Last Time

① Light is made of discrete packets - Photons

$$\boxed{E = hf}$$

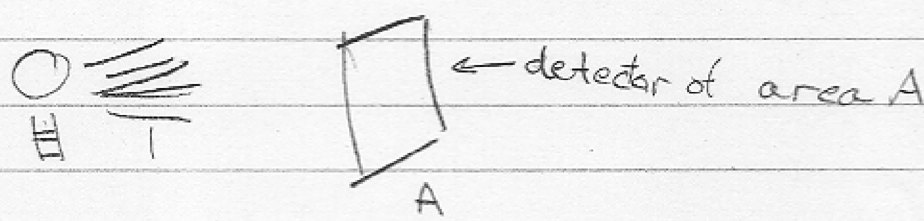
$hc = 1240\, eV/nm$ $\quad$ $2eV \sim 600nm \sim$ red

Visible light $\simeq$ a couple of eV's

② When lot's of photons are measured see the wavelike nature of light

a) -- see slides

b) A relation between the wavelike and Particle like properties:



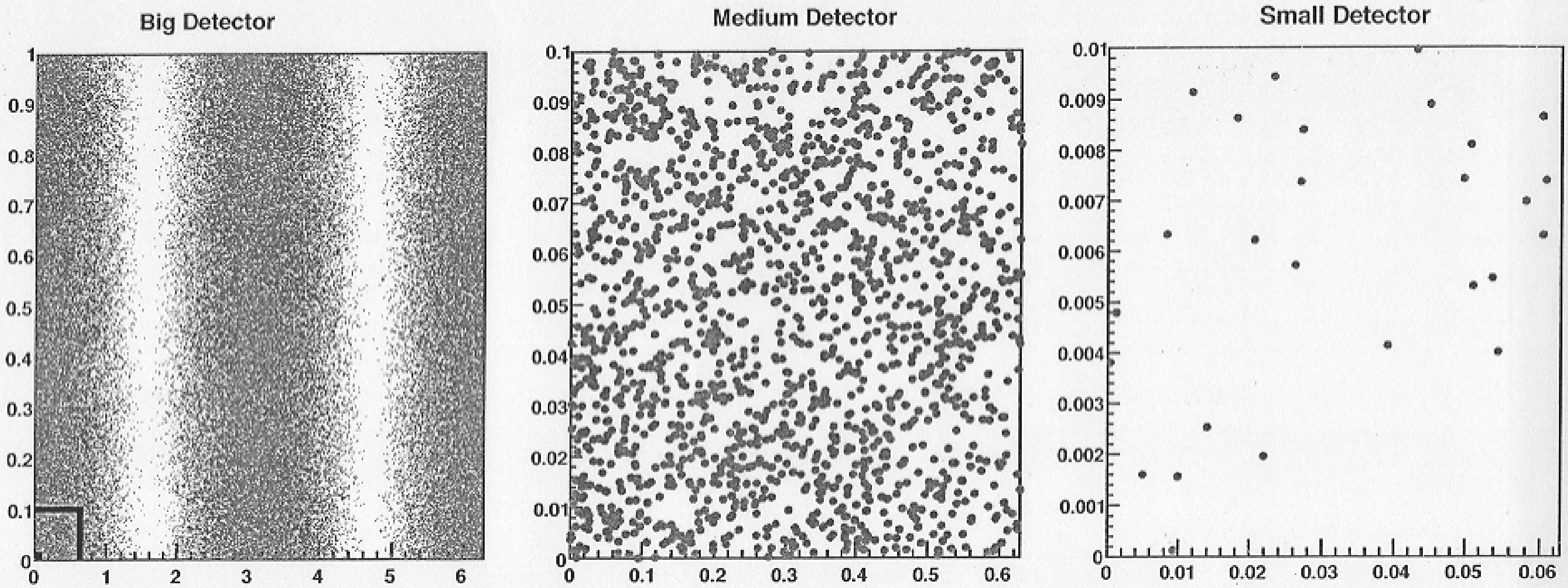

Wave

energy absorbed, area, time

$$\Delta E = I\, A\, \Delta t$$

Intensity of Ligt $I = \langle \vec{S} \rangle \propto \vec{\mathcal{E}} \times \vec{B} \propto \vec{\mathcal{E}}^2$

# Counting Photons



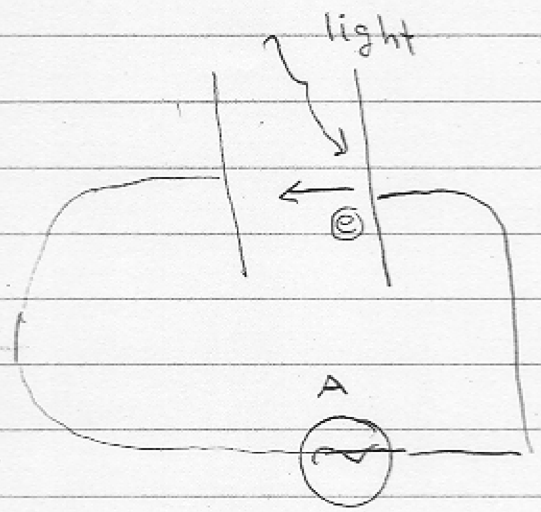
## Particle Picture

$$\underset{\substack{\uparrow \\ \text{energy} \\ \text{absorbed}}}{\Delta E} = \underset{\substack{\uparrow \\ \text{number} \\ \text{absorbed}}}{\Delta N} \; \underset{\substack{\nwarrow \\ \text{Energy per} \\ \text{photon}}}{hf}$$

So,

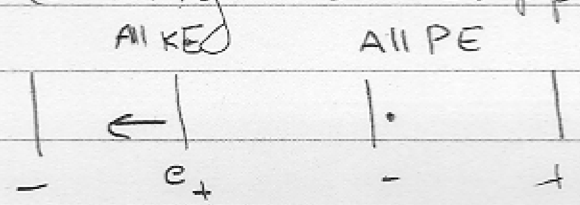
$$\Delta N \propto \frac{I}{hf} A \Delta t$$

③ Photo-Electric Effect – Atoms are small enough to see the discrete natur[e]




$$\underset{\substack{\uparrow \\ \text{KE} \\ \text{of ejected} \\ \text{electron} \\ \sim eV}}{K} = \underset{\substack{\uparrow \\ \text{energy} \\ \text{of light} \\ \sim eV}}{hf} - \underset{\substack{\nwarrow \\ \text{energy} \\ \text{required} \\ \text{to strip} \\ \text{of elect[ron]} \\ \sim eV}}{\phi}$$

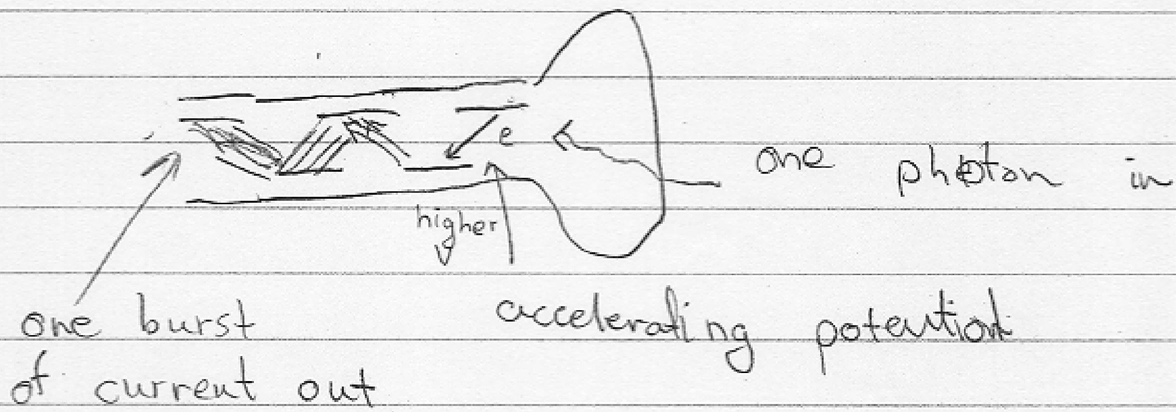
One can measure the electron's energy by turning a stopping voltage




$$\overbrace{eV_{stop}}^{PE} = \overbrace{K}^{KE}$$

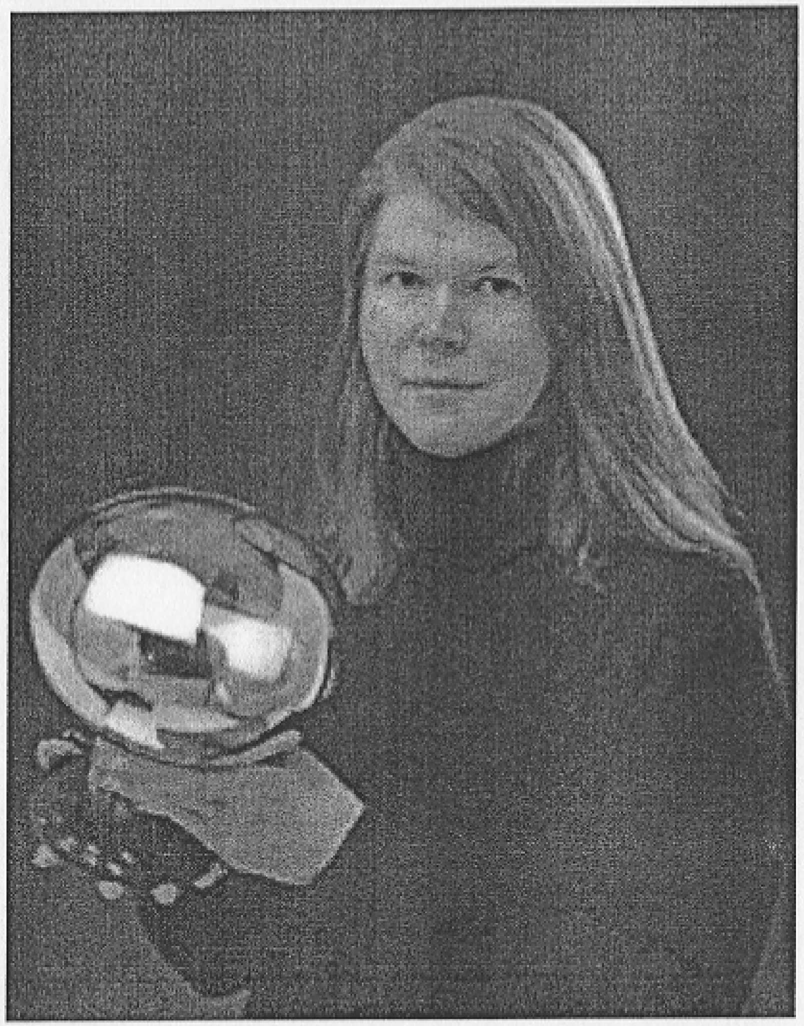
Practical Uses of the Photo-electric effect:

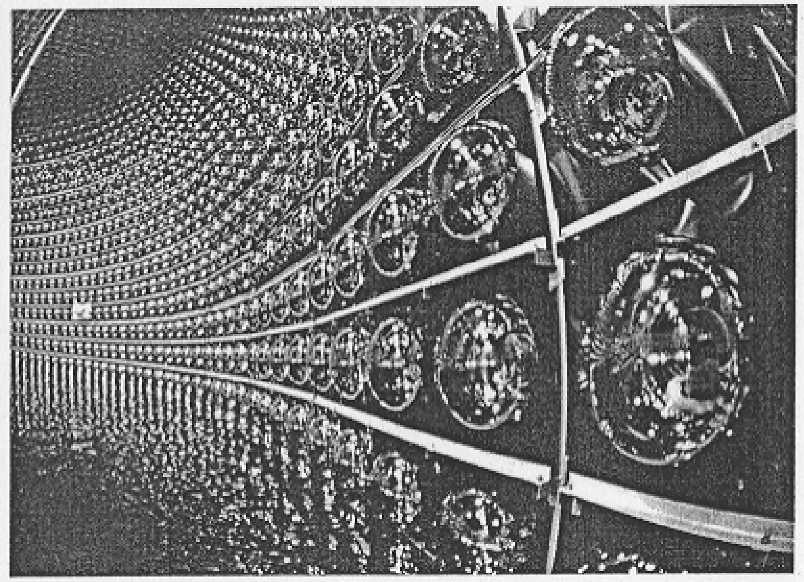
Photomultiplier Tube:



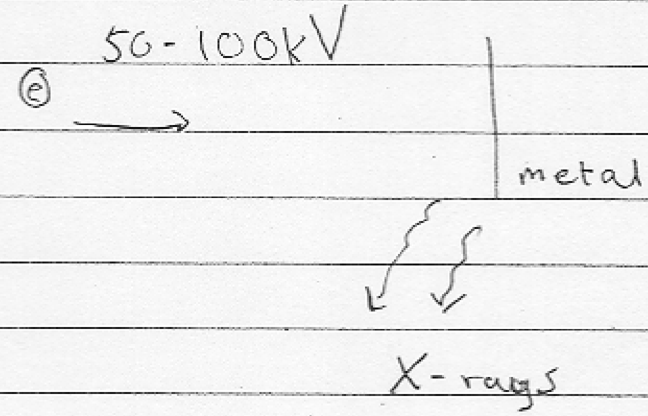
# Photomultiplier Tubes

(Janet Conrad – MIT professor, co-spokesperson for the Miniboone Experiment)



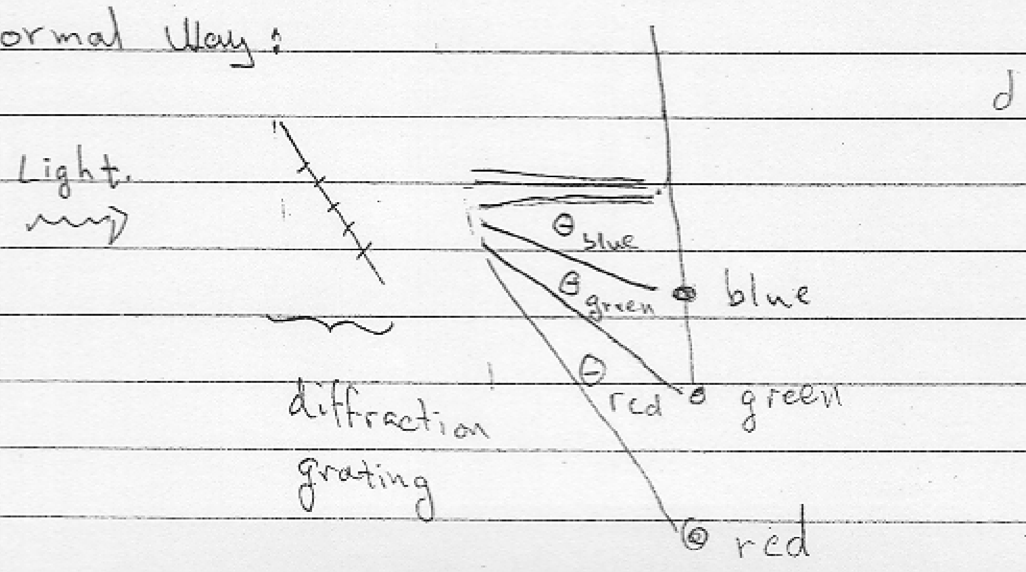


## X-rays




- Very high frequency $\lambda_X \sim 10^{-10}$ m
  - compare $\lambda_{visible} \sim 6000 \times 10^{-10}$ m
- $E = hf$ is a big step! This light is not continuous on human scales
- Problem how to measure the wavelength?

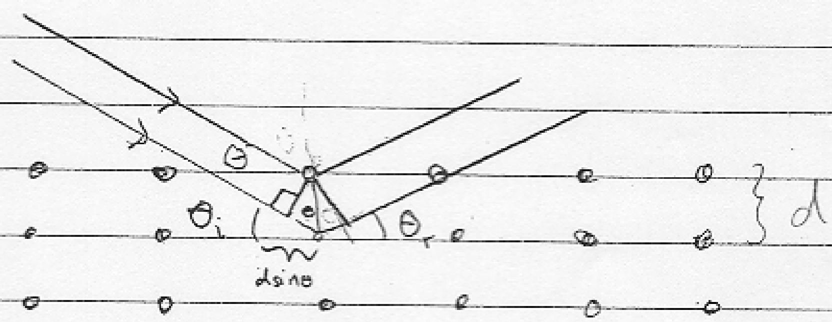
Normal Way:



X-ray Tube – Used for security. 160 Killo Volts

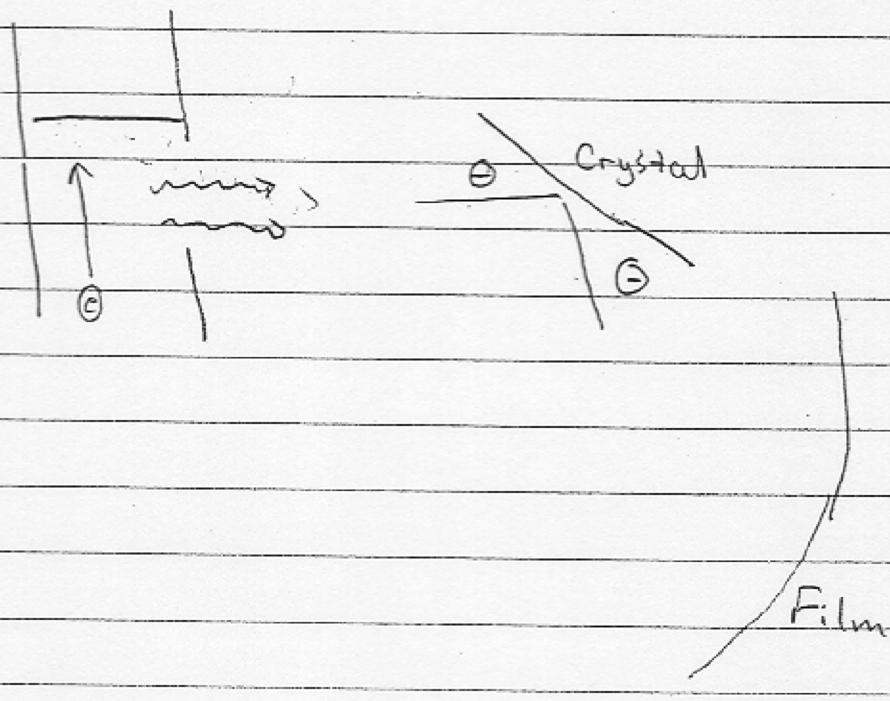
How to make a diffraction grating where the spacing is $\sim\lambda$.

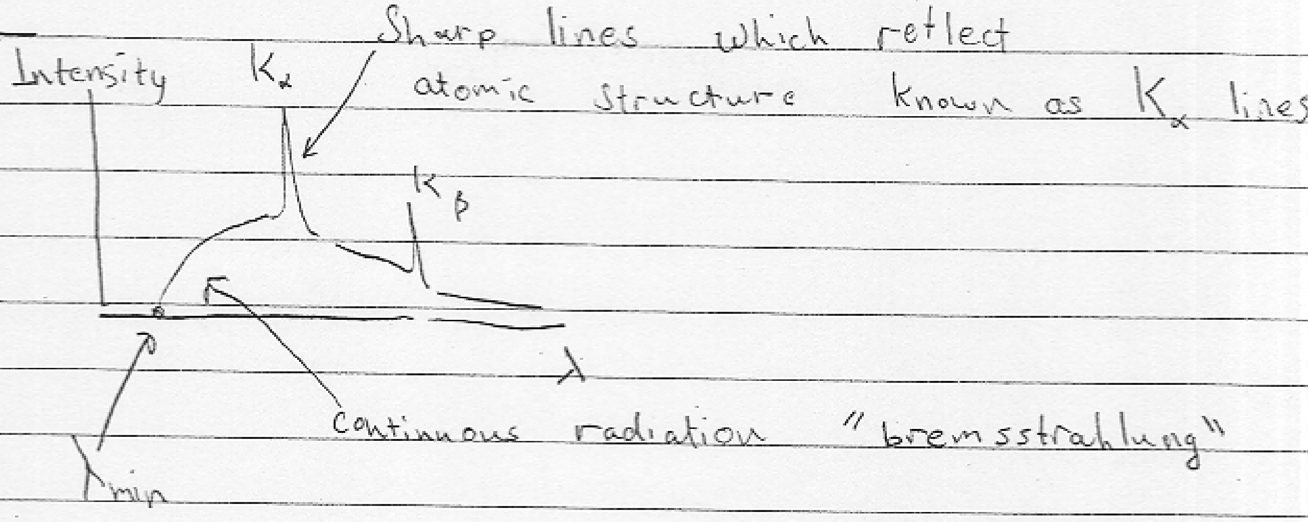
Answer: Make a diffraction grating out of a crystal (Bragg, von Laue)




$$2d\sin\Theta = n\lambda \qquad n = 1, 2, 3, 4, \ldots$$

Result



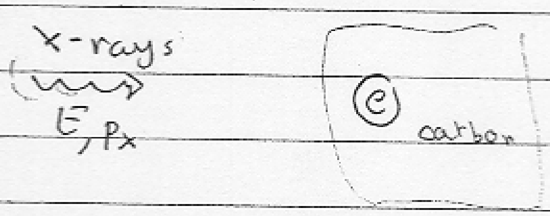



$\lambda_{min}$ is determined when all of then energy of the electron is converted to a single photo

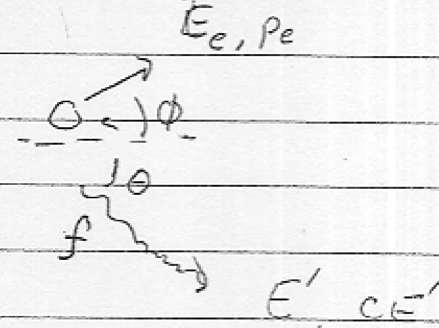
$$eV = hf = h\frac{c}{\lambda_{min}}$$

## The Compton Process (1922)

Before:




After:




Use relativistc energy and momentum conservation and

$$E = hf$$

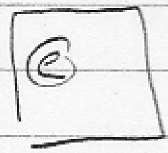
# The Compton Process

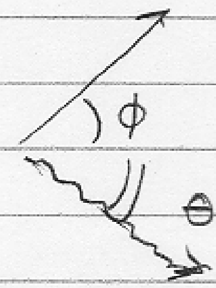
Before:

X-rays

$\lambda$

$p, E$



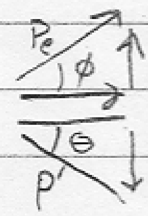



X-rays $\lambda'$

$p', E'$

Particle Description

p-consv:




(2) $$p = p'\cos\theta + p_e \cos\phi$$

(3) $$0 = -p'\sin\theta + p_e \sin\phi$$

E-cosv

(1) $$\underbrace{E}_{cp} + m_e c^2 = \underbrace{E'}_{cp'} + \underbrace{E_e}_{\sqrt{(cp_e)^2 + (m_e c^2)^2}}$$

Knowns:

$\theta, p$

Unknowns:

$\phi, p_e, p'$ $\Rightarrow$ Use equations (1), (2), (3) solve for $\phi, p_e, p'$ in terms of $p, \theta$

Algebra: (See handout)

(*) $$\frac{1}{m_e c}(1-\cos\theta) = \frac{1}{p'} - \frac{1}{p}$$

Now assume

$$E = cp = \frac{hc}{\lambda} \quad \text{and} \quad E' = \frac{hc}{\lambda'}$$

$$p = \frac{h}{\lambda} \quad \text{and} \quad p' = \frac{h}{\lambda'}$$

Then multiplying (*) by $h$

$$\frac{h}{m_e c}(1-\cos\theta) = \frac{h}{p'} - \frac{h}{p}$$

Or

$$\frac{h}{m_e c}(1-\cos\theta) = \lambda' - \lambda$$

Energy (Algebra Skipped if pressed for time)

(1) $E + m_e c^2 = E' + E_e$

Momentum

(2) $p = p'\cos\theta + p_e\cos\phi$

(3) $0 = -p'\sin\theta + p_e\sin\phi$

Also have: $E = cp$ , $E' = cp'$ , $E_e = \sqrt{(cp_e)^2 + m_e c^2}$

Algebra: use the ( ) equation to eliminate $p_e$, $\phi$

$p'\sin\theta = p_e \sin\phi$ (2)

$(p - p'\cos\theta) = p_e \cos\phi$ (3)

Squaring both equations and add:

$$p^2 - 2pp'\cos\theta + p'^2\cos^2\theta = p_e^2\cos^2\phi$$
$$p'^2\sin^2\theta = p_e^2\sin^2\phi$$

$p^2 - 2pp'\cos\theta + p'^2 = p_e^2$ $\Leftarrow$ so far used one equ to elim $\phi$

Now use E-consue

$$(cp - cp') + m_e c^2 = E_e$$

$$\left[(cp - cp') + m_e c^2\right]^2 = E_e^2$$

$$(cp - cp')^2 + 2(cp - cp')(m_e c^2) + (m_e c^2)^2 = E_e^2$$

$$(cp - cp')^2 + 2(cp - cp')\, m_e c^2 = E_e^2 - m_e c^2$$

$$(cp - cp')^2 + 2(cp - cp')\, m_e c^2 = (cp_e)^2$$

$$(p - p')^2 + 2(p - p')\, m_e c = p_e^2$$

Putting in:

$$p^2 - 2pp'\cos\theta + p'^2 = (p - p')^2 + 2(p - p')\, m_e c$$

Working:

$$\frac{1}{m_e c}(1 - \cos\theta) = \frac{1}{p'} - \frac{1}{p}$$

Now assume: $p = h\nu$ $\quad p = h\nu'$

$$\frac{1}{m_e c}(1 - \cos\theta) = \frac{1}{h\nu'} - \frac{1}{h\nu}$$

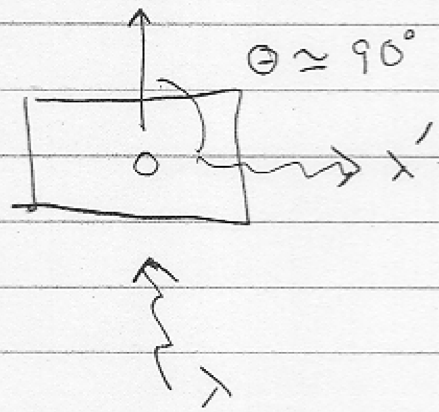
## Important Points

① Used Relativity and Used

$$E = cp = hf$$

and it worked

② The compton wavelength




$\lambda'$ is longer than $\lambda$ :

for 90°

$$\frac{h}{m_e c}(1 - \cos\theta) = \lambda' - \lambda$$

$$\frac{h}{m_e c} = \lambda' - \lambda$$

The wavelength is longer by the compton wavelength

$$\lambda_c \equiv \frac{h}{m_e c} = \text{the intrinsic size of the electron}$$

$$\lambda_c \simeq \frac{h}{m_e c} = \frac{hc}{m_e c^2} = \frac{1240 \text{ eV nm}}{0.511 \times 10^6 \text{ eV}} \simeq 0.025 \text{ Å} = 0.0025$$

$$\lambda \simeq \frac{1}{40} \text{ the size of an atom}$$

(3) Why use X-rays

for $\theta = 90^\circ$

$$\lambda_c (1 - \cancel{\cos}\theta) = \lambda' - \lambda$$

$$\lambda_c = \lambda' - \lambda \Rightarrow \lambda' = \lambda + \lambda_c$$

For $\lambda \sim 500 \text{nm}$ (visible light)

$$\lambda' \simeq 500 \text{nm} + 0.0024 \text{nm} \simeq 500 \text{nm}$$

small shift

Thus it was essential to use X-rays

(4) What is the energy + speed of the recoiling electron when the incoming X-ray has energy $\lambda \simeq \lambda_c$ at $\theta \simeq 90^\circ$

E-consv $\quad E + m_e c^2 = E' + E_e$

Then

$$\lambda_c\,(1-\overset{0 \text{ for } \theta\simeq 90^\circ}{\cancel{\cos\theta}}) = \lambda' - \underbrace{\lambda}_{=\lambda_c \text{ for this problem!}}$$

$$2\lambda_c = \lambda'$$

$$E = hf = h\frac{c}{\lambda} = \frac{hc}{\frac{hc}{m_e c^2}} = m_e c^2$$

$$E' = hf' = h\frac{c}{\lambda'} = \frac{m_e c^2}{2}$$

So $\quad E + m_e c^2 = E' + E_e$

$$m_e c^2 + m_e c^2 = \frac{m_e c^2}{2} + E_e$$

$$\frac{3}{2} m_e c^2 = E_e \quad \Rightarrow$$

$$\frac{3}{2} m_e c^2 = \gamma m_e c^2 \quad \Rightarrow \quad \gamma = 3/2$$

$$\beta \simeq 0.74$$

Thus electrons are pretty relativistic

Q: How important is the binding Energy?