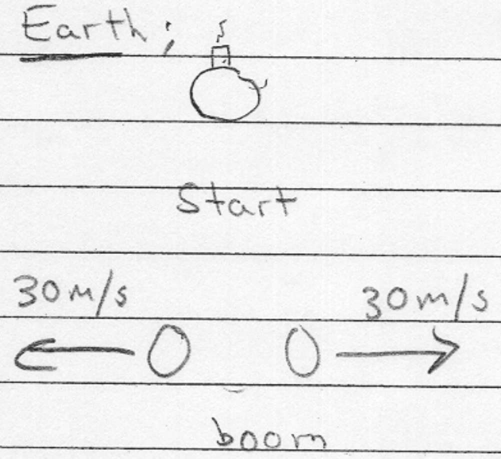
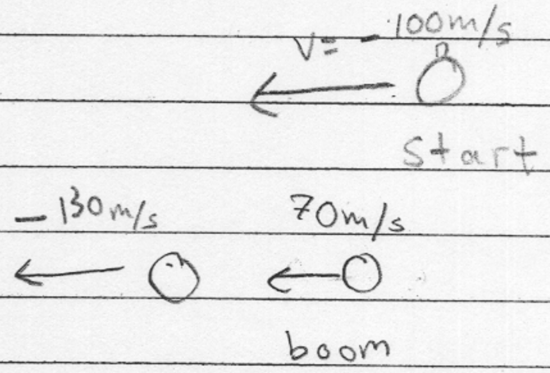
Problem 1

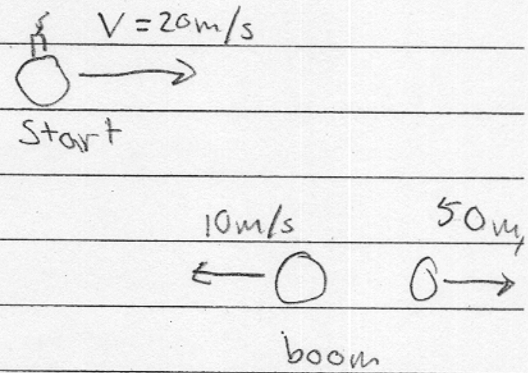
(a) Earth;



Observer B



Observer C :



(b) $u_b' = u_b - V$

u_b: velocity of bomb measured by earth.

V: velocity of B or C, relative to earth.

u_b': velocity of bomb measured by observer B or C.

For B:

$u_b' = u_b - V = 0 - (100ms) = -100m/s$ (see cartoon

For fragments we have

$$u_1' = u_1 - V \quad \text{and} \quad u_2' = u - V$$

- Where u_1 is the velocity of one fragment which moves with $+30\,m/s$ as measured by earth, i.e. the right mover
- $u_2 = -30\,m/s$ as measured by earth, the left mover

$$\left.\begin{aligned} u_1' &= 30\,m/s - 100\,m/s = -70\,m/s \\ u_2' &= -30\,m/s - 100\,m/s = -130\,m/s \end{aligned}\right\} \text{see cartoon}$$

(c) For C:

$$u_b' = u_b - V = 0\,m/s - (-20\,m/s) = 20\,m/s$$

$$u_1' = u_1 - V = 30\,m/s - (-20\,m/s) = 50\,m/s$$

$$u_2' = u_2 - V = -30\,m/s - (-20\,m/s) = -10\,m/s$$

(d) The clock starts at $(t, x) = (0, 0)$
The bomb explodes at $(t, x) = (5s, 0)$

d According to B these events happen at

$t' = t$

$x' = x - vt$

Start : $(t', x') = (0, 0)$

Explode: $t' = t = 5s$

$x' = x - vt = 0 - (100 m/s)(5s) = -500m$

i.e. $(t', x') = (5s, -500m)$

(e) According To C

Start $(t', x') = 0$

Boom:

$t' = t = 5s$

$x' = x - vt = 0 - (-20 m/s)\, 5s = 100m$

$(t', x') = (5s, 100m)$

(f) Bomb according to earth:

$x = 0$ for $t < 5s$

Fragment 1:

$x = (+30 m/s) \cdot (t - 5s)$ $t > 5s$

Fragment 2:

$x = -30 m/s \cdot (t - 5s)$ $t > 5s$

Let explain this more formally

$$X = u\,(t - t_0) + x_0$$

This is the equation of motion for constant velocity. Note we are measuring time starting after t_0. If t_0 is zero we have

$$x = ut + x_0 \qquad (x = x_0 + vt \text{ last year!})$$

Then x_0 is the position of the particle at time t_0. Thus (t_0, x_0) are "when and where" the motion starts

Fragment 1 starts at a time of 5s at a position $x_0 = 0$

$$x = u\,(t - 5s) + 0$$

It moves with a speed of 30m/s. So at a time of 1s after explosion or 6s after start the fragment is 30 m to the right

$$x = 30\,m/s\,(6s - 5s) + 0 = 30\,m$$

The formula works ✓

(g) For B:

$$x' = u'(t' - t_0') + x_0'$$

{ t_0', x_0' are when and where you start.

Bomb:

$$x' = (-100\,m/s)\,t'$$

{ Bomb starts at $(t_0', x_0') = (0,0)$

F1:

$$x' = (-70\,m/s)(t' - 5s) + -500\,m$$

F2:

$$x' = (-130\,m/s)(t' - 5s) + -500\,m$$

Note this result: the explosion starts at at $(t_0, x_0) = (5s, 0)$ on earth but happens at

$(t_0', x_0') = (5s, -500m)$ ← see part (f)

According to B

(h) According to C

Bomb:

$$x' = (20\,m/s)\,t' \qquad (t_0', x_0') = (0,0)$$

F1

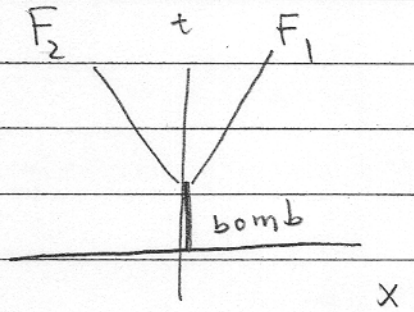
$$x' = (50\,m/s)(t' - 5s) + 100\,m$$

This follows $(t_0', x_0') = (5s, 100m)$ see part (e)

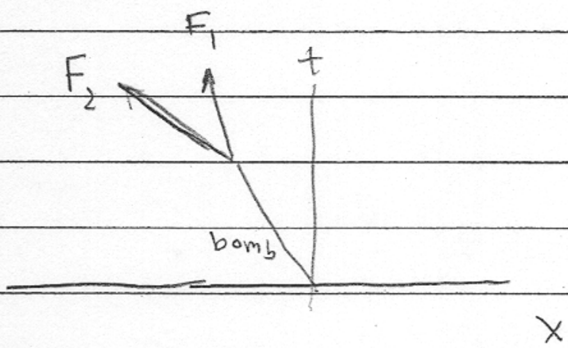
F2

$$x' = -10\,m/s\,(t' - 5s) + 100\,m \qquad \text{for } t > 5s$$

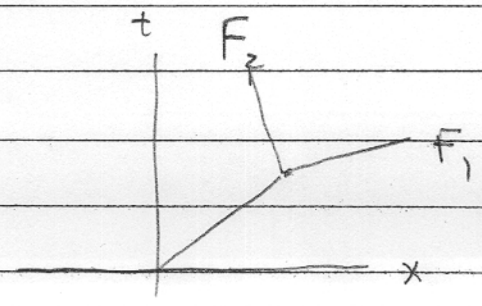
(i) Earth



(j)



(k)



Problem 2

Grader

[redacted]

derek.teaney@gmail.com

$$\Delta t = \gamma\, \Delta\tau$$

time on ground $\quad$ time sitting on clock in airplane

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \simeq 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 \quad \text{fro } v \ll c$$

So

$$\Delta t = \left(1 + \frac{1}{2}\left(\frac{v}{c}\right)^2\right) \Delta\tau$$

So Δt is <u>longer</u> by

$$(\Delta t - \Delta\tau) \simeq \frac{1}{2}\left(\frac{v}{c}\right)^2 \Delta\tau$$

$$\simeq \frac{1}{2} \cdot \left(\frac{400\,m/s}{3\times 10^8\,m/s}\right)^2 (3600\,s)$$

$$\boxed{\Delta t - \Delta\tau \simeq 0.32\times 10^{-8}\,s}$$

~~Problem 2: Not Graded~~

~~Problem 3 and Problem 4.4~~

a) $\gamma = \dfrac{1}{\sqrt{1-(V/c)^2}}$

$\gamma^2 = \dfrac{1}{(1-(V/c)^2)}$ or $\dfrac{1}{\gamma^2} = 1 - (v_p/c)^2$

so, $(v_p/c)^2 = 1 - \dfrac{1}{\gamma^2}$

b) For $\gamma \gg 1$ and $1/\gamma \ll 1$ we have

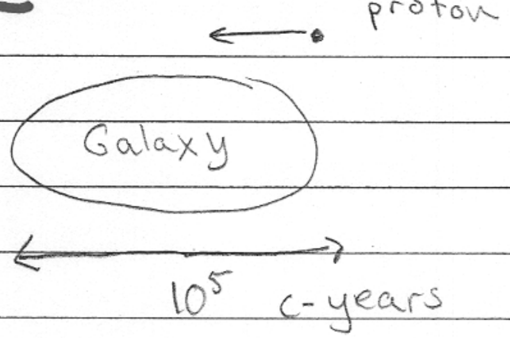
$v_p/c = \sqrt{1 - 1/\gamma^2} \simeq 1 - \dfrac{1}{2\gamma^2}$

where we used

$(1+x)^\alpha \simeq 1 + \alpha x$ for x small with $\alpha = \dfrac{1}{2}$ $x = -\dfrac{1}{\gamma^2}$

c) Problem 3

a)



The proton is moving very close to the speed of light

So

$$t_{galaxy} = \frac{d_{galaxy}}{v_p} = \frac{10^5 \ \ell\text{-years}}{\cancel{c}} = 10^5 \text{ years}$$

b) The particle time is:

$$t = \gamma \Delta\tau$$

Using

$$v_p/c \simeq 1 - \frac{1}{2\gamma^2}$$

$$\frac{1}{2\gamma^2} \simeq 1 - v_p/c$$

$$\gamma = \frac{1}{\sqrt{2(1-v_p/c)}} = \frac{1}{\sqrt{2\,(0.5 \times 10^{-20})}} = 1 \times 10^{10}$$

We find

$$\Delta\tau_{particle} = \frac{t}{\gamma} = \frac{10^5 \text{ years}}{1 \times 10^{10}} = \frac{10^5 \times 3.15 \times 10^7 \text{ s}}{1 \times 10^{10}}$$

1 year = 3.15×10^7 s

$$\Delta\tau_{part} = 3.15 \times 10^2 \text{ s} \simeq 315 \text{ s}$$

d) $1 \text{ c-year} = (3\times10^{8} \text{ m/s})(3.15\times10^{7}\text{ s})$

$= 9.45\times10^{15} \text{ m}$

e) $L \simeq \frac{L_p}{\gamma} \sim \frac{10^{5} \text{ c-years}}{1\times10^{10}} \simeq 9.45\times10^{10} \text{ m}$

$L \simeq 9.45\times10^{7} \text{ km}$

The distance between NY and California ~ 5000 km, Then 1 c-year as measured on earth is approximately 9400 km as measured by proton

Problem 4

- For $t = 1T$, $N = N_0\left(\frac{1}{2}\right)^{1} \Rightarrow N = \frac{10000}{2} = 5000$

That is one-half ✓

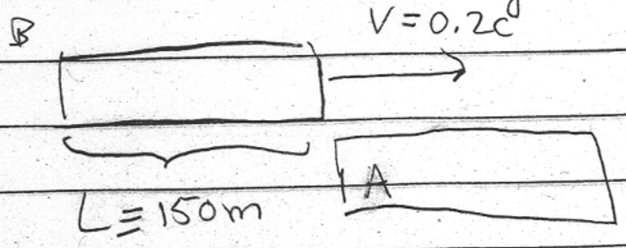
- For $t = 2T$, $N = N_0\left(\frac{1}{2}\right)^{2} \Rightarrow N = 2500$

That is 1/4

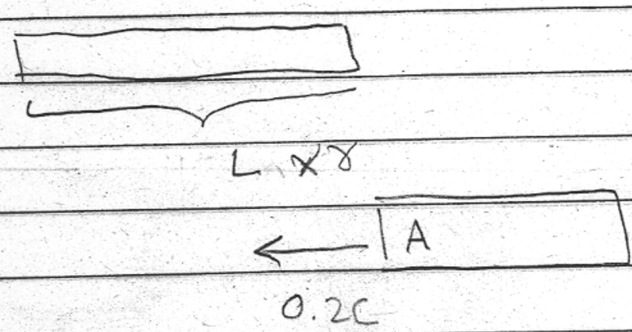
- For $t = 0.5T$, $N = N_0\left(\frac{1}{2}\right)^{0.5} \Rightarrow N = 10000 \times 0.7 \simeq 7071$

Prob 4

A has following Picture



B has picture



a) $L = \frac{L_0}{\gamma} \Rightarrow L_0 = L\gamma$ $\qquad \gamma = \frac{1}{\sqrt{1-\left(\frac{0.2c}{c}\right)^2}}$

$L_0 = (150m)\gamma$

$L_0 = 153$ $\qquad \gamma = 1.02$

b) Time measured by A

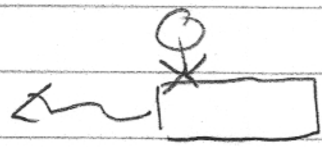
$$t = \frac{L}{V} = \frac{150m}{0.2c} = 750\ m/c$$

c) $$t_B = \frac{L_0}{V} = \gamma \frac{L}{V} = \gamma t_A = 765\ m/c$$

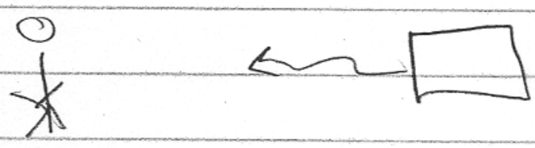
Prob 5

a) $\tau_0 = \frac{1}{f_0}$ $\qquad c = \lambda_0 f_0$

$$\boxed{\text{period} = \frac{1}{f_0}}$$

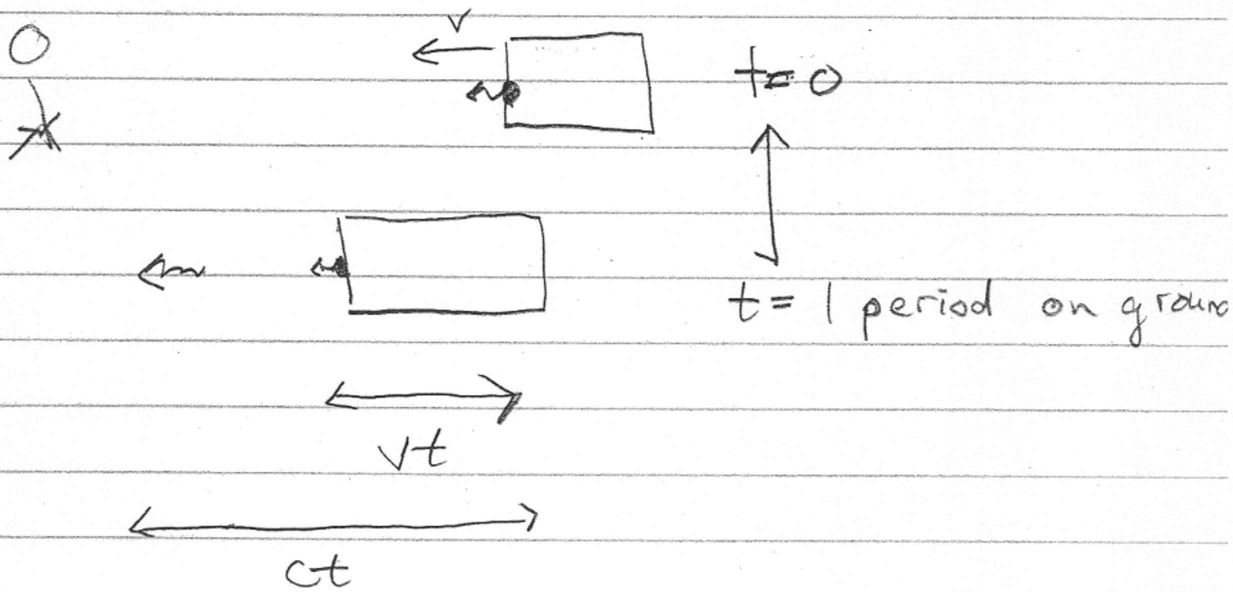


b)



$$\Delta t = \gamma \, \Delta\tau = \frac{1}{\sqrt{1-(v/c)^2}} \, \frac{1}{f_0}$$

c) Sketch

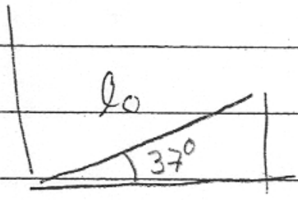


laser moved vt

light moved ct

Prob 6

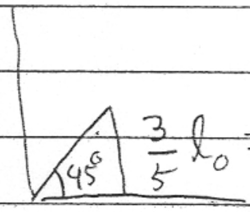
~~The~~ the rod is making a 3-4-5 triangle



$\frac{3}{5} l_0 = L_y$

$L_x = \frac{4}{5} l_0$

Then since the transverse directions are not contracted someone in ~~8~~ O sees



$\frac{3}{5} l_0 = L'_y$

$$L'_x = \frac{L_x}{\gamma} = \frac{3}{5} l_0$$

So

$$L'_x = \frac{L_x}{\gamma} = \frac{3}{5} l_0$$

$$\frac{4/5\, \cancel{l_0}}{\gamma} = \frac{3}{5} \cancel{l_0}$$

$$5/3 \cdot 4/5 = \gamma$$

$$\frac{4}{3} = \gamma$$

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$ or $\beta = \sqrt{1 - 1/\gamma^2} = \left(1 - \left(\frac{1}{4/3}\right)^2\right)^{1/2}$

$\beta = \sqrt{7}/4$ $\boxed{v = \sqrt{7}/4\, c \approx 0.66c}$