Problem I

(1) $F = ma$

$$\frac{e^2}{4\pi\varepsilon}\frac{1}{r^2} = m\frac{v^2}{r} \qquad L = mvr = n\hbar$$

$$\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r^2} = \frac{L^2}{mr^3}$$

$$r = \frac{L^2}{m\left(\frac{e^2}{4\pi\varepsilon_0}\right)}$$

$$r_n = n^2\frac{\hbar^2}{m(e^2/4\pi\varepsilon_0)}$$

Then

$$L = mv_n r_n = n\hbar$$

$$v_n = \frac{n\hbar}{mr_n} = \frac{n\hbar}{m\frac{n^2\hbar^2}{me^2/4\pi\varepsilon_0}}$$

$$v_n = \frac{e^2}{4\pi\varepsilon_0\hbar}\frac{1}{n}$$

(2)

Then consider $n=3$

$$\tau_{orb} = \frac{2\pi r}{v} = \frac{2\pi\, n^2 a_0}{\frac{\alpha}{n} c} = \frac{n^3}{\alpha}\left(\frac{2\pi a_0}{c}\right)$$

So; for $n=3$

$$\tau_{orb} = \frac{3^3}{(1/137)} \cdot \frac{2\pi\,(0.5\times10^{-10}\,m)}{3\times10^{8}\,m/s}$$

$$\tau_{orb} = 1.15\times10^{-15}\,s = 1.15\times10^{-6}\,ns$$

(3) $$\frac{dE}{dt} = \frac{2}{3}\frac{e^2}{4\pi\varepsilon_0}\frac{a^2}{c^3}$$

with $a = \frac{v^2}{r}$ $\Rightarrow$

$$\frac{dE}{dt} = \frac{2}{3}\frac{e^2}{4\pi\varepsilon_0}\frac{v_n^4}{r_n^2 c^3} = \frac{2}{3}\left(\frac{e^2}{4\pi\varepsilon_0}\right)\frac{1}{a_0^2}\frac{(\alpha/n)^4 c^4}{n^4 c^3}$$

$$\frac{dE}{dt} = \frac{2}{3}\frac{c}{a_0}\left(\frac{e^2}{4\pi\varepsilon_0 a_0}\right)\frac{1}{n^8}\alpha^4$$

(4) So the energy lost per orbit

$$\Delta E = \frac{dE}{dt} \tau_{orb}$$

$$\Delta E = \frac{2}{3} \frac{\cancel{c}}{\cancel{a_0}} \left(\frac{e^2}{4\pi\varepsilon_0 a_0}\right) \frac{1}{n^8} \alpha^4 \quad \frac{n^3}{\alpha} \frac{2\pi \cancel{a_0}}{\cancel{c}}$$

$$\Delta E = \frac{\alpha^3}{n^5} \cdot \frac{2}{3} \cdot \left(\frac{e^2}{4\pi\varepsilon_0 a_0}\right) \cdot 2\pi$$

For $n = 3$

$$\Delta E = \frac{1}{(137)^3} \frac{1}{3^5} \frac{4\pi}{3} (27.2\,eV)$$

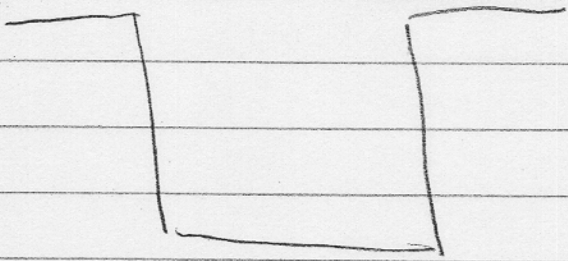
$$\Delta E = 0.18 \times 10^{-6}\ eV$$

So

$$\Delta E \ll E \sim 13.6\,eV$$

The energy lost per orbit is much less than a typical energy spacing

Problem 2



1) $R_A \sim 5\,fm$ $\qquad m_p = 938\ MeV/c^2$

2) $a \sim \pi D \sim 2\pi R_A$

$$E_\gamma = E_2 - E_1 = \frac{\hbar^2 \pi^2}{2ma^2}\left(2^2 - 1^2\right)$$

$$E_\gamma = \frac{\hbar^2 \pi^2}{2m_p a^2} \cdot 3$$

$$= \frac{(\hbar c)^2}{2m_p c^2}\left(\frac{\pi}{2\pi R_A}\right)^2 \cdot 3$$

$$E_\gamma = \frac{(197\ meV\ fm)^2}{2(938\ meV)} \cdot \frac{1}{(5\,fm)^2}\ \frac{1}{4} \cdot 3$$

$$E_\gamma = 0.62\ meV$$

(3) For

$$\psi_2(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right)$$

$$-i\hbar \frac{\partial}{\partial x} \psi_2(x) = \sqrt{\frac{2}{a}} \cos\left(\frac{2\pi x}{a}\right) \cdot \frac{2\pi}{a} \; -i\hbar$$

$$-\hbar^2 \frac{\partial^2}{\partial x^2} \psi(x) = -\hbar^2 \sqrt{\frac{2}{a}} \left(-\sin\left(\frac{2\pi x}{a}\right)\right)\left(\frac{2\pi}{a}\right)^2$$

$$= + \hbar^2 \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right) \left(\frac{2\pi}{a}\right)^2$$

Then

$$\bar{p} = \int_{-\infty}^{\infty} \underbrace{\psi(x)}_{\text{even}} \; \underbrace{-i\hbar \frac{\partial}{\partial x} \psi}_{\text{odd}} = 0$$

The particle is not moving preferrentially to the right or left.

4) $$\overline{p^2} = \int_{-\infty}^{\infty} \psi(x) \; -\hbar^2 \frac{\partial^2}{\partial x^2} \psi(x)$$

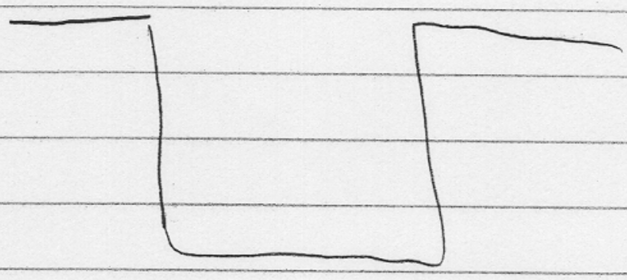
$$\overline{p^2} = \int_{-a/2}^{a/2} \frac{2}{a} \sin\left(\frac{2\pi x}{a}\right) \; \hbar^2 \left(\frac{2\pi}{a}\right)^2 \sin\left(\frac{2\pi x}{a}\right)$$

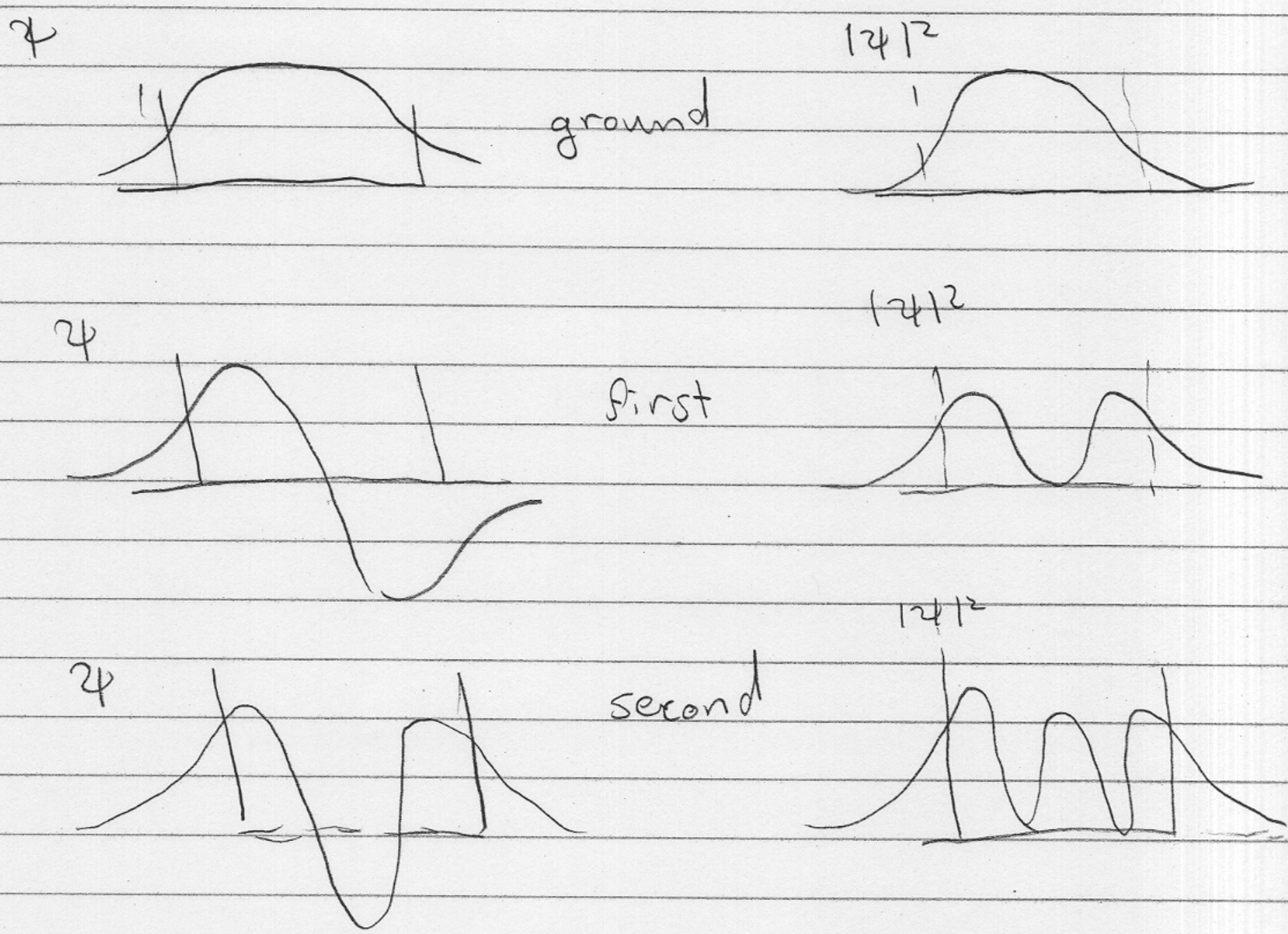
$$\overline{p^2} = \hbar^2 \left(\frac{2\pi}{a}\right)^2$$

So $$\Delta p = \sqrt{\overline{p^2} - \cancel{\bar{p}^2}^{\,0}} = \sqrt{\overline{p^2}} = \hbar\,(2\pi/a)$$

4) From uncertainty expect $\Delta p \sim \frac{\hbar}{\Delta x} \sim \frac{\hbar}{a}$ which is the correct order of magnitude.

5) Now





6) Using perturbation theory

a) if $\Delta V \ll \frac{\hbar^2\pi^2}{2ma^2}$ then we can use perturbation theory

b) $$\delta E = \int_{-\infty}^{\infty} dx\, \psi_n(x)\, \delta V\, \delta\psi(x)$$

$$= \Delta V \int_0^{a/2} \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right) 1 \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right)$$

$$= \Delta V \cdot \frac{2}{a} \overbrace{\int_0^{a/2} \sin^2\left(\frac{2\pi x}{a}\right)}^{\equiv I}$$

$$\delta E = \Delta V \cdot \frac{2}{a} \cdot \underbrace{\frac{1}{2} \cdot \frac{a}{2}}_{I = \langle \sin^2 \rangle \frac{a}{2}}$$

$$\boxed{\delta E = \frac{\Delta V}{2}}$$

Problem3

1)

$$\Delta\Omega = \frac{A}{R^2} = \frac{\pi(D/2)^2}{R^2} = \frac{\pi}{4}\left(\frac{D}{R}\right)^2$$

2) The number per time is

$$\frac{dN}{dt} = \frac{fP}{hc/\lambda}$$

Then this number is spread over 4π

$$\frac{dN}{dt\,d\Omega} = \frac{fP}{hc/\lambda}\frac{1}{4\pi}$$

So the number collected per time is

$$\frac{dN}{dt} = \frac{fP}{hc/\lambda}\frac{\Delta\Omega}{4\pi} = \frac{dN}{dt\,d\Omega}\Delta\Omega = \frac{fP\lambda}{hc\,16}\frac{D^2}{R^2}$$

3)

$$\frac{dN}{dt} = \int \frac{dN}{dt\,d\Omega}\,d\Omega$$

over cone with $\theta < \theta_0$

$$= \int_0^{\theta_0} \underbrace{\frac{dN}{dt\,d\Omega}}_{\text{const func of angles}} 2\pi \sin\theta \, d\theta$$

So

$$\frac{dN}{dt} = \frac{dN}{dt\,d\Omega}\, 2\pi\, (-\cos\theta)\Big|_0^{\theta_0}$$

$$\frac{dN}{dt} = \frac{dN}{dt\,d\Omega}\, 2\pi\, (-\cos\theta_0 + 1) = \frac{fP\lambda}{hc}\frac{1}{2}(1 - \cos\theta)$$

4) For small θ_0

$$\cos\theta \simeq 1 - \frac{\theta_0^2}{2} \quad \text{or}$$

$$\frac{dN}{dt} = \frac{dN}{dt\,d\Omega}\frac{\theta_0^2}{2}\, 2\pi = \frac{fP\lambda}{hc}\frac{1}{4}\theta_0^2$$