**Problems:**

$$4.23, 4.26\ \textbf{(definitely graded)}, 4.28, 4.35, 3.2 \tag{1}$$

**Answers:** (4.23): $n = 1$, $0.5\AA$, $\hbar$, $3.94keV/c$, 43 fempto-Hz, 1/137, 54.4eV/$\AA$, $a = \alpha^2 c^2/a_o$, $KE = 13.6$, $PE = -27.2$, $E = -13.6$, $r \propto n^2$, $E \propto 1/n^2$ (4.26): Six energies in eV: 12.75, 2.55, 0.66, 1.88, 12.088, 10.2., part (c) $v_{\rm recoil} \simeq 3.74\,{\rm m/s}$ . For part ($c$) use momentum conservation and the momentum of the photon $E = h\nu/c$.

**Additional Problems: One of these will be graded**

1. Without looking up the numbers, determine the potential energy of two electrons separated by 3 Bohr Radii in units of eV.
2. In class we have derived a formula for an electron orbiting a proton which says that
$$\frac{v_1}{c} = \alpha \simeq \frac{1}{137} \tag{2}$$
When an electron orbits $Z$ protons as in a nucleus, show that
$$\frac{v_1}{c} = Z\alpha \tag{3}$$
Approximately how many protons are needed in a nucleus before the inner electron is no longer non-relativistic.
3. **Important.** (a) Determine the ratio between the compton waveleght $\lambda_C$ and the Bohr radius and express your result in terms of the fine structure constant. (b) Determine the ratio between the wavelength of light emitted in the $n = 2$ to $n = 1$ transiton of hydrogen and the Bohr radius. Work with symbols rather than numbers. Express your result in terms of the fine structure constant. Draw a schematic picture showing the electron, the atom and the light.
4. **Important.** This problem will derive the Bohr radius in another way
   (a) Consider the n-th orbit. Show (using the fact that the $p = mv$ and $L = mvr$) that the momentum of an electron in the n-th orbit
   $$p = \frac{n\hbar}{r_n} \tag{4}$$
   (b) Starting from Newton's law, show for an electron in circular orbit that
   $$KE = -\frac{1}{2}PE \tag{5}$$
   This result is known as the virial theorem.
   (c) Show that the kinetic energy of the electron can be written
   $$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m} \tag{6}$$
   and use this to find a formula for the kinetic energy in terms of $\hbar, m, r_n$ (Answer: $n^2\hbar^2/(2mr_n^2)$ )
   (d) Use the above relation between $KE$ and $PE$ to determine $r_n$. Verify that this is the same expression for $r_n$ derived in class. The whole set-up is self consistent.
   (e) **Ultra Important** Take the lowest orbit which has radius $a_o$. Show
   $$R_\infty \equiv \frac{1}{4\pi\epsilon_o}\frac{e^2}{2a_o} = \frac{\hbar^2}{2m_e a_o^2} = \frac{1}{2}m_e c^2 \alpha^2$$
   where $R_\infty$ is the Rydberg constant. Give a simple physical explantion for each equality in this set of relationships. Show that the total energy in the ground state $KE + PE$ is
   $$E_1 = -\frac{1}{4\pi\epsilon_o}\frac{e^2}{2a_o}$$
   Determine $R_\infty$ numerically (Answer=13.6 eV).

5. Describe briefly how a spectrometer works. Use this video on you tube here

**Physical Constants**

1. Often we use $\hbar = \frac{h}{2\pi}$. The quantity $\hbar c$ is a very useful conversion factor

$$\hbar c = 197\text{eV nm} \tag{7}$$

2. The fine structure constant is a pure number and is is the only dimensionless quantity that can be made out $\hbar, m_e, c$ and $e$

$$\alpha = \frac{e^2}{4\pi\epsilon_o \hbar c} \simeq \frac{1}{137} \tag{8}$$

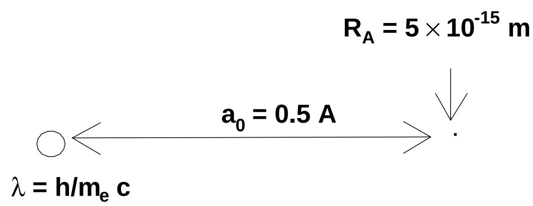
3. The mass of the electron is

$$m_e c^2 \simeq 0.5 MeV \tag{9}$$

4. The Bohr radius is

$$a_0 = \frac{\hbar}{m_e c}\frac{1}{\alpha} = \frac{\hbar^2}{m_e e^2/(4\pi\epsilon_o)} \simeq 0.5\mathring{A} \tag{10}$$

The picture of the atom is the following (the circle is the electron and the dot is the nucleus)




**Bohr Model**

1. Electrons move about the nucleus in circular orbits determined by Newtons Laws
2. Only certain orbits are stable, in these orbits the angular momentum of the electorn is

$$L = m_e v r = n\hbar n = 1, 2, 3, \ldots \tag{11}$$

3. For these orbits (labelled by n) we have

$$\begin{aligned} E_n &= -\overbrace{\frac{1}{4\pi\epsilon_o}\frac{e^2}{2a_o}}^{\equiv R_\infty}\left[\frac{1}{n^2}\right] \Leftrightarrow \text{Energy} && (12)\\ &= -13.6\text{eV}\left[\frac{1}{n^2}\right] && (13) \end{aligned}$$

where $R_\infty$ is the Rydberg constant $R_\infty = 13.6\,\text{eV}$. For the n-th orbit we also have

$$\begin{aligned} \frac{v_n}{c} &= \alpha\left[\frac{1}{n}\right] \Leftrightarrow \text{velocity} && (14)\\ r_n &= a_0\left[n^2\right] \Leftrightarrow \text{radius} && (15)\\ & && (16) \end{aligned}$$

4. An extremely important set of relationships is

$$\frac{1}{4\pi\epsilon_o}\frac{e^2}{2a_o} = \frac{\hbar^2}{2m_e a_o^2} = \frac{1}{2}m_e c^2 \alpha^2 = 13.6\text{eV}$$

5. Light of a given frequency is emitted as the atom makes a transition from one $n$ (say $n = 2$) to another (say $n = 1$). If light is emitted, the change in energy $\Delta E = E_f - E_i$ of the atom is negative sense the atom lowers its energy by emitting light energy which makes up the change. The frequency of the light which is emitted is given by energy conservation.

$$hf = (E_i - E_f) \tag{17}$$

6. The above formulas are for a single electron running around a single proton. When a single electron runs around $Z$ protons the formulas become

$$
\begin{aligned}
L = m_e v r &= n\hbar \qquad n = 1, 2, 3 \ldots && (18)\\
\frac{v_n}{c} &= Z\alpha \left[\frac{1}{n}\right] \Leftrightarrow \text{velocity} && (19)\\
E_n &= -\frac{1}{4\pi\epsilon_o}\frac{e^2}{2a_o} Z^2 \left[\frac{1}{n^2}\right] \Leftrightarrow \text{Energy} && (20)\\
&= -13.6\text{eV} Z^2 \left[\frac{1}{n^2}\right] && (21)\\
r_n &= \frac{a_0}{Z}\left[n^2\right] \Leftrightarrow \text{radius} && (22)
\end{aligned}
$$

**Experiments**

1. Diffraction grating can separate light into its spectrum. The angle of deflection is

$$\sin(\theta) = m\frac{\lambda}{d} \qquad m = 0, 1, 2, 3 \ldots \tag{23}$$

where $m$ is an integer which counts the maximums, and $d$ is the spacing between slits.

**Basic notions of wave functions**

1. DeBroglie says that the momentum is related to the wavelength

$$p = \frac{h}{\lambda} = \hbar\frac{2\pi}{\lambda} = \hbar k \tag{24}$$

2. Similarly the frequency determines energy

$$E = \hbar\omega \qquad \omega = 2\pi\nu \tag{25}$$

where $\nu$ is the frequency.

3. If the typical size of the wave function is $\Delta x$ then the typical spread is in the momentum $\Delta p$ is determined by the uncertainty relation

$$\Delta x \Delta p \gtrsim \hbar/2 \tag{26}$$

4. Similarly if the typical duration of a wave pulse (of e.g. sound, E&M, or electron wave) is $\Delta t$ then its frequency $\omega$ is only determined to within $1/\Delta t$. In quantum mechanics this is written

$$\Delta t \Delta\omega \sim \frac{1}{2} \qquad \text{or} \qquad \Delta t \Delta E \gtrsim \hbar/2$$

i.e. if something is observed for a short period of time its energy can not be precisely known

5. In general an attractive potential energy tends to localize (make smaller) the particles wave function. As the particle is localized the kinetic energy increases. The balance determines the typical extent of the wave function (or the size of the object).