Problems:

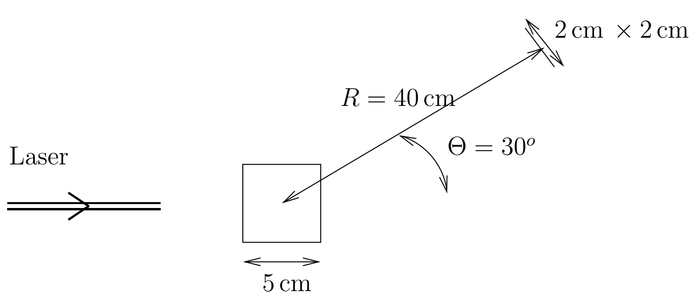
$$4.8, 4.9, 4.10, 4.11 \tag{1}$$

1. Use half angle formulas to show that
$$\int \frac{\sin(\theta)d\theta}{\sin^4(\theta/2)} = -\frac{2}{\sin^2(\theta/2)} \tag{2}$$
which may be useful for problem 4.8.

2. The cross section for backward scattering would be defined as the number of particles which scatter with $\Theta > \pi/2$ divided by the number of α's and Au nuclei which cross per unit area
$$\sigma_{\text{backwards}} = \frac{\text{Number which scatter with } \Theta > \pi/2}{\text{Number of } \alpha \text{ and Au which cross per area}}$$
$$\sigma_{\text{backwards}} = \frac{\text{Number which scatter with } \Theta > \pi/2}{N_\alpha \rho_{Au} t_{\text{foil}}}$$
Show that this is
$$\sigma_{\text{backwards}} = \frac{\pi D^2}{4}$$
where D is the distance of closest approach. Use the result of problem 4.8. Note the cross section is independent of the the details of the target and beam experimental characteristics. Note that it has dimensions of area and is a measure of the size of the interaction area between the α particle and the gold nuclei. Measuring the cross section ultimately measures D.

3. A point source of photons emits photons in all possible directions but not uniformly. In particular the number of photons emitted per unit time per unit solid angle is
$$\frac{dN}{dtd\Omega} = N_o(1 + \cos^2(\Theta)) \tag{3}$$
where $N_o = 2 \times 10^{19}$ photons/sec/str. (a) Determine the total number of photons emitted by the source. The integrals should be easy enough. (b) If all these photons were emitted uniformly what would be $\frac{dN}{dtd\Omega}$. (Answers (a) 0.33×10^{21} photons/sec (b) 0.26×10^{20} photons/sec/str)

4. A glass square cell contains an unkonwn gas. Laser light with $\lambda = 422.7\,\text{nm}$ is beamed at the the glass cell and scattered by the gas uniformly in all directions. The total power emitted by the laser of $30\,\text{mW}$, and 1/10th of the incident power gets scattered by the gas while 9/10th of the power passes through the gas. To detect the scattered radiation a photodetector with collecting area which is $2\,\text{cm} \times 2\,\text{cm}$ is placed 40 cm away from the interaction region at an angle of 30^o. (Answers: (a) 0.0025 str (b) $\Delta\Theta = 2.86^o$ $\Delta\phi = 5.72^o$ (c) $0.51 \times 10^{15}\,1/s$ (d) $1.27 \times 10^{12}\,1/s$)
   (a) Determine the solid angle angle subtended by the detector
   (b) Determine the angles $\Delta\Theta$ and the angle $\Delta\phi$ subtended by the detector. Take the laser to point along the Z-axis and ϕ the angle around this Z-axis (see also Fig. 1).
   (c) Determine the number of photons scattered per unit time per unit solid angle $dN/(dtd\Omega)$.
   (d) Determine the number of photons which are absorbed by the photodector per unit time.



Physical Constants

1. 2π is annoying, often use

$$\hbar = \frac{h}{2\pi} \qquad \hbar c = 197\text{eV nm} \tag{4}$$

2. The fine structure constant is a pure number and is is the only dimensionless quantity that can be made out $\hbar, c$ and e

$$\alpha_{\rm EM} = \frac{e^2}{4\pi\epsilon_o \hbar c} \simeq \frac{1}{137} \tag{5}$$

The coulomb potential between to objects with $Z_1 e$ charge and $Z_2 e$ charge is

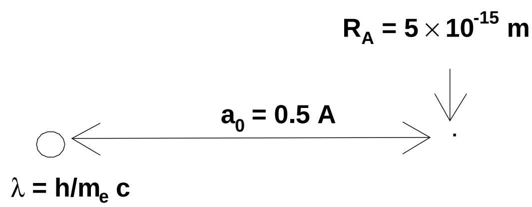
$$U = \frac{1}{4\pi\epsilon_o}\frac{Z_1 e Z_2 e}{r} \tag{6}$$

$$= \alpha_{\rm EM}\frac{Z_1 Z_2\, \hbar c}{r} \tag{7}$$

3. The mass of the electron and protons are

$$m_e c^2 \simeq 0.511\,\text{MeV} \qquad m_p c^2 \simeq 938\,\text{MeV} \tag{8}$$

4. The picture of the atom is the following (the circle is the electron and the dot is the nucleus)



Solid Angles

1. For a patch on the sphere of area A, the solid angle is defined as

$$\Omega \equiv \frac{A}{r^2} \tag{9}$$

in analogy to $\theta = s/r$. Like radians, the units of solid angle are dimensionless. However, we sometimes use the unit ster-radians to denote that a given dimensionless numbers corresponds to a definite solid angle.

2. For a small area dA you should be able to show from the picture below that

$$d\Omega = \frac{dA}{r^2} = \sin(\Theta)\, d\Theta d\phi \tag{10}$$

The area of a little patch of area dA on the sphere is

$$dA = r^2 d\Omega = r^2 \sin(\Theta)\, d\Theta d\phi \tag{11}$$

The volume of the patch is $dV = r^2 \sin(\Theta)\, d\Theta d\phi dr$

3. The integral of some quantity over the sphere is

$$\int_{\rm sphere} d\Omega \text{ (some quantity)} = \int_0^{\pi} d\Theta \sin\Theta \int_0^{2\pi} d\phi \text{ (some quantity)} \tag{12}$$

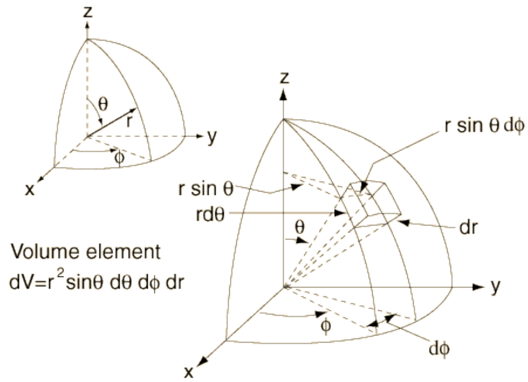


FIG. 1: Figure illustrating spherical coordinates and the concept of solid angle. You should be able to show $d\Omega = \sin(\Theta)d\Theta d\phi$

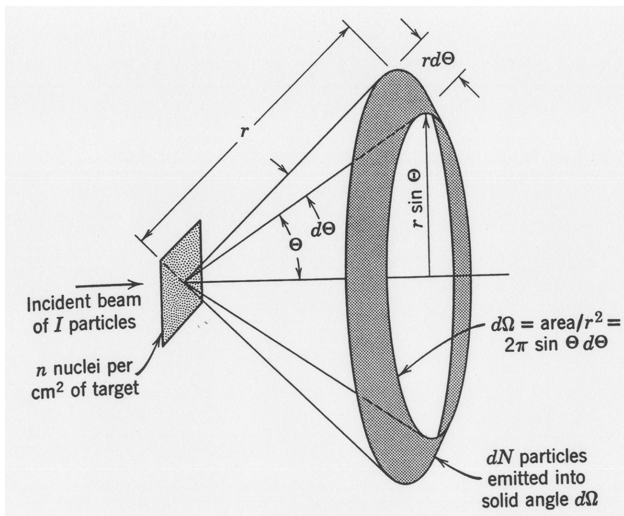


FIG. 2: Figure illustrating the concept of solid angle. You should be able to show that the solid angle subtended by the strip is $2\pi\sin(\Theta)d\Theta$

4. For a small cylindrical stip as shown below you should be able to show that

$$d\Omega = 2\pi\sin(\Theta)d\Theta \tag{13}$$

Rutherford Experiments

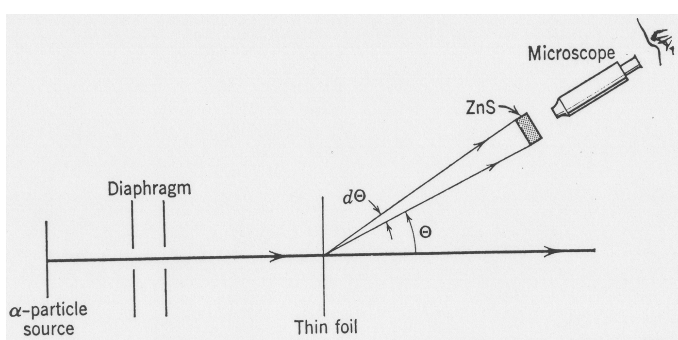


FIG. 3: Schematic of the Rutherford experiment

The Rutherford experiments shot α particles ($^4_2$He) onto gold nuclei $^{197}_{79}$Au. A schematic of the appartus is shown in Fig. 3

1. When alpha ($z_\alpha = +2$) particles impinge on a nucleus of charge $+Ze$ the distance of closest approach D is found by equating the initial kinetic energy with the final potential energy

$$K = \frac{1}{2} m_\alpha v_\alpha^2 = \frac{1}{4\pi\epsilon_o} \frac{(Ze)(z_\alpha e)}{D} \tag{14}$$

$$\frac{1}{2} m_\alpha v_\alpha^2 = \alpha_{EM} \frac{Z z_\alpha \hbar c}{D} \tag{15}$$

   i.e.

$$D = \alpha_{EM} \left(\frac{Z z_\alpha \hbar c}{\frac{1}{2} m_\alpha v_\alpha^2} \right)$$

2. The number of α particles scattered into a given solid angle is

$$dN_{\mathrm{scatt}} = [N_\alpha \rho_{Au} t_{\mathrm{foil}}] \times \frac{D^2}{16} \frac{1}{\sin^4(\Theta/2)} \times d\Omega \tag{16}$$

   where

$$N_\alpha = \text{Number of } \alpha' s \text{ sent in. } \textbf{The book calls this } I \tag{17}$$

$$\rho_{Au} = \text{The number of gold nuclei per volume in the foil} \tag{18}$$

$$t_{\mathrm{foil}} = \text{The thickness of the foil} \tag{19}$$

$$d\Omega = \text{the solid angle of the detector} \tag{20}$$

   For a ring like detector as shown in Fig. (3) the number scattered into the detector is

$$dN_{\mathrm{scatt}} = [N_\alpha \rho_{Au} t_{\mathrm{foil}}] \times \frac{D^2}{16} \frac{1}{\sin^4(\Theta/2)} \times 2\pi \sin(\Theta) d\Theta \tag{21}$$