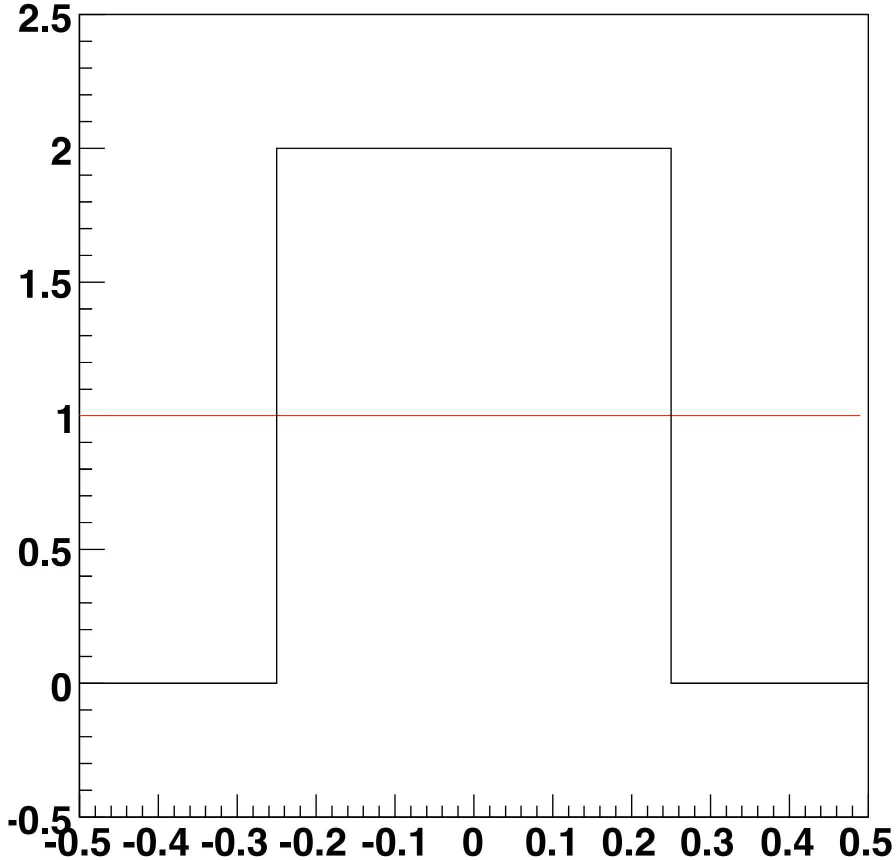
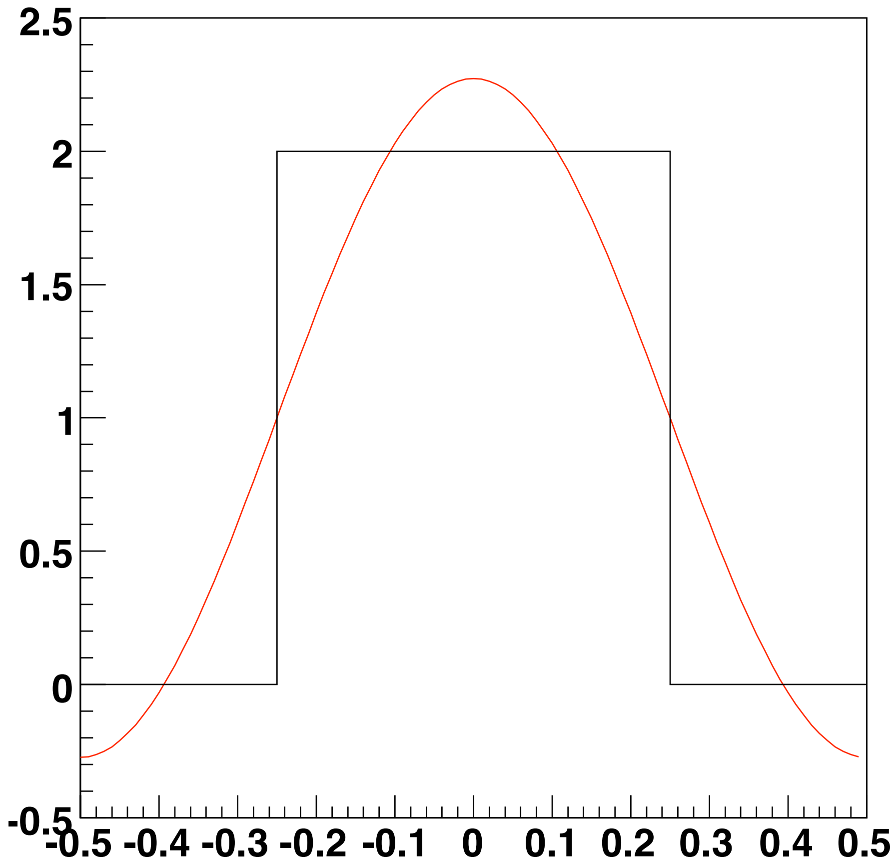
Zero-th Approximation



Approximate function by its average

$$f(x) \simeq 1$$

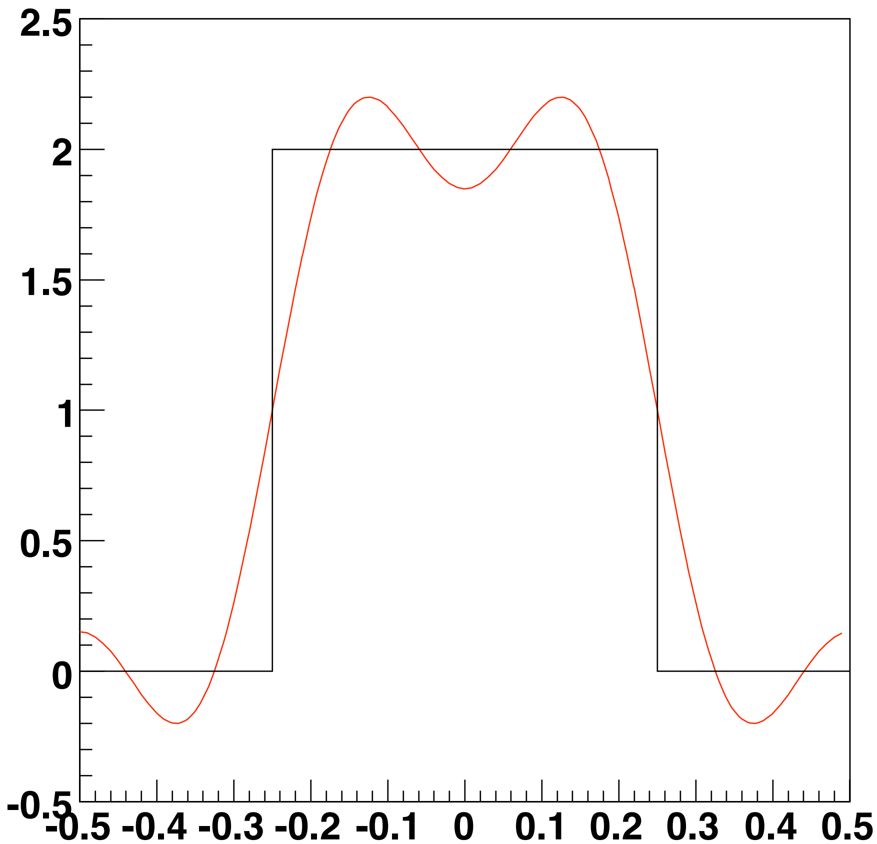
1st Approximation



Approximate function by its average + cos(x)

$$f(x) \simeq 1 + \frac{2}{\pi}\cos(2\pi x)$$

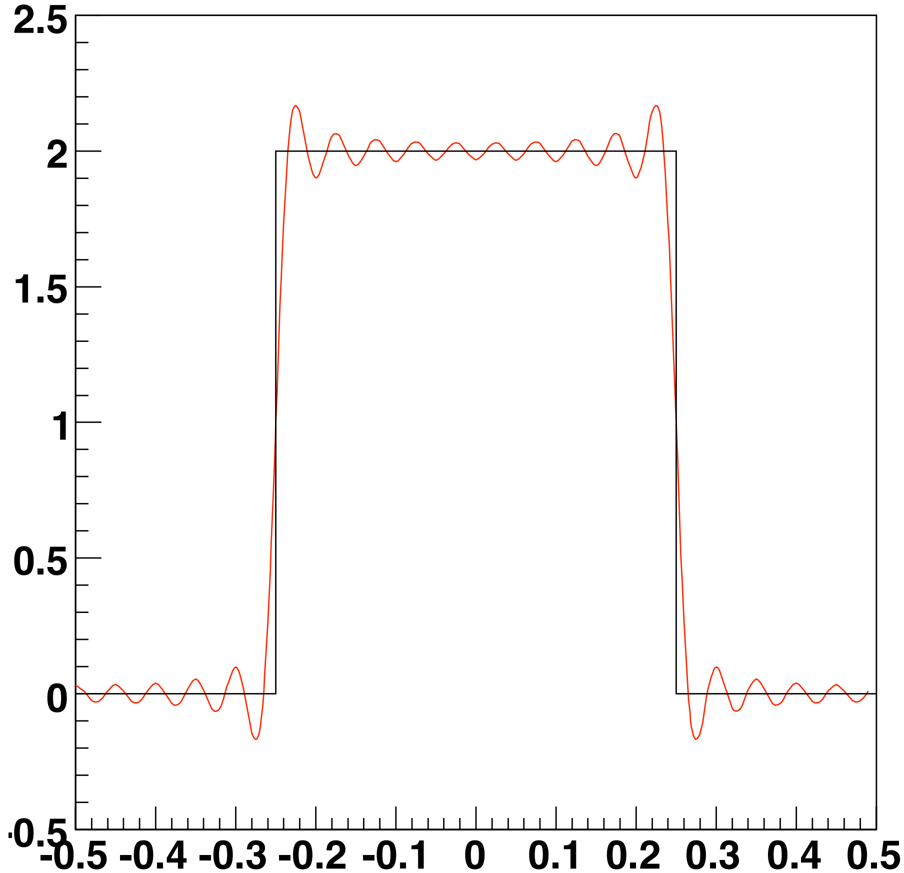
3 term Approximation



Approximate function by its average + cos(x)

$$f(x) \simeq 1 + \frac{2}{\pi}\cos(2\pi x) - \frac{2}{3\pi}\cos(2\pi \cdot 3x)$$

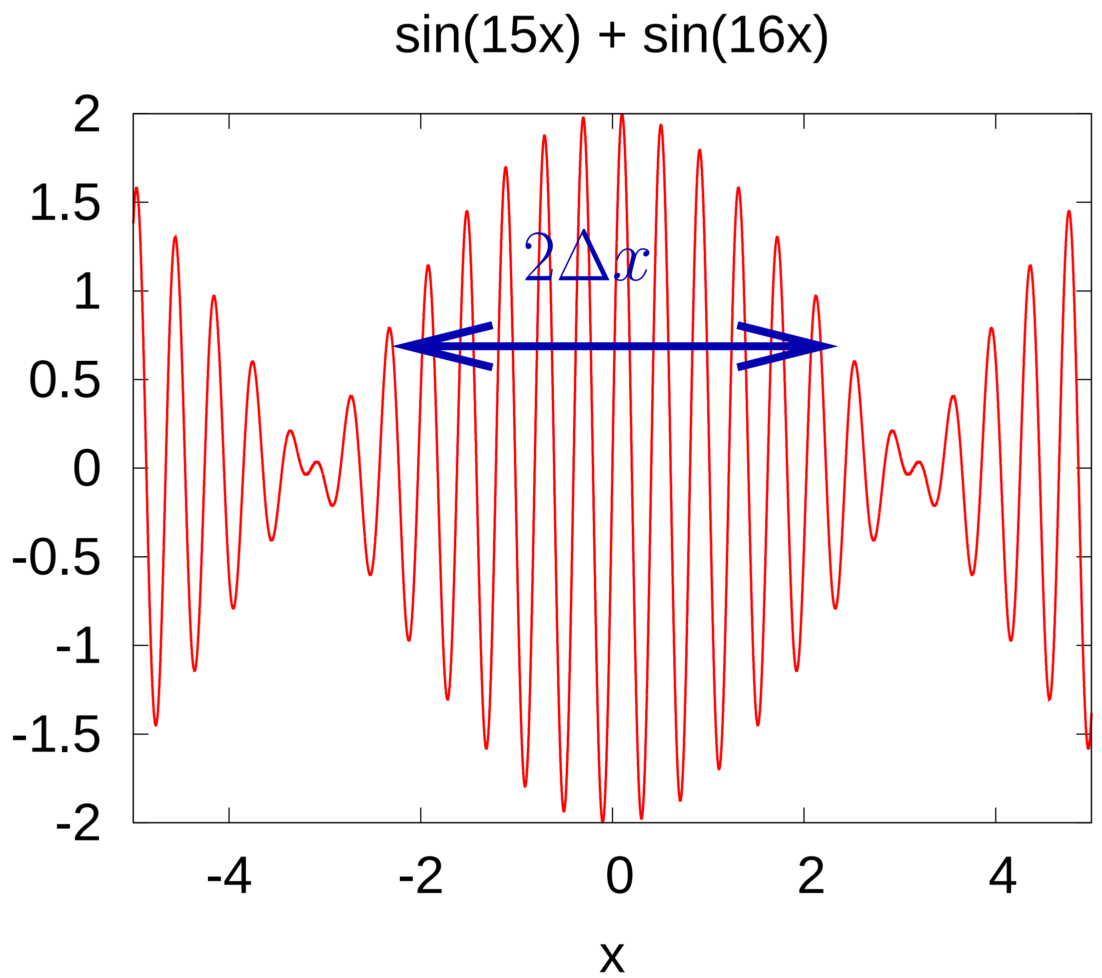
10 term Approximation



Approximate function by its average + cos(x)

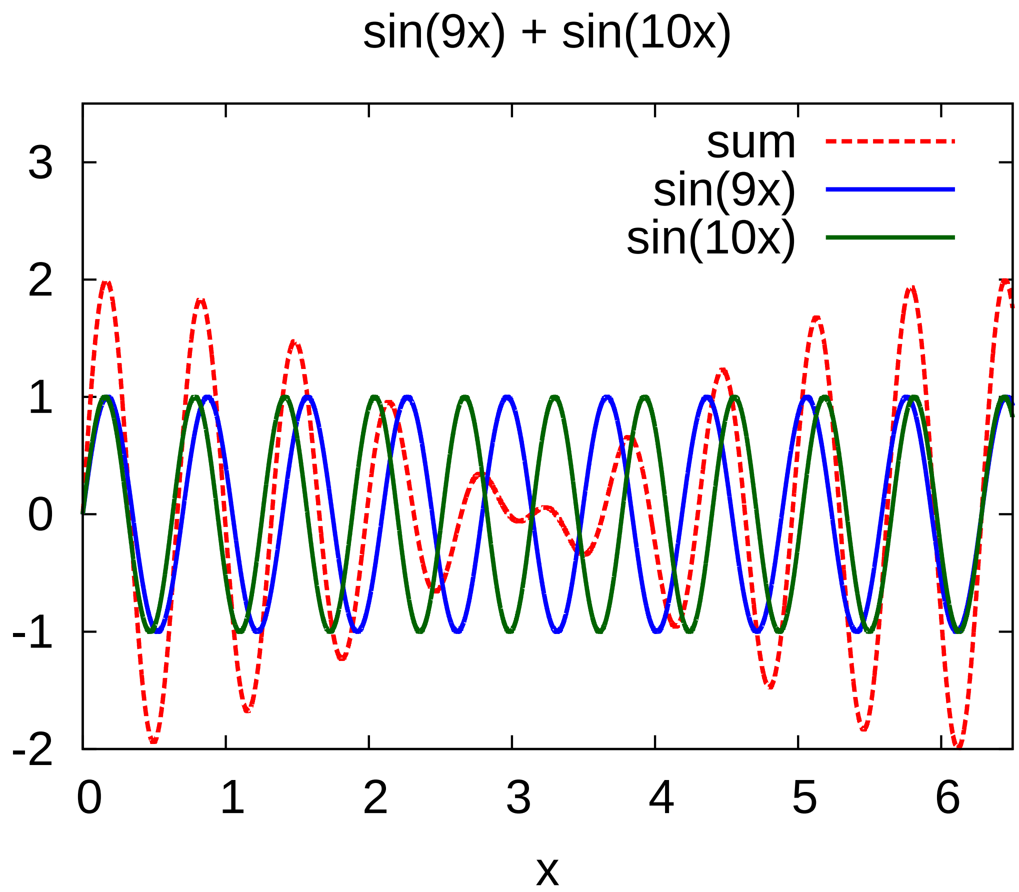
$$f(x) \simeq 1 + \frac{2}{\pi}\cos(2\pi x) - \frac{2}{3\pi}\cos(2\pi \cdot 3x) + \frac{2}{5\pi}\cos(2\pi \cdot 5x) + \ldots$$

Adding two sin waves



Adding two waves of similar frequency makes ”beats”

Beats Explanation

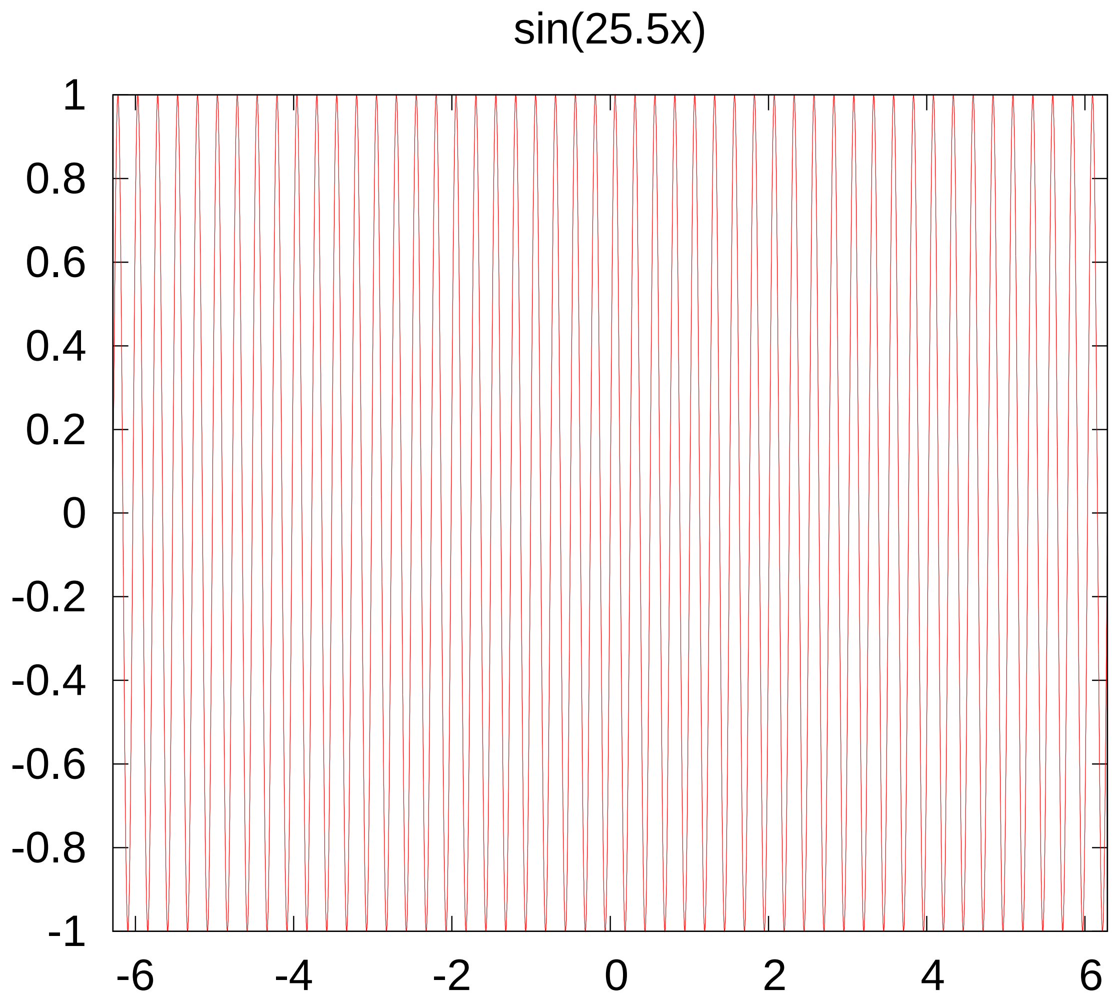


Near $x = 0$ the waves are in phase, but gradually get out of phase near $x = 3$

The larger the difference $\Delta k = (k_2 - k_1)/2 = 0.5$ the more rapidly they dephase

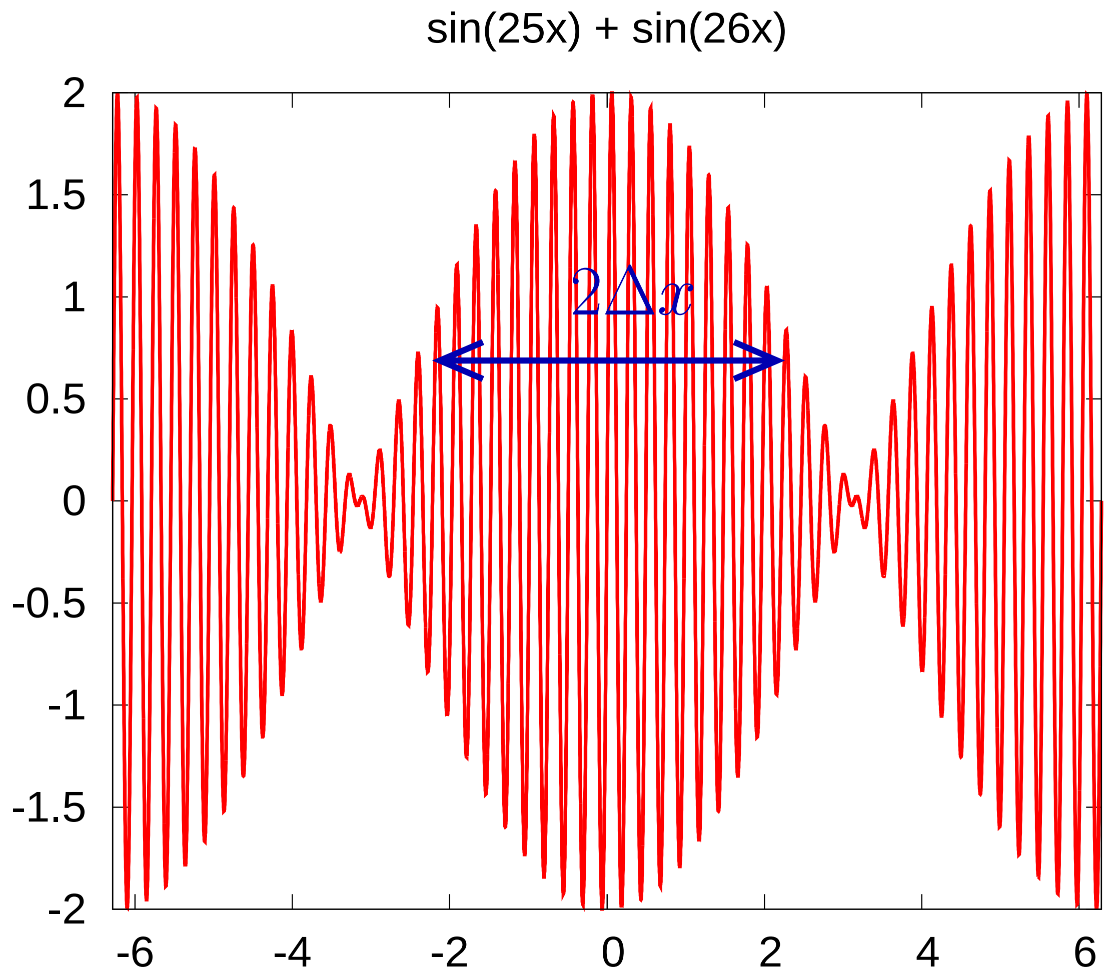
$$\Delta x \sim 1/\Delta k \sim 2$$

Sin wave unlocalized in space



We know the wavelength $\bar{k} = \frac{2\pi}{\lambda} = 25.5$. But wave is delocalized

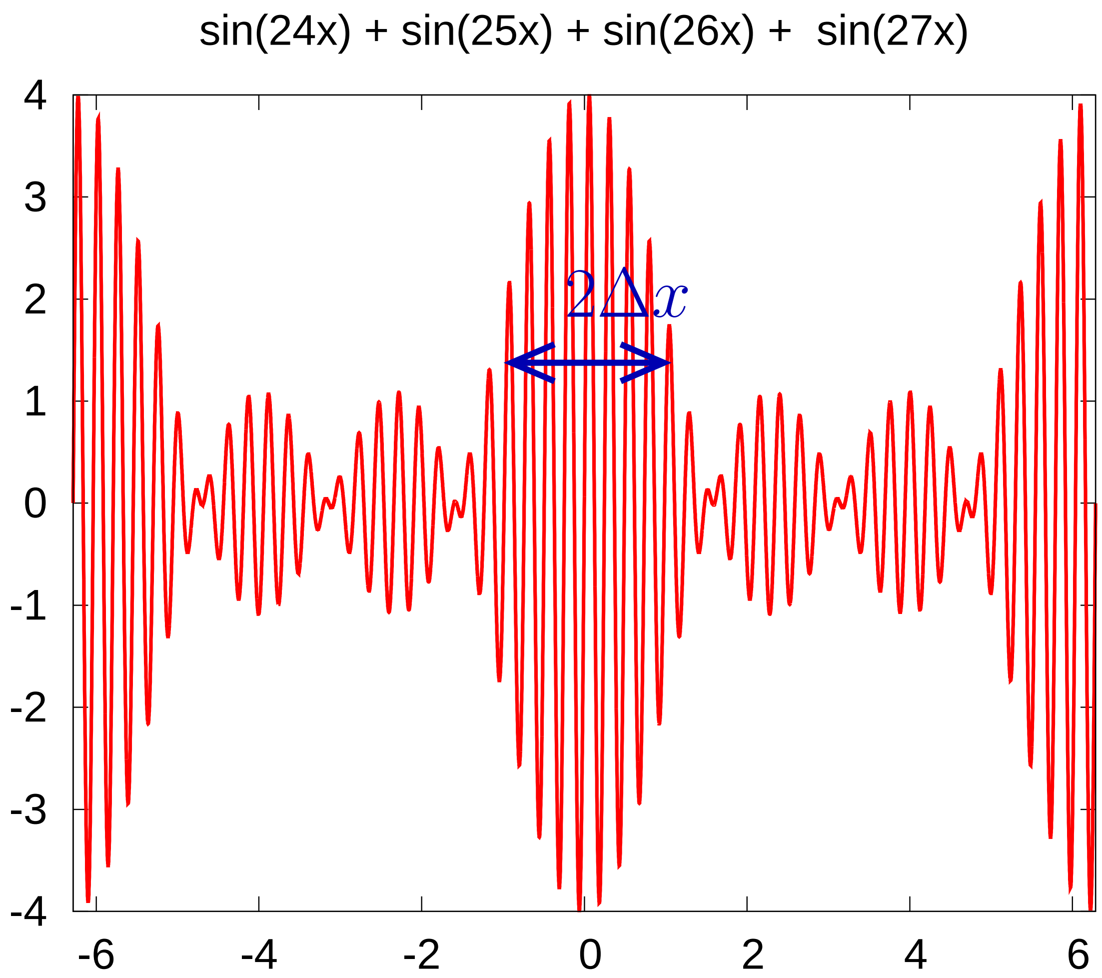
Wave slightly localized in space



$\bar{k} \simeq 25.5$ but uncertain, $\Delta k = 0.5$ and $\Delta x \simeq \frac{1}{0.5} \simeq 2$

$\Delta x \sim \frac{1}{\Delta k}$ is where the 25 and 26 components dephase

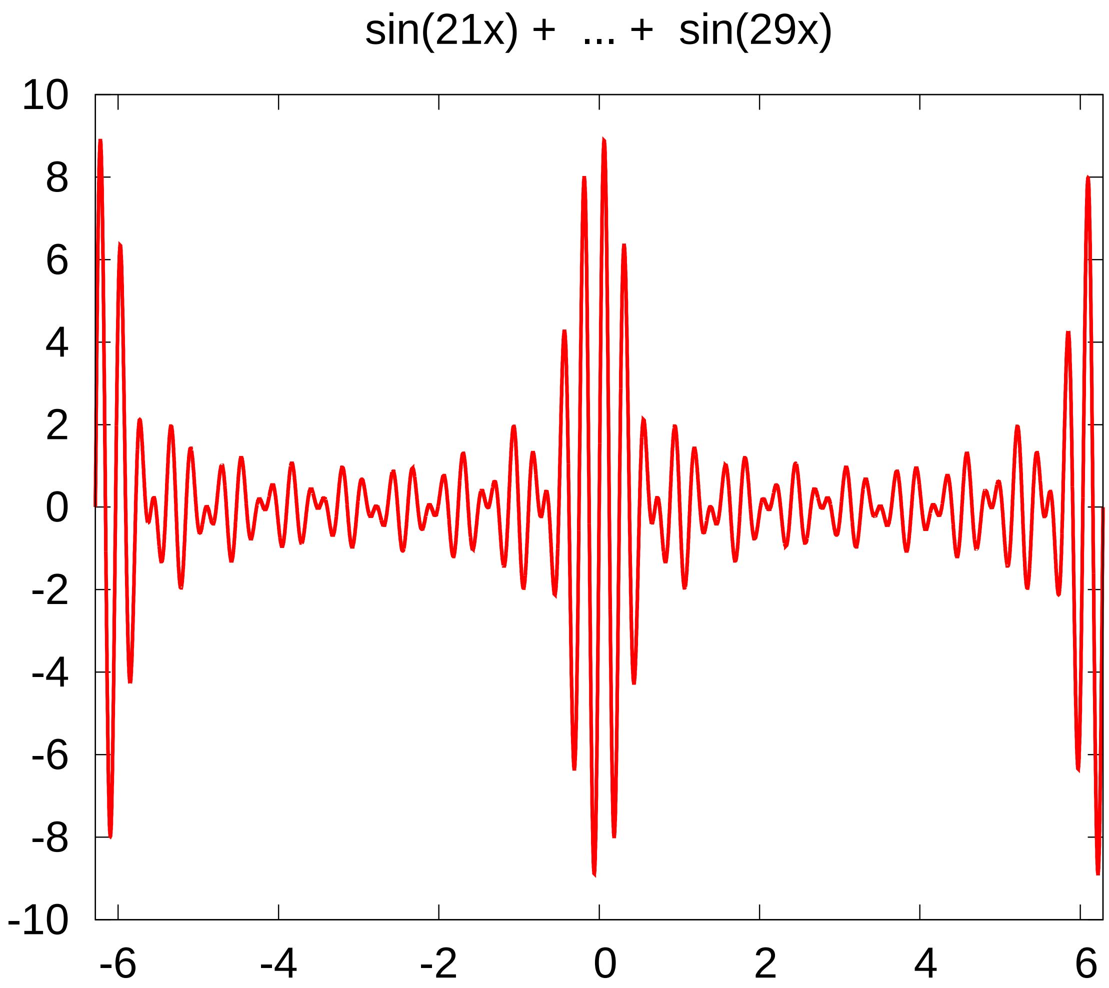
More sin waves better localized



$\bar{k} \simeq 25.5$ but uncertain, $\Delta k = 1.5$ and $\Delta x \simeq \frac{1}{1.5} \simeq 0.66$

$\Delta x \sim \frac{1}{\Delta k}$ is where the 24,25,26, 27 components dephase

More sin waves better localized



$\bar{k} \simeq 25.5$ but uncertain, $\Delta k = 5$ and $\Delta x \simeq 0.2$

$\Delta x \sim \frac{1}{\Delta k}$ is where the $21 \ldots 29$ components dephase

Group velocity – see Michael Fowler's applet