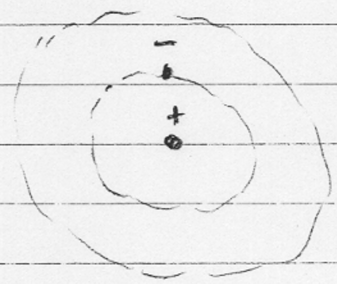
Last Time



- The n-th orbit is characterized by

$$L = n\hbar \qquad + \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r_n^2} = m_e \frac{v^2}{r_n}$$

- Found for $n = 1$

$\rightarrow \frac{v_1}{c} = \alpha \simeq \frac{1}{137}$ $\quad \left(\frac{v_n}{c} = \frac{\alpha}{n}\right)$

- Atoms move non-relativistically

$\rightarrow r_1 = a_0 \simeq 0.5\,A = \frac{\hbar}{m_e c\alpha}$ $\quad (r_n = n^2 a_0)$

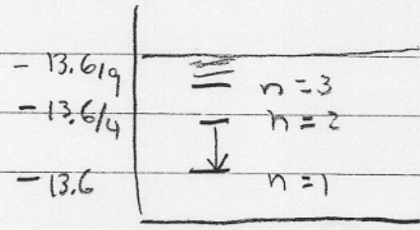
- The orbital radius is much greater than the electron size.

$$a_0 = \frac{\hbar}{m_e c\alpha} \approx 137 \times \left(\frac{\hbar}{m_e c}\right)$$

$\leftarrow$ Compton length

- Examined Energies

$$E_n = -\frac{13.6\ eV}{n^2}$$




Why negative

We also said this a consequence a balance between kinetic and potential energies

- $$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

multiply by r

$$-PE = 2KE$$

$$KE_1 = \frac{1}{2} m (\alpha c)^2 \quad , \quad PE_1 = -2KE = -m(\alpha c)^2$$

So: $$E^1_{TOT} = PE + KE = -KE = -\frac{1}{2} mc^2 \alpha^2 = -13.6\,eV$$

Then consider a $n=2 \rightarrow n=1$ transitio

$$E_i = E_f + h\nu$$

$$h\nu = E_f - E_i = -\frac{1}{2} mc^2 \alpha^2 \left[ -1 - \frac{-1}{4} \right] = \frac{3}{4} mc^2 \alpha^2$$

$\simeq$ 10.2eV deep UV ~ 121 nm

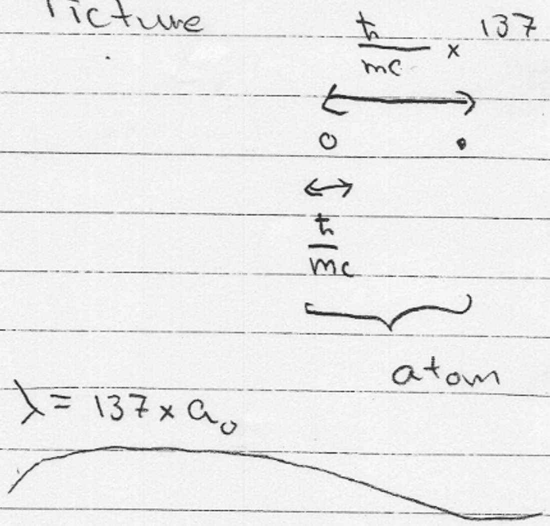
• This wavelength is much larger than the size of the atom

$$\lambda \sim \frac{hc}{E} \sim \frac{hc}{mc^2 \alpha^2}$$

$$\lambda \sim (137) \cdot \overbrace{\frac{h}{mc\alpha}}^{2\pi a_0} \qquad a$$

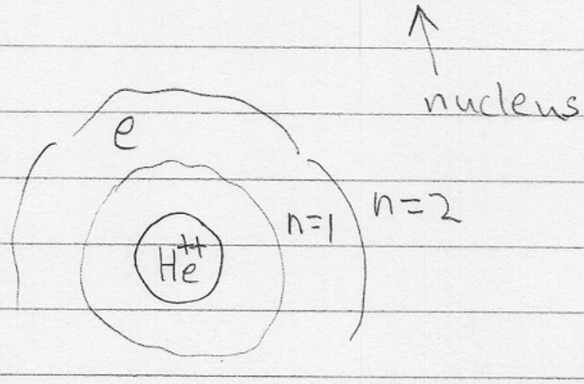
$$\lambda \sim 600\, a_0$$

Picture



# Start of New Material

Extension to $^{++}He$




$L = m_e v r = n\hbar$ ← The same

But coulomb Law is different

$$F = \frac{mv^2}{r}$$

$$\frac{Ze \cdot e}{4\pi\varepsilon_0}\frac{1}{r^2} = \frac{mv^2}{r} \qquad Z = 2$$

Then the formulas are all the same but with replacement $e^2 \rightarrow Ze^2$

$$r_n = n^2 \frac{a_0}{Z} \qquad E_n = -R_\infty \frac{Z^2}{n^2}$$

$$= -13.6\,\text{eV}\,\frac{Z^2}{n^2}$$

→ We will give a heuristic understanding of how Z goes today

De Broglie ~ 1920

- Bohr theory clearly inadequate
  - Want something like: given electron at time one where is electron at time #2
  - Only limitted success at describing multi-electron atoms, Just specifies $L_{TOT}$, not enough to describe each individual electron

Think about light:

$$E = h\frac{c}{\lambda} \qquad E = cp$$

$$\cancel{c}p = \frac{h\cancel{c}}{\lambda} \qquad p = \frac{h}{\lambda} \Rightarrow \boxed{\lambda = \frac{h}{p}}$$

Also remember the Bohr model: $\frac{v}{c} = \alpha \quad v = \alpha c$

$$p = m v = \hbar\underbrace{\left(\frac{m\alpha c}{\hbar}\right)}_{1/a_0} \cancel{m\left(\frac{ke^2}{\hbar c}\right)\cdot c = \hbar\underbrace{\left(\frac{mkc^2}{\hbar^2}\right)}_{1/a_0}}$$

$$p = \frac{\hbar}{a_0}$$

call it wave length

Thin

$$a_0 = \frac{\hbar}{p} \qquad \text{so} \qquad \overbrace{2\pi a_0}^{\lambda} = \frac{h}{p}$$

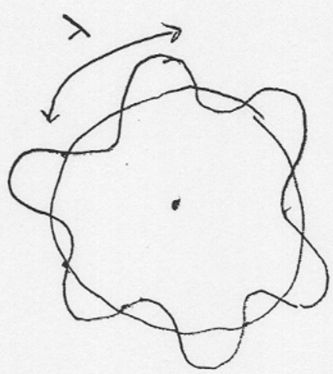
Then Bohr quantization condition

$$\overbrace{mv}\,r = n\,\frac{h}{2\pi}$$

$$2\pi \cdot \underbrace{p}\,r = n\,h$$

$$2\pi\,\frac{\cancel{h}}{\lambda}\,r = n\,\cancel{h} \qquad \boxed{2\pi r = n\lambda}$$

Picture




have to have complete wave ~~to~~ lengths going aroun the circle

$$\lambda = \frac{h}{p} \qquad p = \frac{h}{\lambda} = \frac{h}{2\pi}\,\frac{2\pi}{\lambda} = \boxed{\hbar k = p}$$

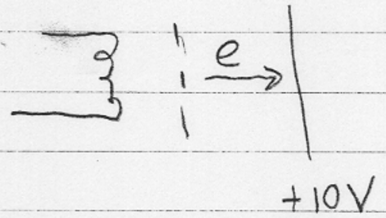
## Problem

- Calc the de Broglie wavelength of a baseball with mass 140g, $v \simeq 27\,m/s$

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\,J.s}{(0.14\,kg)(27\,m/s)} = 1.7\times10^{-34}\,m$$

absurdly short

- Calculate the de Broglie wavelength of a typical electron in the lab



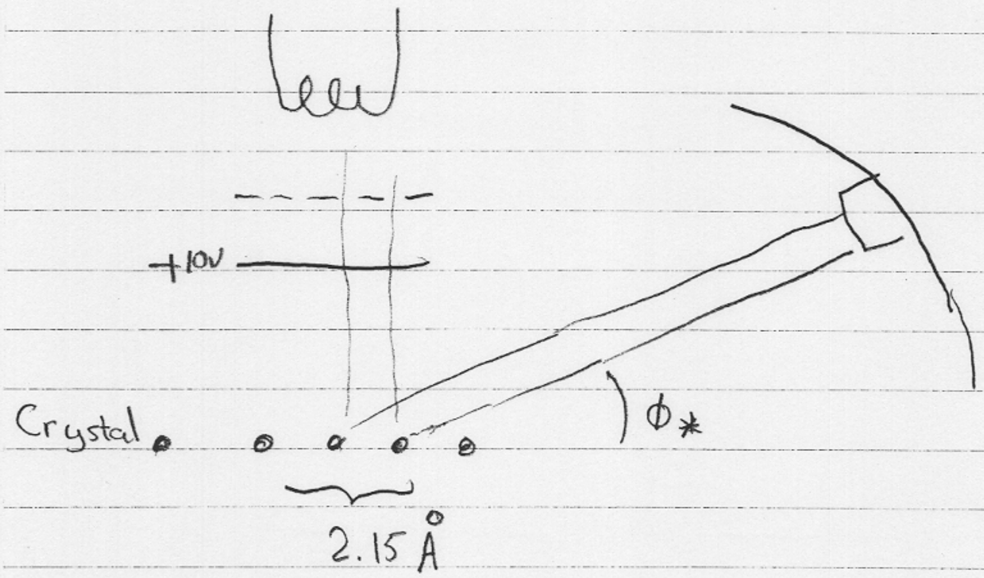

$K = 10eV$ ← Kinetic energy after passing through a 10V potentia

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{hc}{\sqrt{2m_ec^2 K}} \qquad K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK}$$

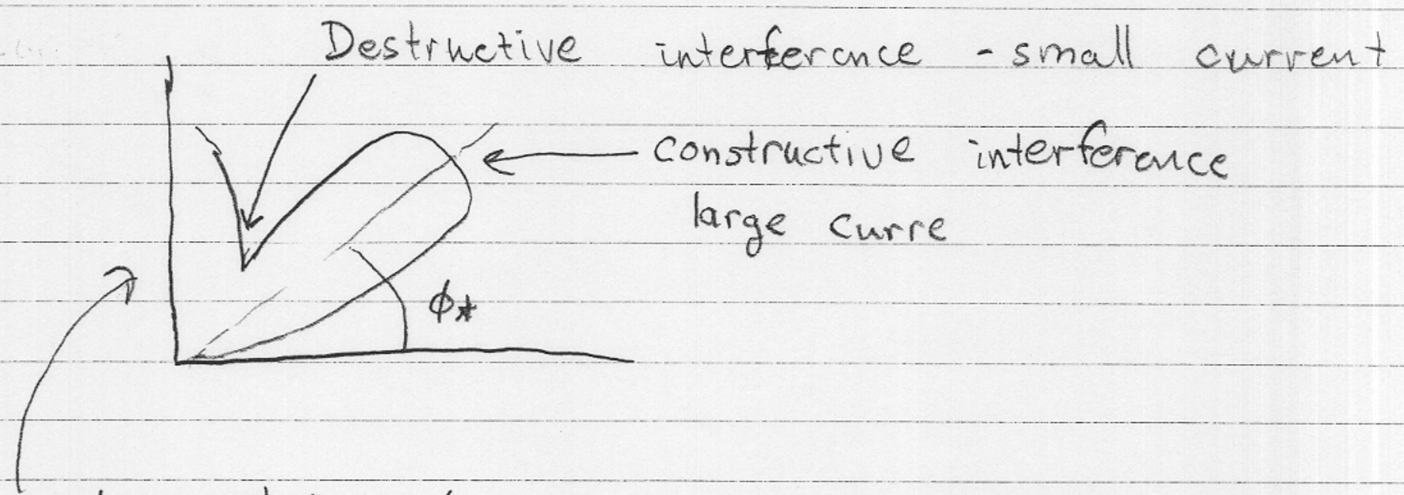
$$\lambda = \frac{1240\,eV\cdot nm}{\sqrt{2\cdot 511000_{eV}\cdot 10eV}} = 0.38\,nm \simeq 3.8\,\mathring{A} \sim 7\,a_0$$

Of order inter-atomic spacing

Experiments - (Davisson Gersemer)




The crystal serves as a diffraction grating. Find




polar Plot of current vs. angle

## The Uncertainty Principle (To Be Refined Later)

$$p = \frac{h}{\lambda}$$

Small sized objects are described by short wavelengths

$$\lambda \sim \Delta x$$

So expect:

$$\Delta p \sim \frac{\hbar}{\Delta x} \implies$$

Usually written

$$\Delta p \, \Delta x \sim \hbar$$

* Note this is an estimate not an equality. Means can drop factors of 2 and $\pi$ and such

# The uncertainty principle and the Bohr Radius

$$KE \sim \frac{1}{2}mv^2 \sim \frac{p^2}{2m} \sim \frac{1}{2m}\left(\frac{\hbar}{a_0}\right) \sim \boxed{\frac{\hbar^2}{2ma_0^2} = 13.6 eV}$$

$$PE = \frac{-e^2}{4\pi\varepsilon_0}\frac{1}{a_0} = -\left(\frac{e^2}{4\pi\varepsilon_0\hbar c}\right)\cdot\frac{\hbar c}{a_0} = \boxed{-\alpha\left(\frac{\hbar c}{a_0}\right) = PE = -27.2eV}$$

Important Edward!

The electron is pulled down to the center of the nucleus. It gets squished into a smaller and smaller size or larger KE. When $KE \sim |PE|$ the process stops

<u>Estimate:</u>

$$|PE| \sim KE$$

$$\alpha\frac{\hbar c}{a_0} \sim \frac{\hbar^2}{2ma_0^2}$$

$$a_0 \sim \frac{\hbar^{\cancel{2}}}{2m\cancel{\hbar}c\alpha}$$

$$a_0 \sim \frac{\hbar}{2mc\alpha}$$

<u>Exact:</u>

$$\frac{1}{2}|PE| = KE \qquad \overbrace{a_0 p}^{L} = \hbar$$

$$p = \frac{\hbar}{a_0}$$

$$\frac{1}{2}\alpha\left(\frac{\hbar c}{a_0}\right) = \frac{p^2}{2m}$$

$$\frac{1}{\cancel{2}}\alpha\left(\frac{\hbar c}{a_0}\right) = \frac{\hbar^2}{\cancel{2}ma_0^2}$$

$$a_0 = \frac{\hbar}{m_e c\alpha}$$

## Bohr model with Z protons

Estimate:

$$PE \sim \frac{Ze^2}{4\pi\varepsilon_0 r} \sim \frac{Ze^2}{4\pi\varepsilon_0 \hbar c} \frac{\hbar c}{r} = Z\alpha \frac{\hbar c}{r}$$

$$KE \sim \frac{p^2}{2m} \sim \frac{\hbar^2}{2mr^2}$$

- Energy Balance gives

$PE \sim KE$

$$Z\alpha \frac{\hbar c}{r} \sim \frac{\hbar^2}{2mr^2}$$

$$r \sim \frac{\hbar}{2mc(Z\alpha)}$$

Exact:

$$r \sim \frac{a_0}{2Z} \qquad r_n = n^2 \frac{a_0}{Z}$$

- Energies

① $PE \sim \frac{-Ze^2}{4\pi\varepsilon_0 r} \sim \frac{-Ze^2}{4\pi\varepsilon_0 \frac{a_0}{Z}} \sim -Z^2\left(\frac{e^2}{4\pi\varepsilon_0 a_0}\right)$

$PE \sim -Z^2 (27.2\text{eV})$

Exact = $PE = -Z^2 \overbrace{\left(\frac{e^2}{4\pi\varepsilon_0 a_0}\right)}^{27.2\text{eV}} \frac{1}{n^2}$

② $KE \sim \frac{\hbar^2}{2mr^2} \sim \frac{\hbar^2}{2m(a_0/Z)^2} \sim Z^2 \frac{\hbar^2}{(2ma_0)^2}$

Exact

$KE = +Z^2 \underbrace{\left(\frac{\hbar^2}{2ma_0^2}\right)}_{13.6\text{eV}} \frac{1}{n^2}$

③ Total

$E \sim PE \sim -KE \sim -Z^2 \frac{\hbar^2}{2ma_0^2}$

Exact: $E = -Z^2\left(\frac{\hbar^2}{2ma_0^2}\right)\frac{1}{n^2} = -Z^2\, 13.6\text{eV}/n^2$