① $PE = \alpha \frac{\hbar c}{(3a_0)} = \frac{1}{137} \cdot \frac{197\,eV\,nm}{3 \times (0.05\,nm)}$

$PE = 9.58\,eV$

also $PE = \frac{e^2}{4\pi\varepsilon_0 (3a_0)} = \frac{1}{3}\left(\frac{e^2}{4\pi\varepsilon_0 a_0}\right) = \frac{27.2\,eV}{3}$

② $F = ma$

$\frac{Ze^2}{4\pi\varepsilon_0} \frac{1}{r^2} = m\left(\frac{v^2}{r}\right)$ $\qquad L = mvr = \hbar$

$v = \frac{\hbar}{mr}$

$\frac{Ze^2}{4\pi\varepsilon_0} \frac{1}{r^2} = \frac{\hbar^2}{mr^3}$

$\frac{v}{c} = \frac{\hbar c}{mc^2 r}$

$Z\alpha \frac{\hbar c}{r^2} = \frac{(\hbar c)^3}{mc^2 r^3}$

$v/c = \frac{\hbar c}{mc^2 \frac{\hbar c}{mc^2 Z\alpha}}$

$r = \frac{\hbar c}{mc^2 \alpha Z}$

$\boxed{v/c = Z\alpha}$

(3) $a_0 = \frac{\hbar}{mc\alpha}$

$$a_0 = \frac{1}{2\pi}\left(\frac{h}{mc}\right)\frac{1}{\alpha} = \frac{137}{2\pi}\overbrace{\left(\frac{h}{mc}\right)}^{\lambda_c}$$

a) $\frac{a_0}{\lambda_c} = \frac{137}{2\pi} \simeq 21$

b) $h\nu = E_1 - E_2 = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{2a_0}\left[-\frac{1}{2^2} - \frac{1}{1^2}\right]$

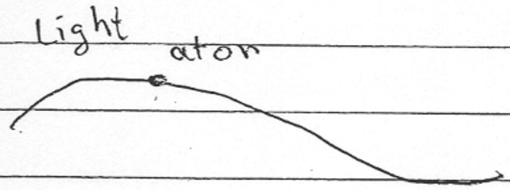
$$\frac{hc}{\lambda} = \frac{e^2}{4\pi\varepsilon_0 \hbar c}\frac{\hbar c}{2a_0}\cdot\frac{3}{4}$$

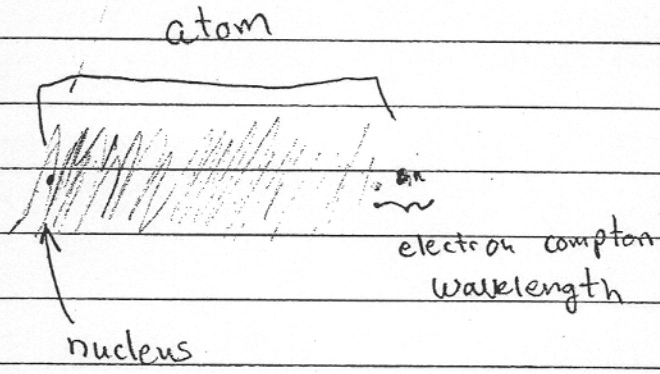
$$\frac{hc}{\lambda} = \alpha\,\frac{hc}{a_0}\,\frac{3}{4}\,\frac{1}{2\pi}$$

$$\frac{2\pi\cdot 4/3}{\alpha} = \frac{\lambda}{a_0}$$

$$\frac{1}{\alpha}(8\pi/3) = \frac{\lambda}{a_0} \simeq 1147$$

Picture:





(4) $L = mvr = n\hbar$

(a) $p = n\frac{\hbar}{r}$

(b) $\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r^2} = m\frac{v^2}{r}$

$$\left(\frac{e^2}{4\pi\varepsilon_0 r}\right) = \left(mv^2\right)$$

$-PE = 2KE$

(C) $KE = \frac{1}{2} m v^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$

$KE = \frac{(n\hbar/r)^2}{2m} = n^2 \frac{\hbar^2}{2mr^2}$

d) $\frac{1}{2} \frac{e^2}{4\pi\varepsilon_0 r_n} = \frac{n^2 \hbar^2}{2mr_n^2}$

$r_n = n^2 \frac{\hbar^2}{\left(2m \frac{e^2}{4\pi\varepsilon_0}\right)} = n^2 \frac{\hbar^2}{\left(2m \left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right) \hbar c\right)}$

$$\boxed{r_n^2 = n^2 \frac{\hbar}{mc\alpha}}$$

e) For the lowest orbit this equation reads

$\frac{1}{2} \frac{e^2}{4\pi\varepsilon_0 a_0} = \frac{\hbar^2}{2ma_0^2}$

Thus this equality is a reflection of the balance between kinetic and potential

With $a_0 = \frac{\hbar}{mc\alpha}$ $\quad \frac{\hbar^2}{2ma_0^2} = \frac{(\hbar c)^2}{2mc^2 \left(\frac{\hbar c}{mc^2 \alpha}\right)^2} = \frac{1}{2}(mc^2)$

Problem 5

$\sin\theta = n \frac{\lambda}{d}$ $\qquad n = 1$

$\frac{\lambda}{d} = \left(\frac{\lambda}{100 nm}\right) \frac{100 nm}{\left(\frac{2.54 cm}{10,000}\right)} \approx \left(\frac{\lambda}{100 nm}\right) 0.039$

Now for small angles $\sin\theta \approx \theta$

θ in rad $= \left(\frac{\lambda}{100 nm}\right) 0.039$

θ in degrees $= \frac{180}{\pi}(\theta \text{ in rad})$

$\theta \approx 2.25^\circ \left(\frac{\lambda}{100 nm}\right)$

So

$\theta_{uv} = 5.5^\circ$ $\qquad \lambda = 200 nm$

$\theta_{green} = 2.25^\circ \left(\frac{540}{100}\right) = 12^\circ$ $\qquad \lambda = 540 nm$

$\theta_{Red} = 15.8^\circ$ $\qquad \lambda = 700 nm$

UV light is more energetic and is deflected less

~~Thus the last equality is a reflection of the fact that the velocity is $\alpha \cdot c$~~

~~i.e. the electrons are non-relativistic~~

~~4) — skipped (It had some numerical inconsistencie~~

~~(5) See next page~~

4.23

a) $n=1$

b) $a_0 = 0.5\,\mathring{A}$

c) $\hbar$

d) $mv = \frac{\hbar}{a_0} = \frac{\hbar c}{c a_0} = \frac{197\ eV\cdot nm}{c(0.05\,nm)} = \frac{3.94\ keV}{c}$

e) $\omega = \frac{v}{r} = \frac{\alpha c}{r} = \frac{1}{137} \cdot \frac{3\times 10^8\,m/s}{(0.5\times 10^{-10}\,m)} = 43 \times 10^{15}\ \frac{cycles}{s}$

$= 43$ femto Hz

f) $v = \frac{1}{137} c$

g) $F = \frac{mv^2}{r} = \frac{mc^2\alpha^2}{a_0} = \frac{27.2\ eV}{0.5\ \mathring{A}} = 54.4\ eV/\mathring{A}$

$= \frac{54.4 \times 1.6\times 10^{-19}\,J}{1\times 10^{-10}\,m} = 87 \times 10^{-9}$

h) $a = \frac{v^2}{r} = \alpha^2 \left(\frac{c^2}{a_0}\right)$

i) $KE = \frac{\hbar}{2ma_0^2} = 13.6eV$

j) $PE = \frac{-e^2}{4\pi\varepsilon_0 a_0} = -27.2eV$

k) $Tot = PE + KE = -27.2eV + 13.6eV = -13.6eV$

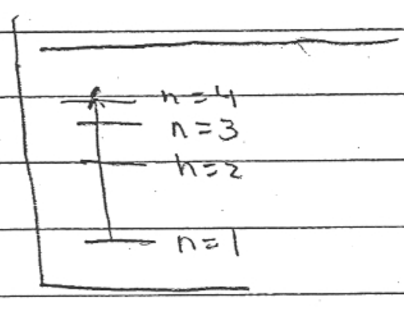
b) $r \propto n^2$

$$KE = \frac{L^2}{2mr^2} = \frac{n^2 \hbar^2}{2m(n^2 a_0)^2} = \frac{\hbar^2}{2ma_0^2} \frac{1}{n^2}$$

$$PE = \frac{-e^2}{4\pi\varepsilon_0 r} = \frac{-e^2}{4\pi\varepsilon_0 a_0 n^2} = \frac{-e^2}{4\pi\varepsilon_0 a_0} \frac{1}{n^2}$$

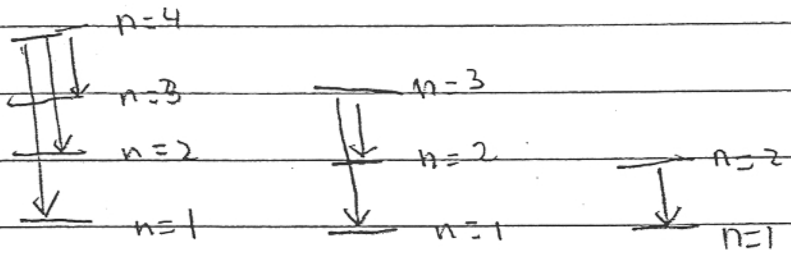
$$Tot \propto \frac{1}{n^2}$$

4.26)



$$E_{abs} = E_4 - E_1 = \frac{-13.6eV}{4^2} - \frac{-13.6}{1^2} = 13.6eV \cdot \frac{15}{16} = 12.75$$

4.26 - Continued



There are a total of six lines

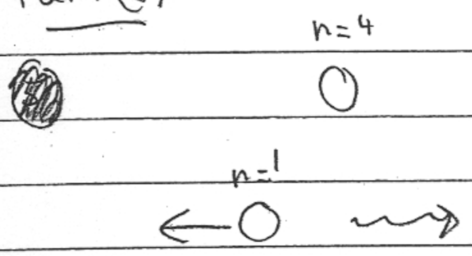
		E (in eV)
① (4,1)	=	12.75
② (4,2)	=	2.55
③ (4,3)	=	0.66
④ (3,2)	=	1.88
⑤ (3,1)	=	12.088
⑥ (2,1)	=	10.2

Example for (4,3):

$E = E_4 - E_3$

$E = \frac{-13.6}{4^2} + \frac{+13.6}{3^2} = 12.75 eV$

Part (c)



$P_{Atom} = P_\gamma = \frac{E_\gamma}{c} = \frac{1}{2}\frac{m_e c^2 \alpha^2}{c}\left[\frac{-1}{4^2} + \frac{1}{1^2}\right]$

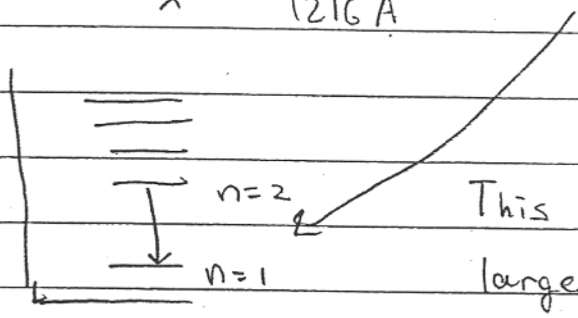
$M_p c\left(\frac{V_p}{c}\right) = \frac{1}{2} m_e c \alpha^2 \frac{15}{16}$

$\frac{V_p}{c} = \frac{m_e}{M_p}\alpha^2 \frac{15}{32} \Rightarrow V_p = 3\times10^8\left(\frac{1}{2000}\right)\left(\frac{1}{137}\right)^2 \cdot \frac{15}{32}$

$v_p = 3.74 m/s$

4.28

$$E = \frac{hc}{\lambda} = \frac{12400 eV \mathring{A}}{1216 \mathring{A}} = 10.2 eV$$



This is the largest possible transition energy

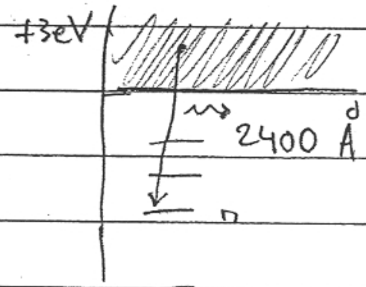
4.35

2400 Å

e = K = 3eV

• He^{++}

$$E_\gamma = \frac{hc}{\lambda} = \frac{12400 eV \mathring{A}}{2400 \mathring{A}} = 5.16 eV$$



$$E_i = E_f + \frac{hc}{\lambda}$$

$$K = -\frac{R_\infty Z^2}{n^2} + \frac{hc}{\lambda}$$

$$\frac{Z^2 R_\infty}{n^2} = \frac{hc}{\lambda} - K$$

$$\sqrt{\frac{Z^2 R_\infty}{E_\gamma - K}} = n = \sqrt{\frac{4 (13.6 eV)}{5.16 eV - 3 eV}} =$$