

1 Ideal Gasses and Equipartition.

1. For any ideal gasses we have the equation of state

$$PV = Nk_B T \quad (1)$$

Here P is the pressure, V is the volume (for a fixed particle number N), T is the temperature in Kelvin and k_B is the Boltzmann constant

$$k_B = 1.38 \times 10^{-23} \frac{J}{^\circ K} \quad (2)$$

2. A microscopic analysis shows that the pressure is related to the average translational kinetic energy of the molecules (see below)

$$PV = \frac{2}{3} N \times \overline{\frac{1}{2} m v^2} \quad (3)$$

3. Usually instead of expressing the total number of particles with N , we express it in moles n , which counts particles in units of Avagadros number 6.02×10^{23}

$$N = nN_A \quad (4)$$

Then we have the following facts about moles and Avagadros number:

- (a) Then the ideal gas constant is defined as

$$R \equiv N_A k_B \quad R = 8.3 \frac{J}{^\circ K} \quad (5)$$

So that the ideal gas equation of state is

$$PV = nRT \quad (6)$$

- (b) The mass of a n moles of a substance is

$$M = n\mathcal{M} \quad (7)$$

where \mathcal{M} is the molar mass, i.e. the mass of the one avagadros number of the molecule in question.

- (c) The molar mass is easy to estimate. The weight of an avagadros number of protons is approximately 1 gram, i.e. $\mathcal{M} = 1g$. The mass of a neutron is approximately the mass of a proton $m_n \approx m_p$. The mass of an electron is negligible $m_e/m_p \approx 1/2000$. Thus oxygen which has 8 protons and 8 neutrons and 8 electrons the molar mass is approximately $\mathcal{M} = 16g$.
4. The energy per particle of a general (i.e. non-ideal) gas is a function (which can be measured by measuring specific heats) of temperature and density $U/N = f(N/V, T)$. But the energy of an ideal gas (where there is no interaction between the molecules) is a function of temperature only

$$U/N = f(T) \quad (8)$$

5. The equipartition theorem states:

The average energy of each degree of freedom is $\frac{1}{2}k_B T$

- (a) The degrees of freedom of a mono-atomic are simply the motion in x, y, z

$$\overline{\frac{1}{2} m v_x^2} = \overline{\frac{1}{2} m v_y^2} = \overline{\frac{1}{2} m v_z^2} = \frac{1}{2} k_B T \quad (9)$$

- (b) The degrees of freedom for a diatomic gas are simply motion in x, y, z and rotation around the two axes x, y

$$\overline{\frac{1}{2}mv_x^2} = \overline{\frac{1}{2}mv_y^2} = \overline{\frac{1}{2}mv_z^2} = \overline{\frac{1}{2}I\omega_x^2} = \overline{\frac{1}{2}I\omega_y^2} = \frac{1}{2}k_B T \quad (10)$$

6. From the equipartition theorem we conclude that for a mono-atomic or a diatomic gas we have

$$\overline{\frac{1}{2}mv^2} = \frac{3}{2}k_B T \quad (11)$$

i.e. the root-mean square velocity is

$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_B T}{m}} \quad (12)$$

This is typically of order of the speed of sound $v_{\text{rms}} \sim 400$ m/s at room temp.

7. The total energy of an ideal gas is simply the kinetic energies of all the molecules.

- (a) (Mono-atomic ideal gas only) For a monoatomic gas this is simply the translational kinetic energy $\frac{1}{2}mv^2$ per molecule

$$U = \frac{3}{2}Nk_B T = \frac{3}{2}nRT \quad (13)$$

- (b) (Diatomic ideal gas only) For a diatomic gas the kinetic energy per molecule is translation + rotational, $\frac{1}{2}mv^2 + \frac{1}{2}I\omega_x^2 + \frac{1}{2}I\omega_y^2$.

$$U = \frac{5}{2}Nk_B T = \frac{5}{2}nRT \quad (14)$$

2 Heat and Specific Heats:

1. Heat is the transfer of microscopic forms of energy from one system to another. When heat is added to a system which is not changing phase the temperature rises. Normally one would add heat at constant pressure and the temperature rises as

$$dQ_p = Mc_p dT \quad (15)$$

Here the subscript p means at constant pressure, and c_p is the specific heat per unit mass. The specific heat per mole is indicated with a capital C_p

$$dQ_p = nC_p dT \quad (16)$$

You should be able to show $C_p = Mc_p$.

2. When working problems on calorimetry, since temperature differences are the same in Kelvin and Celsius $\Delta T = \Delta t$. One can often use Celsius when writing $\Delta Q = nC_p \Delta t$
3. When heat is added this sometimes causes the material to change phase (e.g. ice melts and water vaporizes) rather than to rise in temperature. The amount of heat required to change the phase of a substance of mass M is

$$Q = ML \quad (17)$$

where L is the heat of sublimation (melting) or the heat of vaporization, etc.

4. Sometimes heat is added while keeping the volume of the gas constant

$$dQ_v = nC_v dT \quad dQ_v = Mc_v dT \quad (18)$$

with $C_v = Mc_v$. These specific heats are not the same as c_p and C_p .

5. (1G only) For an ideal gas an analysis of the first law of thermodynamics, together with Eq. ??, show that

$$C_p = C_v + R \quad (19)$$

One also defines the adiabatic index, which is useful for an adiabatic expansion discussed below

$$\gamma = \frac{C_p}{C_v} \quad (20)$$

6. For an ideal gas you should be able to show using the first law of thermodynamics (see below) that:

- (a) (MIG only) For a mono-atomic ideal gas we have

$$C_v = \frac{3}{2}R \quad C_p = \frac{5}{2}R \quad \gamma = \frac{5}{3} = 1.66 \quad (21)$$

- (b) (DIG only) For a diatomic ideal gas we have

$$C_v = \frac{5}{2}R \quad C_p = \frac{7}{2}R \quad \gamma = \frac{7}{5} = 1.4 \quad (22)$$

3 Work and the First Law

1. During an expansion of a substance (e.g. a gas), a certain amount of macroscopic work is done as the substance increases its volume

$$dW = p dV \quad \text{or} \quad W = \int_{V_i}^{V_f} p dV \quad (23)$$

Generally we draw the pressure vs. volume, and the area under this curve is the work done by the gas. A negative work means that we did positive work on the gas rather than the gas doing positive work on us.

2. During an expansion, a certain amount of heat flows into the gas and a certain amount of macroscopic work is done, energy of the gas changes accordingly

$$dU = dQ - p dV \quad \text{or} \quad \Delta U = Q - W \quad (24)$$

which is the first law of thermodynamics.

3. For processes occurring at constant volume you should show that

$$dU = nC_V dT \quad (\text{any subst., const vol only}) \quad (25)$$

For an ideal gas where the energy is independent of the volume, one has for an arbitrary expansion

$$dU = nC_V dT \quad (\text{Ideal gas, arbitrary expansion only}) \quad (26)$$

4. We considered various types of thermodynamic processes:

- (a) An *isothermal expansion* is an expansion at constant temperature. For an ideal gas, you should be able to show that

$$W = nRT \log \frac{V_f}{V_i} \quad (27)$$

For an ideal gas, we have $\Delta U = 0$ (why?), that the heat which enters the system is $Q = W$

- (b) In a *isobaric expansion* pressure is constant and the work done is

$$W = p(V_f - V_i) \quad (28)$$

For an ideal mono-atomic gas, or a diatomic gas, you should be able to calculate the change in temperature, energy etc.

(c) For a heat exchnage at *constant volume* we have $\Delta V = 0$ and therefore

$$W = 0 \quad (29)$$

So $\Delta U = Q$.

(d) For an *adiabatic expansion*, the heat which flows into the system is zero, $Q = 0$.

i. (IG only) Then for an ideal gas we have

$$PV^\gamma = C \quad \text{where } C \text{ is a constant} \quad (30)$$

Similarly from this you should be able to show that

$$TV^{\gamma-1} = C' \quad \text{where } C' \text{ is a different const} \quad (31)$$

ii. (IG only) You should be able to show that the work in this case

$$W_{12} = \frac{P_1 V_1}{\gamma - 1} \left[1 - \left(\frac{V_1}{V_2} \right)^{\gamma-1} \right] \quad (32)$$

$$= \frac{P_1 V_1}{\gamma - 1} - \frac{P_2 V_2}{\gamma - 1} \quad (33)$$

iii. (IG only) In the adiabatic expansion $U = -W$, for an ideal gas one has

$$\Delta U = nC_v(T_2 - T_1) \quad (34)$$

You should be able to show the equivalence of Eq. 32,Eq. 33, and Eq. 34.

4 Engines

1. An engine is a cyclical machine which takes a certain amount of heat in Q_{in} at high temperature and expels a certain amount of heat Q_{out} at low temperature performing useful work W in the process. The net heat input (what we called Q above) is $Q = Q_{\text{in}} - Q_{\text{out}}$
2. The otto cycle is an idealized model of an internal combustion engine. In an engine the pistons fire, this turns the crankshaft. As the crankshaft turns, it turns the camshaft (or shafts) which is connected by the timing belt. The camshaft opens and closes the valves at the right time to let gas/air in and exhaust out. Look at the web site for some videos of this.
3. In a complete cycle $\Delta U = 0$ (or $W = Q$) because the energy only depends on the state variables $T, V/N$ which are the same at the start and end of the cycle.
4. The efficiency of the engine is the work per intake heat

$$\epsilon = \frac{W}{Q_{\text{in}}} = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} \quad (35)$$

5. In an engine heat is taken a hottest point T_{max} and a coldest point T_{min} . The maximum possible efficiency of an engine acting between these two extremes is carnot efficiency.

$$\epsilon_{\text{max}} = 1 - \frac{T_{\text{min}}}{T_{\text{max}}} \quad (36)$$

5 Sound

1. Sound is a pressure wave. One often is interested in the (time-averaged) intensity of the wave where the intensity is the energy per area per time.

$$\bar{I} = \frac{\Delta E}{A\Delta t} \quad (37)$$

2. The intensity is how loud the sound is. This is measured with the decibel scale

$$\bar{I} = I_0 \times 10^{\beta/10} \quad I_0 = 1 \times 10^{-12} \frac{W}{m^2} \quad (38)$$

and β is the sound level in dB. This can be written

$$\beta = 10 \log_{10} \left(\frac{I}{I_0} \right) \quad (39)$$

I_0 is inaudible, 20 dB is normal, 120 dB is the threshold of pain. Every 10 dB corresponds to a factor of ten. So an increase of 30 dB is an increase of a 1000 times more intensity.

3. The (time averaged) intensity of the pressure wave can be related to the (time average) energy density \bar{u}_E of the sinusoidal wave, and the sound velocity v_s

$$\bar{I} = \bar{u}_E v_s. \quad (40)$$

Can you derive this last formula. The energy density (and intensity) can be related to the maximum amplitude of the sinusoidal wave ΔP

$$\bar{u}_E = \frac{(\Delta P)^2}{2v_s^2 \rho} \quad (41)$$

Here ΔP is the maximum of the sinusoidal pressure wave, $\rho = 1.29 \text{ kg/m}^3$ is the mass density of air, and $v_s = 343 \text{ m/s}$ is the speed of sound.

4. In a spherical wave the source emits with total power \mathcal{P} in all directions. The intensity of the wave decreases as

$$\bar{I} = \frac{\mathcal{P}}{4\pi r^2} \quad (42)$$

5. When two sinusoidal waves are added with different frequencies f_1 and f_2 we have the following identity which is responsible for beats

$$A \sin(2\pi f_1 t) + A \sin(2\pi f_2 t) = 2A \sin(2\pi \bar{f} t) \cos(2\pi \frac{\Delta f}{2} t) \quad (43)$$

You should be able to show this by writing f_1 and f_2 in terms of average and difference: $f_1 = \bar{f} + \Delta f/2$ and $f_2 = \bar{f} - \Delta f/2$ where $\bar{f} = (f_1 + f_2)/2$ is the average and $\Delta f = f_1 - f_2$ is the difference.