frac{d^2x}{dt^2} \quad (3)$$

After some more work we come up with a equation for the position as a function of time:

$$y(t) = y_o \cos(\omega_o t) + \frac{v_o}{\omega_o} \sin(\omega_o t) \quad (4)$$

Some things to keep in mind:

$$y_o = d \quad (5)$$

$$\omega_o = \sqrt{\frac{k}{m}} \quad (6)$$

$$k = \frac{ma}{d} \quad (7)$$

$$v_o = \sqrt{2gh} \quad (8)$$

$$y(t) = y(0.148s) = 0 \quad (9)$$

$$y(t_o) = y(0) = 19.6cm = 0.196m \quad (10)$$

Working with these equations we can get a value for  $\omega_o$

$$\omega_o = \sqrt{\frac{mg}{dm}} = \sqrt{\frac{g}{d}} = \sqrt{\frac{9.8m/s^2}{0.196m}} = \sqrt{50} s^{-1}$$

Solving equation (4) for  $v_o$  we get

$$v_o = -\frac{dw_o \cos(w_o t)}{\sin(w_o t)}$$

Plugging in our values for  $w_o$ ,  $t$ , and  $d$  we get

$$v_o = -\frac{(0.196m)(\sqrt{50}s^{-1})\cos(\sqrt{50}s^{-1})(0.148s)}{\sin(\sqrt{50}s^{-1})(0.148s)} = -\frac{0.69378m/s}{0.86569}$$

$$v_o = -0.80142m/s$$

Using equation (8) we can solve for  $h$  and get

$$h = \frac{v_o^2}{2g} = \frac{(-0.80142m/s)^2}{2(9.8m/s^2)}$$

$$h = 0.032769m$$