7.10

a) $E_A = K + mc^2$

$E_B = m_c^2$

$E_C = E_A + E_B = K + 2mc^2$

b) $cp_A = E_A^2 - mc^2$

$= (K + mc^2)^2 - mc^2$

$cp_A = K(K + 2mc^2)$

$cp_c = cp_A = K(K + 2mc^2)$

c) $m_c c^2 = E_c^2 - (cp_c)^2$

$= (K + 2mc^2)^2 - (K^2 + 2mc^2 K)$

$= 2mc^2 K + (2mc^2)^2$

$(m_c c^2)^2 = (2mc^2)^2 \left(1 + \frac{K}{2mc^2}\right)$

d) non-rel:

$m_c c^2 = 2mc^2 \qquad K \ll 1$

There is no limit

rel:

$(m_c c^2)^2 \simeq 2mc^2 K \Rightarrow m_c c^2 \simeq \sqrt{2mc^2 K}$

P1

$$E_{TOT} = E_1 + E_2 = \boxed{200\ \text{MeV} + 100\ \text{MeV} = E}$$

$$p^x_{TOT} = p^x_1 + \cancel{p^x_2}^{\,0} = 200\ \text{MeV/c}$$

$$p^y_{TOT} = \cancel{p^y_1}^{\,0} + p^y_2 = 100\ \text{MeV/c}$$

- Note we have used $E = cp$ so for the photon moving in the x direction

$$p = \frac{E}{c} = 200\ \frac{\text{MeV}}{c}$$

$$|\vec{p}_{TOT}| = \sqrt{(p^x_{TOT})^2 + (p^y_{TOT})^2} = \sqrt{(200)^2 + (100)^2}$$

$$\boxed{p \simeq 223\ \text{MeV/c}}$$

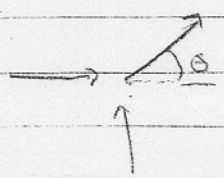
- Its mass is:

$$E^2 = (mc^2)^2 + (cp)^2$$

$$mc^2 = \sqrt{E^2 - (cp)^2} \simeq 200\ \text{MeV}$$

$$\boxed{m = 200\ \text{MeV}/c^2}$$

- Direction



$$\tan\theta = \frac{p^y}{p_x} = \frac{100\ \text{MeV/c}}{200\ \text{MeV/c}} = \frac{1}{2} \qquad \boxed{\theta \simeq 27^\circ}$$

Then speed

$$\frac{v}{c} = \frac{cp}{E} = \frac{\cancel{c} \cdot 223\ \text{MeV}/\cancel{c}}{300\ \text{MeV}} = 0.74$$

$$\boxed{v = 0.74c}$$

P2 First lets determine its mass;

$$E^2 = (mc^2)^2 + (cp)^2$$

$$mc^2 = \sqrt{E^2 - (cp)^2} \simeq \sqrt{5^2 - 3^2} \doteq 4\ \text{GeV}$$

$$\boxed{m \simeq 4\ \frac{\text{GeV}}{c^2}}$$

$$1\ \text{GeV}/c^2 \simeq 1.07\ \text{amu}$$

$$m \simeq 4.29\ \text{amu}$$

Part b

$$\boxed{m = 4.29\ \text{amu}}$$

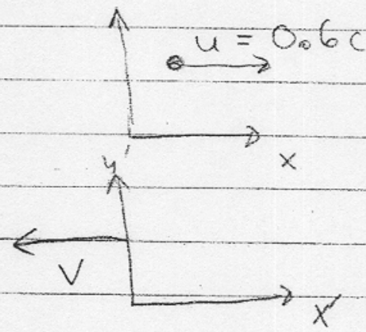
$$E = \sqrt{(mc^2)^2 + (cp)^2} = \sqrt{(4\ \text{GeV})^2 + (4\ \text{GeV})^2} \simeq 5.62\ \text{GeV}$$

The velocity in the first frame

$$u = c\left(\frac{cp}{E}\right) = c \cdot \frac{(3\,\mathrm{GeV})}{(5\,\mathrm{GeV})} = 0.6c$$

$$u' = c\left(\frac{cp}{E}\right) = c\left(\frac{4\,\mathrm{GeV}}{5.62\,\mathrm{GeV}}\right) = 0.711c$$

The picture: is the first frame measures



Then an observer moving to the left with relative speed v measures

$$u' = 0.711c$$

Then

$$u' = \frac{u - v}{1 - uv/c^2} \qquad v < 0$$

$$u' = \frac{u + |v|}{1 + u|v|/c^2}$$

Solving for $|v|$

$$u' + u'u|v|/c^2 = u + |v|$$

$$u' - u = |v| - uu'/c^2\,|v|$$

or

$$|v| = \frac{u' - u}{1 - uu'/c^2} = \frac{0.711c - 0.6c}{1 - (0.711c)(0.6c)/c^2}$$

$$\boxed{|v| = 0.19c}$$

<u>P6.4</u>

E $\rightsquigarrow$ m_o

a) $E_f = E + m_o c^2 = E + m_o c^2$

$p_f = p_1 + \cancel{p_2} = \frac{E}{c}$ $\qquad cp = E$

The velocity:

$$\frac{u}{c} = \frac{(cp_f)}{E_f} = \frac{E}{E + m_o c^2}$$

b) $m_o \ u_1 = 4/5c$ $\qquad m_o \ u_2 = 0$

$E_f = \gamma_1 m_o c^2 + \gamma_2 m_o c^2$ $\qquad \gamma_1 = \frac{1}{\sqrt{1 - (4/5c/c)^2}}$

$E_f = \frac{5}{3} m_o c^2 + m_o c^2 = \frac{8}{3} m_o c^2$ $\qquad \gamma_1 = \frac{5}{3}$

$p_f = p_1 + \cancel{p_2}$

Then

$$p_f = \gamma m_0 u_1 = \frac{5}{3} m_0 \cdot \left(\frac{4}{5} c\right) = \frac{4}{3} m_0 c$$

The final velocity

$$u_f = \frac{c^2 p_f}{E_f} = c^2 \frac{4/3\, m_0 c}{8/3\, m_0 c^2} = \boxed{\frac{1}{2} c}$$

$$m_f c^2 = \sqrt{E_f^2 - (cp_f)^2} = \sqrt{(8/3\, m_0 c^2)^2 - (4/3\, m_0 c^2)^2} = \boxed{\frac{4\sqrt{3}}{3} m_0 c^2}$$

P3

Part a:

$K_\pi = 1\,\text{GeV}$ $\qquad E_2 p_2 \qquad E_1 p_1$

- The energy of the pion is $E_\pi = K + m_0 c^2 = 1.135\ \text{GeV}$
- The momentum of the pion is

$$E^2 = (cp)^2 + (mc^2)^2$$

$$cp = \sqrt{E^2 - (mc^2)^2} \simeq 1.127\ \text{GeV}$$

Energy and momentum give:

$$E_\pi = E_1 + E_2 \qquad p_1 = E_1/c$$

for photons

$$p_\pi = p_1 - p_2 \qquad p_2 = E_2/c$$

So

$$E_\pi = E_1 + E_2$$

$$cp_\pi = cp_1 - cp_2 = E_1 - E_2$$

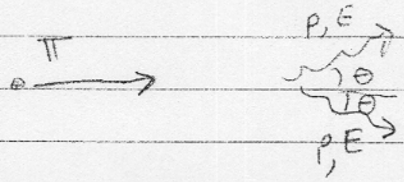
Solving for E_1 and E_2

$$\frac{E_\pi + cp_\pi}{2} = E_1 = \frac{1.135 + 1.127}{2} \text{ GeV} \simeq \boxed{1.131 \text{ GeV} = E_1}$$

$$\frac{E_\pi - cp_\pi}{2} = E_2 = \frac{1.135 - 1.127}{2} \text{ GeV} \simeq 0.004 \text{ GeV}$$

$$\simeq \boxed{4 \text{ MeV} = E_2}$$

(b)



The symmetry dictates that the two photons have equal momenta:

- $E_\pi = E + E$
- $p_\pi = p\cos\theta + p\cos\theta$
- $0 = p\sin\theta - p\sin\theta$

Then:

$$E_\pi = 2E$$

$\boxed{\Theta \simeq 6.9^\circ \leftarrow \text{The Book quotes } 2\Theta \simeq 14^\circ}$

7.1 $\qquad K \longrightarrow \qquad \underset{\pi}{\overset{E_2}{\circ}} \ \underset{\pi}{\overset{E_1}{\bullet\!\!\longrightarrow}}$

Energy and momentum:

(1) $\quad E_K = E_1 + m_\pi c^2$

(2) $\quad cp_K = cp_1 + \cancelto{0}{cp_2}$

Now count: two equations (1) + (2) and

(3) $\quad E_K^2 = (cp_K)^2 + (m_K c^2)^2 \qquad$ (4) $\quad E_1^2 = (cp_1)^2 + (m_\pi c^2)^2$

Unknowns:

E_K and E_1, cp_K and cp_1 and

four equations (1), (2), (3), (4)

Problem 4

$E = 50 \text{ meV}$

$m = 0.511 \frac{\text{MeV}}{c^2}$

$\gamma = \frac{E}{mc^2} = \frac{50 \text{ meV}}{0.511 \text{ meV}} = 97.84$

$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$

$(v/c)^2 = 1 - \frac{1}{\gamma^2}$ $\qquad v/c \simeq 1 - \frac{1}{2\gamma^2}$

$v/c = 1 - 0.522 \times 10^{-4}$

Prob 5

$\beta = 0.998$ $\qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = 15.8$

$E = \gamma mc^2$ $\qquad p = \gamma mv$

$E = (15.8)(0.511 \text{ MeV})$

$cp = \gamma mc^2 \frac{v}{c}$

$E = 8.08 \text{ MeV}$

$cp = 8.0674 \text{ MeV}$

$K = E - mc^2$

$p = 8.0674 \text{ meV}/c$

$K = (8.08 \text{ MeV}) - (0.511 \text{ meV})$

$K = 7.5 \text{ MeV}$

6

$$K = \gamma mc^2 - mc^2$$

$$\gamma = \frac{1}{(1-\beta^2)^{1/2}}$$

$$\gamma \approx 1 + \left(\frac{-1}{2}\right)(-\beta^2) + \frac{-\frac{1}{2} \cdot -\frac{3}{2}(-\beta^2)^2}{2!}$$

$$\gamma \simeq 1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4$$

$$K = \left(1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4\right)mc^2 - mc^2$$

$$K = \frac{1}{2}\frac{v^2}{c^2}mc^2 + \frac{3}{8}\beta^4 mc^2$$

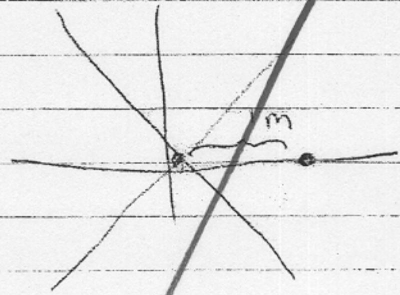
$$K = \frac{1}{2}mv^2 + \frac{3}{8}mv^2\beta^2$$

$$\%\text{err} = \frac{\left(K - \frac{1}{2}mv^2\right)}{\frac{1}{2}mv^2} = \%\text{ deviation of } K \text{ from } \frac{1}{2}mv^2$$

$$\%\text{err} = \frac{3/8\, mv^2\beta^2}{1/2\, mv^2} = \frac{3}{4}\beta^2 = 0.01$$

$$\beta \cong 0.12c$$

c)



$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

$$(\Delta s)^2 = (0)^2 - (1m)^2 = -1m^2$$

d)

$$E' = \gamma E - \gamma\beta cp$$

$$cp' = -\gamma\beta E + \gamma cp$$

$$(E')^2 - (cp')^2 = (\gamma E - \gamma\beta cp)^2 - (-\gamma\beta E + \gamma cp)^2$$

$$= \gamma^2 E^2 - 2\gamma^2 \beta E cp + \gamma^2\beta^2 (cp)^2$$

$$- \gamma^2\beta^2 E^2 + 2\gamma^2 \beta E cp - \gamma^2 (cp)^2$$

$$(E')^2 - (cp')^2 = E^2 - (cp)^2$$

Problem 7

a) Binding Energy $\simeq 16 \times \frac{BE}{\text{nucleon}} \simeq 16 \times 8 \frac{MeV}{\text{nucleons}}$

$$BE \simeq 128 \text{ MeV}$$

b) $^{7}_{3}Li$ 3 protons and 4 neutrons

(c) $\ ^{2}_{1}H + \ ^{1}_{1}H \longrightarrow \ ^{3}_{2}He$ + energy

Then we have the energy required to tear apart the $H^2_1 + H_1$ into 2 protons and two neutrons is the binding energy of H^2_1

$$BE\ H^2_1 = 2 \times (1\ MeV) = 2\ MeV$$

The energy gained by assembling the two protons and neutron into $\ ^{3}_{2}He$ Is

$$3 \times (2.5\ MeV) \simeq 7.5\ MeV$$

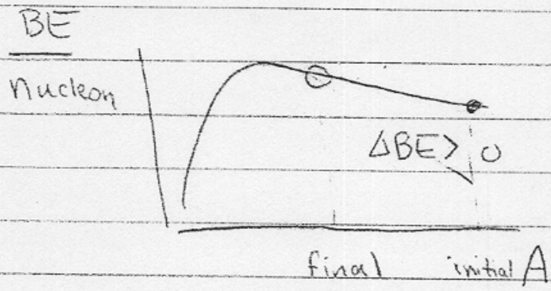
So the net energy released is

$$\boxed{\simeq 5.5\ MeV}$$

(d) By Fusing two ^{56}Fe nuclei the energy balance is:

$$\begin{pmatrix} \text{mass of} \\ 2\times 56 \text{ protons} \\ \text{+ neutrons} \end{pmatrix} - \begin{pmatrix} \text{Binding Energy} \\ \text{of two } ^{56}Fe \end{pmatrix} \longrightarrow \begin{pmatrix} \text{mass of} \\ 2\times 56 \text{ protons} \\ \text{+ neutrons} \end{pmatrix} + \begin{pmatrix} - \text{ BE of nucleus} \\ \text{w 112 protons} \\ \text{+ neutron} \end{pmatrix} + \text{Energy released}$$

Then for fission you move down the



Thus for the following process you move down the x-axis and $\Delta BE > 0$.

Since $Q = \Delta BE$ the energy released is positive

See part e