Numerical Solution

1. Start at $x = -x_{\rm max}$.

   - Specify the wave function and derivative:

$$\Psi(x) = 10^{-6} \qquad \Psi'(x) \equiv \frac{d\Psi}{dx} = 10^{-6}$$

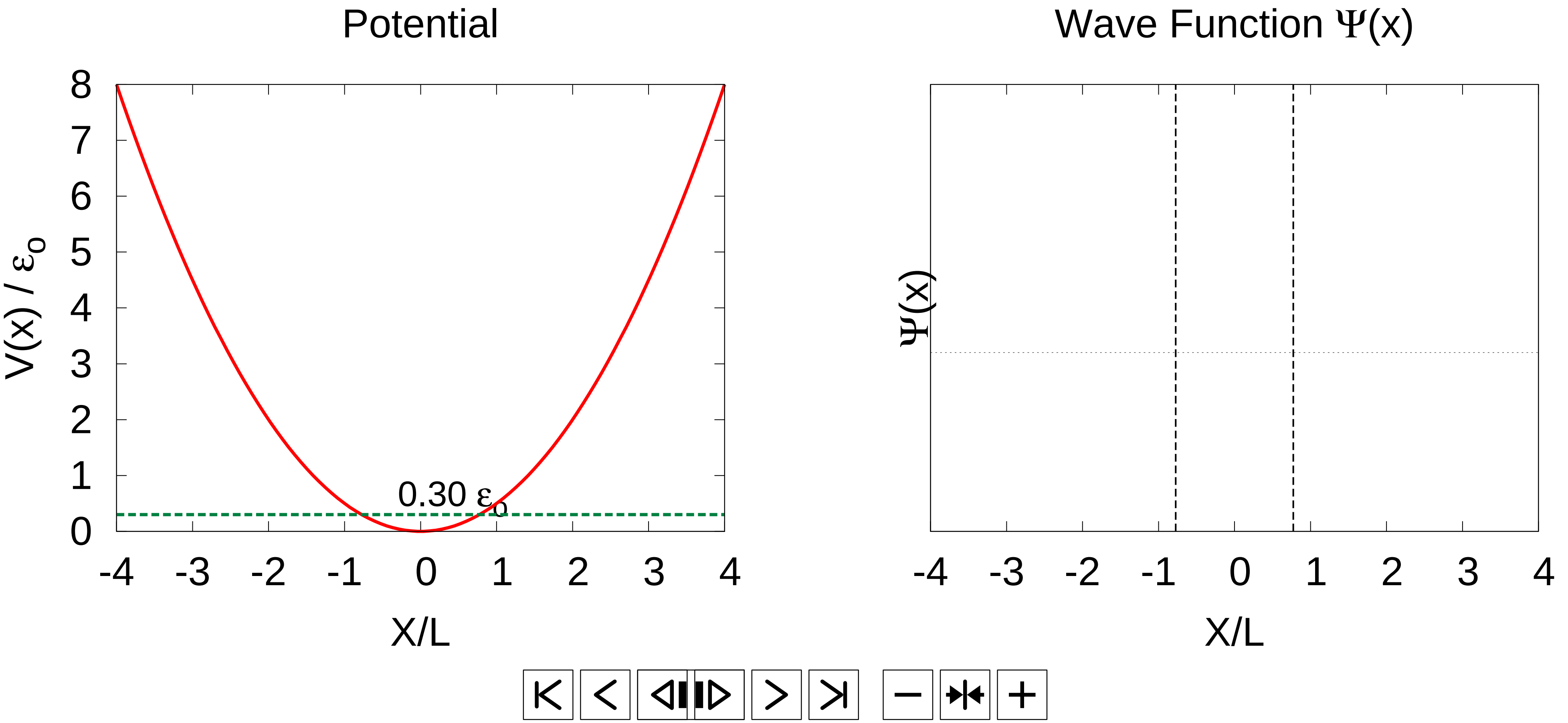
   - Choose an energy arbitrarily $E = 0.3\,\epsilon_{\rm o}$

2. Integrate forward with $\Delta x = 0.01$ until $x_{\rm max}$

$$\Psi(x + \Delta x) \simeq \Psi(x) + \Psi'(x)\,\Delta x \qquad (1)$$

$$\Psi'(x + \Delta x) \simeq \Psi'(x) + \frac{d\Psi'(x)}{dx}\,\Delta x \qquad (2)$$

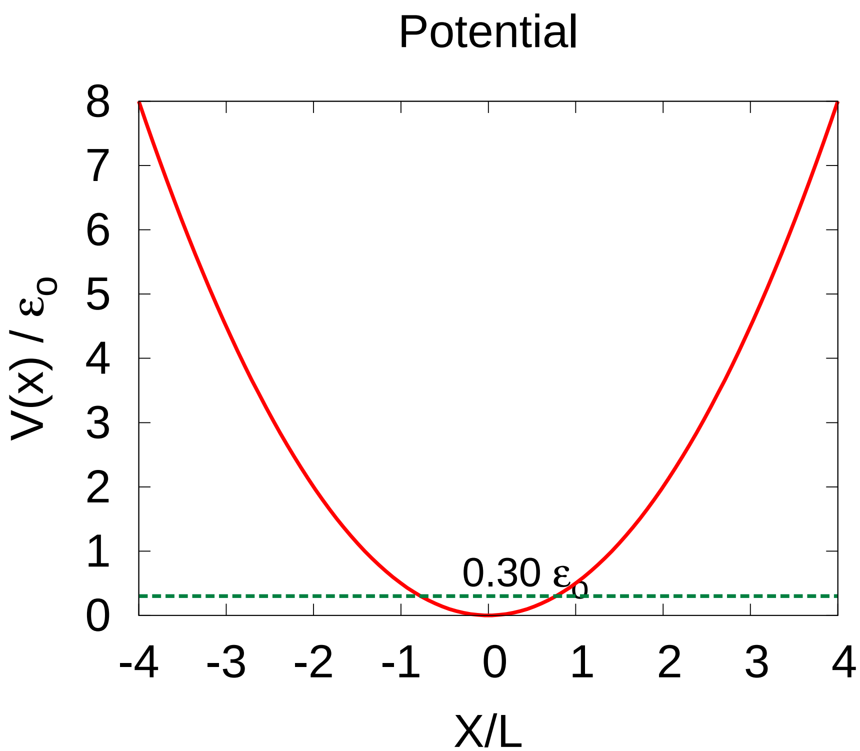
$$\frac{d\Psi'(x)}{dx} = -2(\bar{E} - v(x))\Psi(x) \Leftarrow \text{The Schrodinger equation} \qquad (3)$$

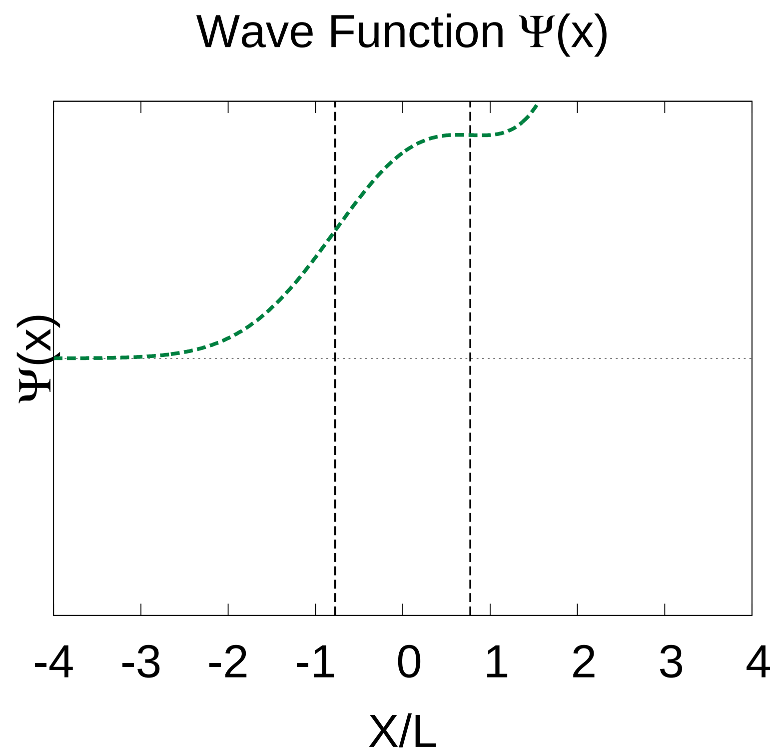

Potential
Wave Function Ψ(x)
V(x) / ε0
Ψ(x)
0.30 ε0
X/L
X/L
8
7
6
5
4
3
2
1
0
-4
-3
-2
-1
0
1
2
3
4

Numerical Solution

1. Choose energy E
   (a) Start at left end.
   (b) Integrate forward to right end.
2. Change the Energy and repeat step 1
   (a) For most energies: $\Psi(x) \to \pm\infty$ as $x \to \infty$. This is unphysical.

Want to find the discrete E where: $\Psi(x) \to 0$ as $x \to +\infty$





Find physical wave functions when

$$E_n = \frac{1}{2}\epsilon_o,\ \frac{3}{2}\epsilon_o,\ \frac{5}{2}\epsilon_o \ldots \qquad \epsilon_o \equiv \hbar\omega_o$$

$$= \left(n + \frac{1}{2}\right)\epsilon_o \qquad n = 0, 1, 2, 3 \ldots$$