

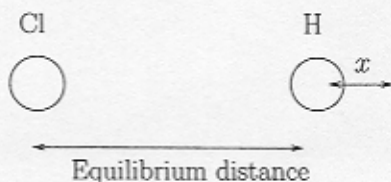
Problems:

5.1, 5.9, 5.12, 5.13

1. Stationary States:

(a) Take the stationary states of the a particle in the box $\Psi_n(x, t) = \Psi_n(x)e^{-iE_n t/\hbar}$. Show that that $\bar{E} = E_n$, $\overline{E^2} = E_n^2$, and $\Delta E = 0$

2. Graded Consider a (pretty good!) model of the vibrations of HCl made up of hydrogen and chlorine atoms in an ionic bond shown below.



In general the the hydrogen atom vibrates around its equilibrium position since it can never be exactly at one spot. Make a model of these vibrations by considering the chlorine nucleus to be very heavy and therefore fixed in space (it is 35 times heavier than the hydrogen atom.) When discussing the vibrations of the hydrogen atom neglect any effects due to the motion of the electrons (electrons are very light compared to nucleus - e.g. when two trucks collide the motion of the mirror is irrelevant. The electrons move much more quickly than the nuclei too as we will see below.) For small displacements of the hydrogen atom from its equilibrium position, the force will be proportional to the displacement x . This means that the potential energy is like a spring $\frac{1}{2}Cx^2$. We recall that the classical oscillation frequency is

$$\omega_o = 2\pi f = \sqrt{\frac{C}{m}}$$

where m is the mass of the hydrogen atom. Classically the hydrogen atom would arrive at the bottom of the potential well and have no displacement from equilibrium. Quantum mechanically the potential tries to suck the wave function to the bottom of the well. But this is balanced by the kinetic energy associated with the uncertainty principle, i.e. if a wave is small in size its typical momentum is very large

(a) Estimate the size of the ground state wave function by balancing the kinetic and potential energies. (Answer

$$: L \sim \left(\frac{\hbar^2}{mk}\right)^{1/4} \sim \left(\frac{\hbar}{m\omega_o}\right)^{1/2}$$

(b) Estimate the kinetic, potential, and total energies in terms of ω_o

(c) The normalized ground state wave function of the harmonic oscillator is

$$\Psi_0(x) = \left(\frac{1}{\sqrt{\pi}L}\right)^{1/2} e^{-\frac{x^2}{2L^2}} \quad (1)$$

where

$$L = \left(\frac{\hbar^2}{mk}\right)^{1/4} = \left(\frac{\hbar}{m\omega_o}\right)^{1/2}$$

Show that this wave function is a solution to the time independent Schrödinger equation and determine the energy associated with this state.

(d) Determine \bar{x} , $\overline{x^2}$, \bar{p} , $\overline{p^2}$.

(e) Show that the ground state of the harmonic oscillator saturates the uncertainty principle:

$$\Delta x \Delta p = \frac{\hbar}{2}$$

(f) Determine the average kinetic, potential and total energies using the results of part (d)