Scales to remember

1. Electron and proton mass

$$m_e c^2 = 0.511\,\text{MeV} \qquad m_p c^2 = 938.\,\text{MeV}$$

2. Speed of sound ($\sim$ typical molecular velocity) and light

$$\text{sound} \sim 300\,\text{m/s} \qquad c = 3 \times 10^8\,\text{m/s}$$

3. Typical wavelength of visible light.

$$\lambda_{\text{red}} \sim 600\,\text{nm} \qquad \lambda_{\text{blue}} \sim 400\,\text{nm}$$

4. Typical wavelength of X-rays. Energy $\sim 50\,\text{kilo} - \text{Volts}$
5. Conversion factor $hc = 1240\,\text{eV nm}$
6. Atomic Size $\sim 1\,$Å. $1\,\mathring{A} = 0.1\,\text{nm}$
7. Compton wavelength $\frac{h}{m_e c} \sim 0.0024\,\text{nm}$

Basic Classical Relativity

1. An observer measures coordinates of events

$$t, x, y, z$$

Another observer moving with velocity v in the x direction sees a different set of coordinates. These coordinates are related the first set by a Gallilean transformation

$$\begin{aligned} t' &= t && (1) \\ x' &= x - vt && (2) \\ y' &= y && (3) \\ z' &= z && (4) \\ & && (5) \end{aligned}$$

2. The first observer measures the velocity of and object to be (u_x, u_y, u_z) where u_x is the x component of the velocity, etc. Another observer moving with velocity v in the x direction relative to the first observer measures a different velocity (u'_x, u'_y, u'_z) which is related to (u_x, u_y, u_z) by the transformation

$$\begin{aligned} u'_x &= u_x - v && (6) \\ u'_y &= u_y && (7) \\ u'_z &= u_z && (8) \end{aligned}$$

3. The two observers measure the same forces and the same accelerations.
4. For an object moving with constant velocity u the equation of motion according to one observer (on earth say) is

$$\Delta x = u\Delta t \qquad \text{or} \qquad x = x_o + u(t - t_o) \tag{9}$$

where x_o is the position at time t_o. For an observer moving with velocity v in the positive x-direction relative to the first observer the equation of motion is the same

$$x' = x'_o + u'(t' - t'_o) \tag{10}$$

Here, for example $x'_o = x_o - vt$ is the relation between x'_o (the starting position as measured by Jupiter) and x_o (the starting position measured by earth.

Basic Special Relativity

1. The speed of light is constant in all reference frames
2. For an observer moving with velocity v relative to a "lab" we use two symbols alot

$$\gamma \equiv \frac{1}{\sqrt{1-(v/c)^2}} \qquad\qquad \beta \equiv \frac{v}{c}$$

3. Moving clocks are time dilated. A time interval $\Delta\tau$ in the rest frame of the clock is measured to be

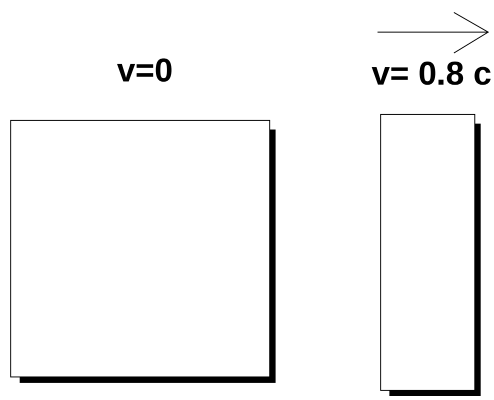
$$\Delta t = \gamma\, \Delta\tau \tag{11}$$

according to an observer moving relative to the clock.

4. An observer moving relative to a ruler stick with rest length L_p will see it length contracted by a factor γ.

$$L = L_p/\gamma \tag{12}$$

The transverse directions are not affected by the motion, i.e. a square becomes a rectangle according to an observer moving quickly with respect to the square.



Mathematical Relations and Taylor Series

1. The Taylor series for any function around a point x_o is

$$f(x) = f(x_o) + f'(x_o)\,\Delta x + \frac{1}{2!} f''(x_o)\,\Delta x^2 + \ldots \tag{13}$$

with $\Delta x = (x - x_o)$.

2. The following Taylor series comes up frequently in relativity

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}\,x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \ldots \tag{14}$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots \tag{15}$$

3. In relativity the following Taylor series comes up a lot for small v (with respect to c)

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \approx 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 \tag{16}$$

Lorentz Transformations

1. An event at space time point (t, x, y, z) will appear at a different space time point (t', x', y', z') according to an observer moving with velocity v in the positive x direction.

$$
\begin{aligned}
ct' &= \gamma\,(ct) - \gamma\beta\,x \qquad (17)\\
x' &= -\gamma\beta\,(c\,t) + \gamma\,x \\
y' &= y \\
z' &= z
\end{aligned}
$$

For an observer moving in the negative x direction the same formula holds with the replacement $\beta \to -\beta$

2. For a given observer in a fixed frame the normal rules of classical physics apply. Thus as in classical physics the postion of an object moving with constant speed u versus time is given by

$$\Delta x = u\Delta t \qquad \text{or} \qquad x = x_o + u(t - t_o) \qquad (18)$$

where x_o is the position at time t_o. For an observer moving with velocity v in the positive x-direction relative to the first observer the equation of motion is the same

$$x' = x'_o + u'(t' - t'_o) \qquad (19)$$

But the coordinates (x', ct') and (x'_o, ct'_o) are related by lorentz transformations to (x, ct) and (x_o, ct_o). The velocity $u^{'}$ is related by the velocity u transformation rule in Eq. (21)

3. The Lorentz transformation leaves the following unchanged

$$(\Delta x')^2 - (c\Delta t')^2 = (\Delta x)^2 - (c\Delta t)^2 \qquad (20)$$

Other results

1. A space ship moves with velocity $\mathbf{u} = (u_x, u_y, u_z)$ in the "lab" frame. According to an observer moving with speed v in the positive x direction relative to the lab, the spaceship moves with a different velocity $\mathbf{u}'$. $\mathbf{u}'$ is related to $\mathbf{u}$ by

$$
\begin{aligned}
u'_x &= \frac{u_x - v}{1 - u_x v/c^2} \qquad (21)\\
u'_y &= \frac{u_y}{\gamma(1 - u_x v/c^2)} \qquad (22)\\
u'_z &= \frac{u_z}{\gamma(1 - u_y v/c^2)} \qquad (23)
\end{aligned}
$$

If the observer is moving to the left then the same formula applies with the substitution $v \to -v$. If the space-ship is moving to the left then u_x is negative in this formula.

2. A source emits light waves with frequency f_o. According to an observer moving directly toward the source with speed v the source has a frequency which is blue shifted

$$f = f_o\sqrt{\frac{1 + v/c}{1 - v/c}} \qquad (24)$$

For an observer moving away from the source make the replacement $v \to -v$.

Dynamics

1. The momentum of a particle moving with velocity $\mathbf{u}$ is

$$\mathbf{p} \equiv \gamma m\mathbf{u} \qquad \text{with} \qquad \gamma \equiv \frac{1}{\sqrt{1 - u^2/c^2}} \qquad (25)$$

For a massless particle $u = c$ and $E = c|\mathbf{p}|$.

2. The energy of a particle moving with velocity $\mathbf{u}$ is

$$E \equiv \gamma mc^2 \qquad \text{with} \qquad \gamma \equiv \frac{1}{\sqrt{1 - u^2/c^2}} \tag{26}$$

3. The rest energy is the energy of a particle when it is not moving, and the kinetic energy is the energy minus the rest energy

$$E_{\text{rest}} = mc^2 \qquad K = \gamma mc^2 - mc^2 = E - mc^2 \tag{27}$$

4. The energy and momentum and velocity given above are related by the formulas

$$E^2 = (cp)^2 + (mc^2)^2 \qquad \mathbf{u} = c^2 \frac{\mathbf{p}}{E} \tag{28}$$

   For a photon we have $E = cp$ and $u = c$ (mass is zero)

5. Total energy and momentum (the sum of the energy's and momenta of all the particles) are always conserved before and after the collision

$$E^{\text{before}}_{\text{tot}} = E^{\text{after}}_{\text{tot}} \qquad \mathbf{p}^{\text{before}}_{\text{tot}} = \mathbf{p}^{\text{after}}_{\text{tot}} \tag{29}$$

6. If the energy and momentum $(E, c\mathbf{p})$ of a particle according to one observer is known, then according to an observer moving to the right with speed v, the energy and momentum of the particle is $(E', c\,\mathbf{p}')$ with

$$E' = \gamma\, E - \gamma\beta\, (cp_x) \tag{30}$$
$$cp'_x = -\gamma\beta\, E + \gamma\, (cp_x) \tag{31}$$
$$cp'_y = cp_y \tag{32}$$
$$cp'_z = cp_z \tag{33}$$

   with $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ and $\beta = v/c$. Of course for a left moving observer we have $v \to -v$. We multiply c times momentum so that $c\,p$ has the same units of energy. The change of frames mixes energy and momenta.

Quantum Nature of Light

1. Light comes in discrete units called photons. The energy and momentum of the photon is related to the frequency (given symbol ν or sometimes f).

$$E = h\nu \qquad c\,p = E \qquad c = \lambda\nu$$

2. h is planck's constant and is

$$hc = 1240\, eV\, \text{nm}$$

3. The intensity of the light is the energy moving across a screen per area per time

$$I = \left\langle \frac{E}{A\Delta t} \right\rangle \tag{34}$$

   The brackets $\langle\rangle$ denote averages over a sufficiently long time or large area so that many photons are involved.

4. For monochromatic (one frequency) light this is

$$I = h\nu \left\langle \frac{N}{A\Delta t} \right\rangle \tag{35}$$

   $N/A\Delta t$ is the number of photons per area per time.

material	Work function(eV)	material	Work Function(eV)	material	Work Function(eV)
Ag	4.26	Al	4.28	As	4.79
Au	5.1	Ba	2.52	Bi	4.34
Ca	2.87	Co	4.97	Cr	4.44
Cs	1.95	Cu	4.65	Fe	4.6
Ga	4.35	Ge	5.15	In	4.08
K	2.3	Mn	4.08	Mo	4.49
Na	2.36	Ni	5.15	Pb	4.25
Pd	5.4	Pt	5.63	Rb	2.05
Ru	4.71	Sb	4.56	Si	4.95
Sn	4.28	Ta	4.3	Ti	4.33
U	4.33	W	4.55	Zn	3.63

TABLE I: Work function for various metals

5. If for a dector of area A and time resolution Δt the number of photons N crossing the detector is very large (compared to one) then classical electromagnetism is a good approximation and we have that the (average) intensity is the time average of the poynting vector

$$I = \left\langle \frac{\mathcal{E} \times \mathcal{B}}{\mu_0} \right\rangle \tag{36}$$

$$\tag{37}$$

For a plane wave, $\mathcal{E} = \mathcal{E}_o \cos(kx - \omega t)$ and then $\mathcal{B} = \mathcal{E}/c$

$$I = \sqrt{\frac{\epsilon_o}{\mu_o}} \left\langle \mathcal{E}^2 \right\rangle = \sqrt{\frac{\epsilon_o}{\mu_o}} \frac{1}{2} \mathcal{E}_o^2 \tag{38}$$

where $\mathcal{E}$ and $\mathcal{B}$ are the electric and magnetic fields respectively.

Photoelectric Effect and Compton Scattering

1. The photo-electric effect is when photons tear away an electron from a metal. The kinetic energy of the electron after it is ripped away is

$$K_e = h\nu - w_o \tag{39}$$

Here w_o is the work function of the metal given and is the energy required to tear the electron away from the atom. By appylying a retarding voltage V_o we can stop the flow of electrons from the metal. When the voltage is

$$|e|V_o = K_e \tag{40}$$

the electrons will not have enough energy to make it to the collector and current will stop.

2. X rays are produced by colliding electrons into a metal foil:

   - When then electrons decelerate they radiate many different frequencies. The total power P emitted by the electron is related to the decceleration a by the *Larmour* formula which is stated without proof

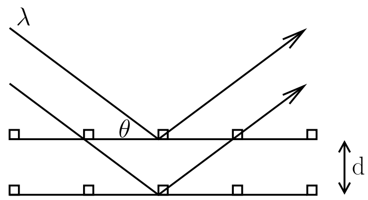
$$P = \frac{2}{3} \frac{e^2}{4\pi\epsilon_o} \frac{a^2}{c^3}$$

   - The shortest wavelength of light produced during the decceleration process is when all of the electrons kinetic energy gets converted into a single photon. If the electrons initial kinetic energy is K

$$K = \frac{hc}{\lambda_{\min}} \tag{41}$$

3. The wavelength of an X-ray may be determined with Bragg's crystal. The condition for constructive interference is

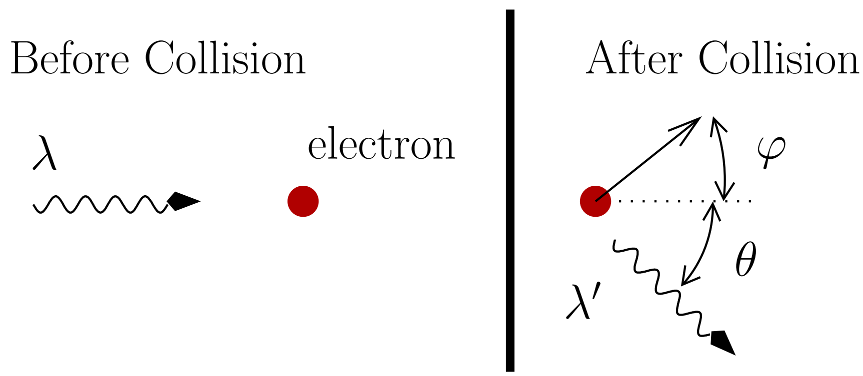
$$2d\sin(\theta) = n\lambda \qquad n = 1, 2, 3, \ldots \tag{42}$$



4. The compton process is the scattering of light (X-rays) on electrons nearly at rest. The following formula results (after algebra) from energy and momentum conservation and $E = hf$

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \tag{43}$$

To determine the electron momentum and direction one would write down relativistic energy and momentum conservation and use the above formula. energy and momentum conservation and use the above formula to determine



5. The compton wavelength is a measure of the size of the "fuzziness" of the electron

$$\lambda_C = \frac{h}{m_e c} = 0.0024\text{nm} \tag{44}$$