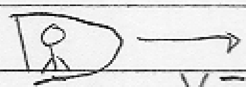
Last time

- Consider a clock with the time between tick and tock = 1s

tick - tock



$V = 4/5 c$ $\qquad \gamma = \frac{1}{\sqrt{1-(v/c)^2}} = 5/3 \qquad \beta = \frac{v}{c} = \frac{4}{5}$

- The coordinates of tock as measured by <u>ear th frame</u>

$$(ct, x) = (1\ cs, 0) \qquad 1cs = 3\times 10^8 \frac{m}{s} \cdot 1s = 3\times 10^8 m$$

- The space time coordinates of tock as measured by the <u>spaceship frame</u> are

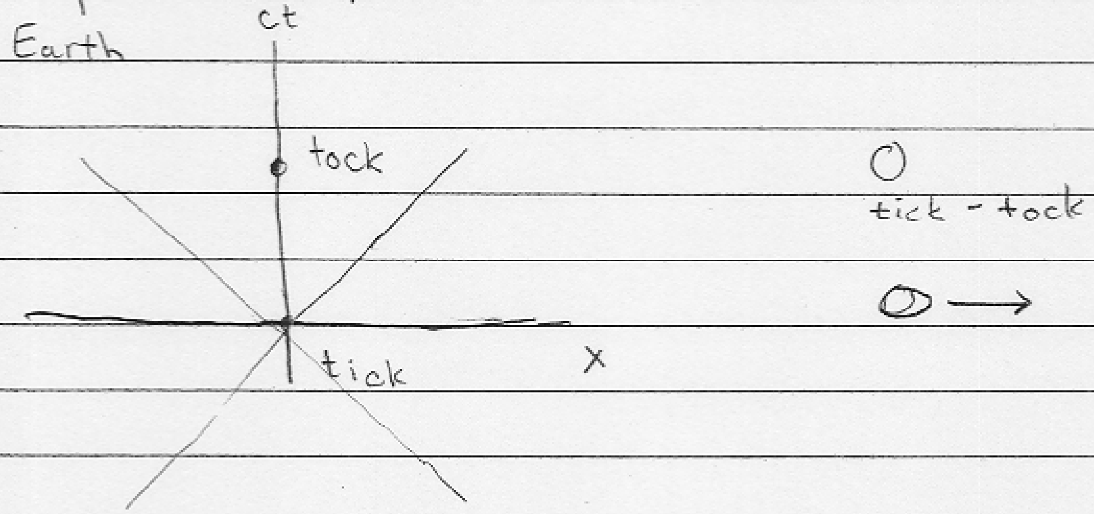
$$ct' = \gamma \cdot ct \mp \gamma\beta \cdot x$$
$$x' = \mp \gamma\beta \cdot ct + \gamma \cdot x$$

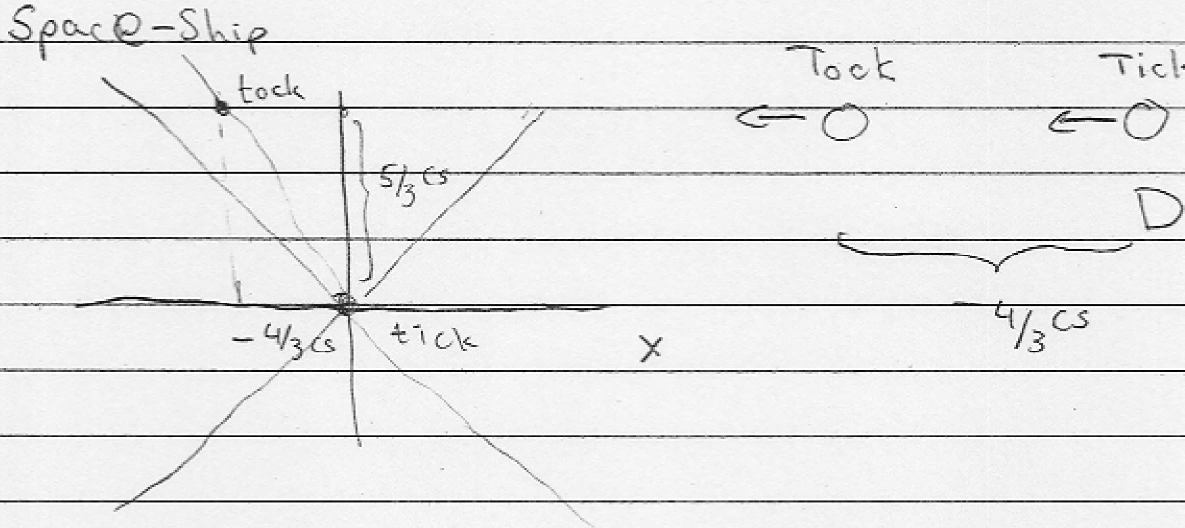
where $(-)$ is for a right moving space ship and $(+)$ is for a left moving space ship

$$ct' = \left(\frac{5}{3}\right)(1cs) - \gamma\beta \cdot (0) = \frac{5}{3} cs$$

$$x' = -\left(\frac{5}{3}\right) \cdot \frac{4}{5}(1cs) + \gamma(0) = -4/3\ cs$$

Space Time:



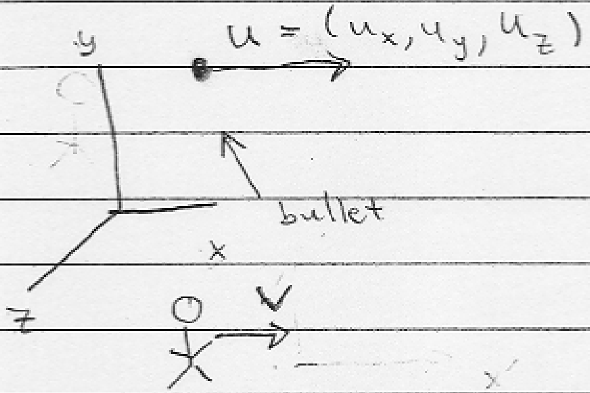


- So far had left out transverse directions
- if include y and z have

$$ct' = \gamma \cdot ct - \gamma\beta \cdot x$$
$$x' = -\gamma\beta \cdot ct + \gamma \cdot x$$
$$y' = y$$
$$z' = z$$

Addition of Velocities

Cartoon



$u =$ Is the bullet velocity as accordin to one observer

According to observer moving to right with speed V the speed is u'

Classically $\quad u'_x = u_x - V \quad u'_y = u_y \quad u'_z = u_z$

Relativistically:

$$u'_x = \frac{u_x - V}{1 - u_x V/c^2}$$

$$u'_y = \frac{u_y}{\gamma(1 - u_x V/c^2)}$$

$$u'_z = \frac{u_z}{\gamma(1 - u_x V/c^2)}$$

Proof: (x coords only)

Suppose a ship moves with speed u_x to the right

$$u_x = \frac{\Delta x}{\Delta t}$$

Then:

$$u_x' = \frac{\Delta x'}{\Delta t'} \qquad \cancel{c}\Delta t' = \gamma \cancel{c}\Delta t - \gamma\beta\, \Delta x / c$$
$$\Delta x' = -\gamma\beta c \Delta t + \gamma \Delta x$$

So

$$u_x' = \frac{\Delta x'}{\Delta t'} = \frac{-\overbrace{\gamma\beta c}^{V} \Delta t + \gamma \Delta x}{\gamma \Delta t - \gamma \frac{\beta}{c} \Delta x} \qquad \beta = V/c$$

$$= \frac{-V \Delta t + \Delta x}{\Delta t - \frac{V}{c^2} \Delta x}$$

cancel γ from each term use $\beta = V/c$

$$u_x' = \frac{-V + \overbrace{\Delta x / \Delta t}^{u_x}}{1 - \frac{V}{c^2} \underbrace{\Delta x / \Delta t}_{u_x}} = \frac{u_x - V}{1 - u_x V / c^2}$$

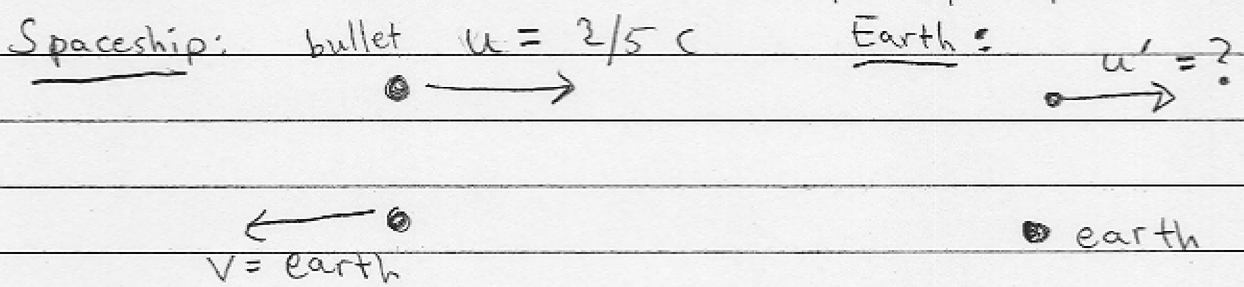
Another example:

- A ship moves at $+\frac{4}{5}c$ relative to earth.

A astronaut on the ship launces a bullet with a speed of $2/5\,c$ as measured by him. What is the speed as measured by earth of the bullet

Classical Answer $\left(\frac{4}{5}c + \frac{2}{5}c = \frac{6}{5}c \quad \text{Wrong!}\right)$

Look at this from ships perspective:

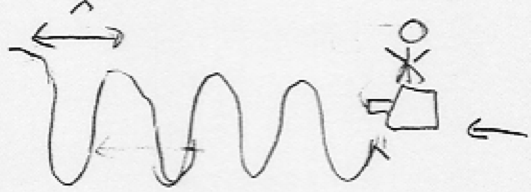


$$u' = \frac{u - v}{1 - u_x v/c^2} = \frac{2/5c - (-4/5c)}{1 - \left(\frac{2}{5}c\right)\left(-4/5c\right)/c^2}$$

$$u' = \frac{6/5c}{1 + 8/25}$$

$$u' = \frac{10}{11}c$$ less than c

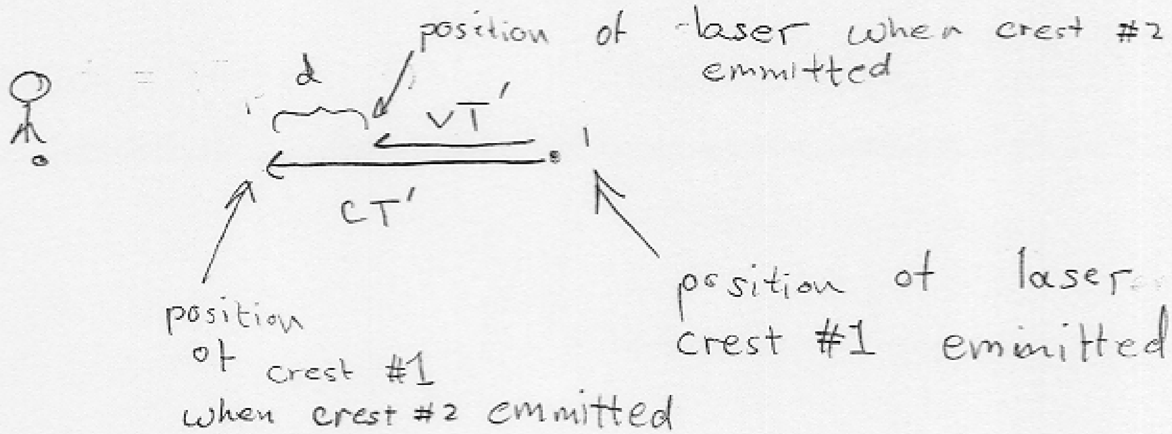
Doppler Shift



- For someone on the laser The crests of the waves emmitted with period τ_p:

$$c = \lambda f \qquad f = \frac{1}{\tau_p}$$

Observer O′ sees a higher laser frequency



distancen between crests $= \lambda' = cT' - vT'$

T' = time between crests as seen by O′

time between crests for a lassar fixed in space

$$T' = \gamma \tau_p$$

$$\lambda' = (c - v)T' = (c - v)\gamma \tau_p$$

<u>Now</u>

$$c = \lambda' f' \quad \Rightarrow \quad f' = c/\lambda'$$

$$f' = \frac{c}{\lambda'} = \frac{c}{(c - v)\gamma \tau_p}$$

$$f' = \frac{1}{(1 - v/c)\gamma \tau_p} = \frac{\sqrt{1-(v/c)^2}}{1 - v/c} f_0$$

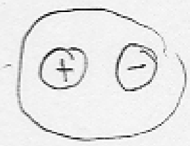
$$\sqrt{1-(v/c)^2} = \sqrt{(1-v/c)(1+v/c)} \qquad \text{so}$$

$$\boxed{f' = f_0 \sqrt{\frac{1+v/c}{1-v/c}}}$$

Energy and Momentum:

Rest Energy:

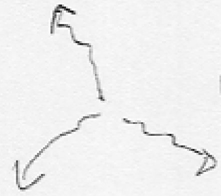
Consider positron + electron which annihilate into 3 photons



annihilate

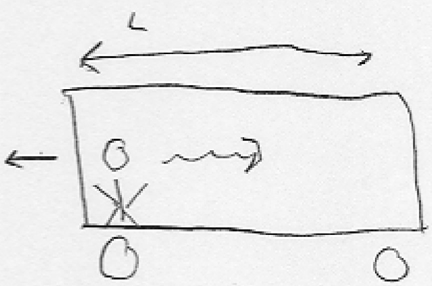
Before:

After:



Energy

Einsteins Box



$$P_{Box} = P_{Light}$$

$$Mv \simeq \frac{E}{c} \leftarrow \text{energy of plot}$$

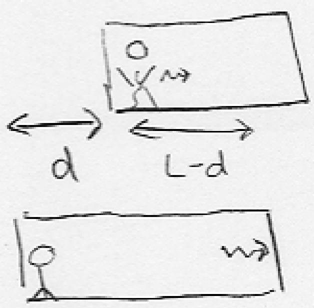
$$v = E/Mc$$

Now the system moves a distance

$$d = v\,t = v\,\frac{L}{c} = \frac{E}{Mc}\,\frac{L}{c} = \frac{E}{Mc^2}\,L \quad \text{(very sma}$$

time it takes light to move from one end to the other

But there are no external Forces so the CM can not change.



$M_{TOT} X_{cm} = M_1 x_1 + M_2 x_2$

Einsteins answer:

- photon carries away some of carts mass as energy

$$(M - \Delta m)(-d) + \Delta m (L - d) = 0$$

$$M d = \Delta m L$$

$$M \frac{E_\gamma L}{M c^2} = \Delta m L$$

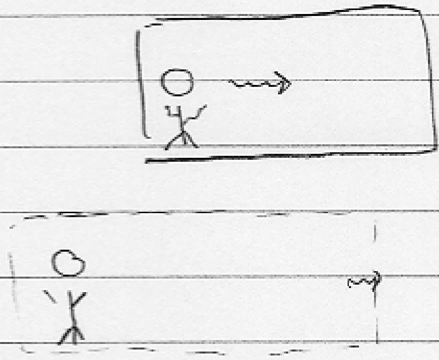
$$\boxed{E_\gamma = \Delta m c^2}$$

Energy of matter in its own rest frame

Lecture 4

Last Time

- Einsteins Box:



$$\boxed{E_{rest} = mc^2}$$

Example:

- $^{236}_{92}U \longrightarrow {}^{90}_{37}Rb + {}^{143}_{55}Cs + 3\,{}^{1}_{0}n$

Determine the energy released by a single Decay in MeV

Units

Energy = Joules or $\underbrace{eV}_{\text{energy of an electron passing through 1 V}} = 1.6 \times 10^{-19}\,J$

mass = $\dfrac{\text{Energy}}{c^2}$ $\Rightarrow$ $\dfrac{eV}{c^2}$ is a unit of mass $= 1.783 \times 10^{-36}$ kg

$$m_p = \overbrace{938.3 \frac{MeV}{c^2}}^{1.007\,u} \qquad m_n = \overbrace{939.5 \frac{MeV}{c^2}}^{1.008\,u}$$

Also use $u = 1$ atomic mass unit $= 931 \frac{MeV}{c^2} \approx m_p \approx m_n$

$N_A u = 1$ gram , $N_A = 6 \times 10^{23}$

$\boxed{\text{One Avagadro's \# of protons and neutrons weighs approximately 1g}}$

What you should have learned in HS

Problem: (Part A)

Energy Released:

$$Q = \underbrace{M_u c^2}_{\text{initial energy}} - \underbrace{(M_{Rb} c^2 + M_{Cs} c^2 + 3 m_n c^2)}_{\text{final energy}}$$

$$= 236.04\, u c^2 - (89.91 + 142.92 + 3(1.008))\, uc^2$$

$$Q = 0.177\, u c^2 = 164 \text{ MeV}$$

One kg of ^{236}U is converted, how much energy is released

- $1\, N_A$ of ^{236}U is 236 g
- $\frac{1}{236} N_A$ is 1 g
- $\frac{1000}{236} N_A$ is 1 kg

So the total energy is 4.1×10^{26} MeV

$1\,\text{MeV} = 4.4 \times 10^{-20}$ kWh

Total $E = 7.48 \times 10^{6}$ kWh

$$\underbrace{NYS = 36 \times 10^{9}\ \text{kWh}}_{\text{Electricity used by New York State in 1991}}$$