Recitation Problem October 22, 2009

A mass \( m \) is dropped from a height \( h \) onto a platform resting on top of a spring at equilibrium. We say that \( t_0 \) is when the mass comes into first contact with the platform. After a change in time \( t \) of 0.148s the platform passes the new equilibrium point a distance \( d \) of 19.6cm from its initial position at \( t_0 \). Find a function for the position \( y \) as a function of time, the height \( h \) it was released from above the platform, and its initial velocity \( (v_o) \) at time \( t_0 \).

\[
mg - kx = m\frac{d^2x}{dt^2} \tag{1}
\]

\[
x = \frac{mg}{k} + y \tag{2}
\]

Plugging equation (2) into equation (1) we get:

\[
mg - k\left(\frac{mg}{k} + y\right) = m\frac{d^2x}{dt^2} \tag{3}
\]

After some more work we come up with an equation for the position as a function of time:

\[
y(t) = y_0\cos(w_o t) + \frac{v_o}{w_o}\sin(w_o t) \tag{4}
\]

Some things to keep in mind:

\[
y_o = d \tag{5}
\]

\[
w_o = \sqrt{\frac{k}{m}} \tag{6}
\]

\[
k = \frac{ma}{d} \tag{7}
\]

\[
v_o = \sqrt{2gh} \tag{8}
\]

\[
y(t) = y(0.148s) = 0 \tag{9}
\]

\[
y(t_o) = y(0) = 19.6cm = 0.196m \tag{10}
\]

Working with these equations we can get a value for \( w_o \)

\[
w_o = \sqrt{\frac{mg}{dm}} = \sqrt{\frac{9.8m/s^2}{0.196m}} = \sqrt{50}s^{-1}
\]
Solving equation (4) for $v_o$ we get

$$v_o = -\frac{dw_o \cos(w_o t)}{\sin(w_o t)}$$

Plugging in our values for $w_o$, $t$, and $d$ we get

$$v_o = -\frac{(0.196 m)(\sqrt{50 s^{-1}}) \cos(\sqrt{50 s^{-1}})(0.148 s)}{\sin(\sqrt{50 s^{-1}})(0.148 s)} = \frac{0.69378 m/s}{0.86569}$$

$$v_o = -0.80142 m/s$$

Using equation (8) we can solve for $h$ and get

$$h = \frac{v_o^2}{2g} = \frac{(-0.80142 m/s)^2}{2(9.8 m/s^2)}$$

$$h = 0.032769 m$$