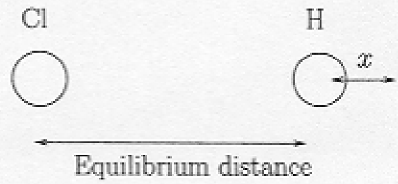
**Problems:**

5.1, 5.9, 5.12, 5.13

1. Stationary States:

   (a) Take the stationary states of the a particle in the box $\Psi_n(x,t) = \Psi_n(x)e^{-iE_nt/\hbar}$. Show that that $\overline{\mathrm{E}} = E_n$, $\overline{\mathrm{E}^2} = E_n^2$, and $\Delta E = 0$

2. **Graded** Consider a (pretty good!) model of the vibrations of HCl made up of hydrogen and clorine atoms in an ionic bond shown below.




In general the the hydrogen atom vibrates around its equilibrium position since it can never be exactly at one spot. Make a model of these vibrations by considering the chlorine nucleus to be very heavy and therefore fixed in space (it is 35 times heavier than the hydrogen atom.) When discussing the vibrations of the hydrogen atom neglect any effects due to the motion of the electrons (electrons are very light compared to nucleus – e.g. when two trucks collide the motion of the mirror is irrelevant. The electrons move much more quickly than the nuclei too as we will see below.) For small displacements of the hydrogen atom from its equilibrium position, the force will be proportional to the displacemnet $x$. This means that the potential energy is like a spirng $\frac{1}{2}Cx^2$. We recall that the classical oscillation frequency is

$$\omega_o = 2\pi f = \sqrt{\frac{C}{m}}$$

where $m$ is the mass of the hydrogen atom. Classically the hydrogen atom would arrive at the bottom of the potential well and have no displacement from equilibrium. Quanum mechanically the potential tries to suck the wave function to the bottom of the well. But this is balanced by the kinetic energy associated with the uncertainty principle, i.e. if a wave is small in size its typical momentum is very large

(a) Estimate the size of the ground state wave function by balancing the kinetic and potential energies. (Answer : $L \sim \left(\frac{\hbar^2}{mk}\right)^{1/4} \sim \left(\frac{\hbar}{m\omega_o}\right)^{1/2}$)

(b) Estimate the kinetic, potential, and total energies in terms of $\omega_o$

(c) The normalized ground state wave function of the harmonic oscillator is

$$\Psi_0(x) = \left(\frac{1}{\sqrt{\pi}L}\right)^{1/2} e^{-\frac{x^2}{2L^2}} \qquad (1)$$

where

$$L = \left(\frac{\hbar^2}{mk}\right)^{1/4} = \left(\frac{\hbar}{m\omega_o}\right)^{1/2}.$$

Show that this wave function is a solution to the time independent Schrödinger equation and determine the energy associated with this state.

(d) Determine $\overline{x}$, $\overline{x^2}$, $\overline{p}$, $\overline{p^2}$.

(e) Show that the ground state of the harmonic oscillator saturates the uncertainty principle:

$$\Delta x \Delta p = \frac{\hbar}{2}$$

(f) Determine the average kinetic, potential and total energies using the results of part ($d$)

(g) Measured spring constants are typically $C = 29.6\,\mathrm{eV}/\text{Å}^2$. Substitute numbers into parts ($a$), ($b$) and compare your results to the Bohr radius and a typical atomic-electronic energy $\sim 2\,\mathrm{eV}$. You should find that the energies associated with molecular vibrations are about 1/20th of electronic excitation energies.

(h) Finally determine the root-mean-square velocity of the hydrogen nulceus

$$v_{\rm rms} \equiv \sqrt{\frac{\overline{p^2}}{m^2}}\,,$$

and compare your velocity to the velocity of an electron in the lowest Borh orbit. You should find $v \sim 3600 m/s$ or about 1/1000th of the bohr orbital velocity.

(i) You may find the following integrals useful.

$$\int_{-\infty}^{\infty} du\, e^{-u^2} = \sqrt{\pi} \qquad \int_{-\infty}^{\infty} du\, u^2 e^{-u^2} = \frac{\sqrt{\pi}}{2} \tag{2}$$

3. Consider making a simple model of an atom.

(a) Estimate the size an atom

(b) Suppose we place an electron in a (1 Dimensional) box with which has a Length $L$ exactly equal to the size you estimated in part (a). Determine the energy of the photon emitted when the electron in the box decays from its first excited state down to its ground state. Work symbolically and then substitute numbers.

(c) Make a sketch of the ground state and first and second excited state wave functions of this electron in the box. Also make a sketch of the associated probability density $\mathrm{P}(x)$ for the ground, first, and second excited states. (There is a total of six graphs in this problem).

## Wavefunctions

1. The electron wave function squared $|\Psi(x,t)|^2 = \mathrm{P}(x,t)$ is *a probability per unit length* to find the particle at time t. Thus the probability $\mathrm{d}\mathscr{P}$ to find a particle between $x$ and $x + dx$ at time $t$

$$\mathrm{d}\mathscr{P} = \mathrm{P}(x,t)dx = |\Psi(x,t)|^2 dx \tag{3}$$

2. The electron must be somewhere so

$$\int_{-\infty}^{\infty} dx\, |\Psi(x,t)|^2 = 1 \tag{4}$$

3. The average position at time $t$

$$\overline{x} = \int dx\, x\, |\Psi(x,t)|^2 \tag{5}$$

4. The average position squared at time $t$ is

$$\overline{x^2} = \int dx\, x^2\, |\Psi(x,t)|^2 \tag{6}$$

5. The uncertainty squared in position $(\Delta x)^2$ (or standard deviation squared) is defined to be

$$(\Delta x)^2 \equiv \overline{x^2} - \overline{x}^2 = \overline{(x-\overline{x})^2} \tag{7}$$

If the average position is zero $\bar{x} = 0$ then $(\Delta x) \equiv \sqrt{\overline{x^2}}$ is the "root mean square" position. This gives a measure of how spread out is the wave function

## Momentum Averages

Prob 5.1

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}(c_1\psi_1 + c_2\psi_2 + c_3\psi_3) + V(x)(c_1\psi_1 + c_2\psi_2 + c_3\psi_3)$$

$$= +i\hbar\frac{\partial}{\partial t}(c_1\psi_1 + c_2\psi_2 + c_3\psi_3)$$

$$\underbrace{c_1\left(-\frac{\hbar^2}{2m}\frac{\partial^2\psi_1}{\partial x^2} + V(x)\psi_1 - i\hbar\frac{\partial\psi_1}{\partial t}\right)}_{0} + \underbrace{c_2(\text{same but } \psi_2)}_{0} + \underbrace{c_3(\text{same but } \psi_3)}_{0} = 0$$

Since $\psi_1$ is a solution to the Schrödinger Equ.

5.9

$$\psi(x,t) = \overbrace{A\sin\left(\frac{2\pi x}{a}\right)}^{\psi_2(x)} e^{-iEt/\hbar}$$

$$+i\hbar\frac{\partial}{\partial t}\psi(x,t) = +i\hbar\frac{\partial}{\partial t}\psi_2(x)e^{-iEt/\hbar} = +i\hbar\,\psi_2(x)\,\frac{-i}{\hbar}E\,e^{-iEt/\hbar}$$

$$= E\,\psi_2(x)\,e^{-iEt/\hbar}$$

Now we need

$$\boxed{\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}}$$ ← The Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{\partial x^2} + V(x)\right]\Psi_n(x)\cancel{e^{-iEt/\hbar}} = E\,\Psi_n(x)\cancel{e^{-iEt/\hbar}}$$

$$\boxed{\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right)\Psi_n(x) = E\,\Psi_n(x)}$$ ← Time indep Schrödinger equ.

Now $V(x) = 0$ for P in Box:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi_n(x)}{\partial x^2} = E\,\Psi_n(x)$$

$$\Psi_n = A\sin\left(\frac{2\pi x}{a}\right) \Rightarrow \frac{d^2}{dx^2}\sin\left(\frac{2\pi x}{a}\right) = -\left(\frac{2\pi}{a}\right)^2\sin\left(\frac{2\pi x}{a}\right)$$

So

$$-\frac{\hbar^2}{2m}\,-\sin\left(\frac{2\pi x}{a}\right)\left(\frac{2\pi}{a}\right)^2 = E\sin\left(\frac{2\pi x}{a}\right)$$

$$\left(\frac{\hbar^2}{2m}\frac{\pi^2}{a^2}\cdot 2^2\right)\Psi_n(x) = E\,\Psi_n(x)$$

So

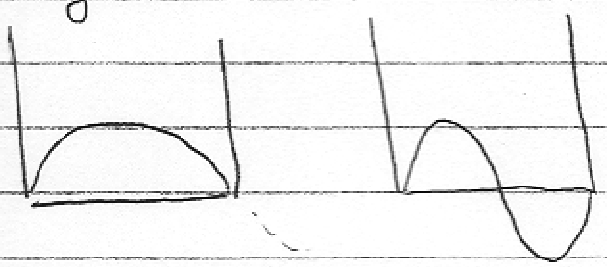
$\Psi_n(x)$ is a solution provided $E = \dfrac{\hbar^2\pi^2}{2ma^2}\cdot 2^2$

(b) $E_2 = \frac{\hbar^2 \pi^2}{2ma^2} \cdot 2^2$

The first excited state is 4x more energetic

$E_1 = \frac{\hbar^2 \pi^2}{2ma^2} \cdot 1^2$

(c) ground first



The first excited state has a shorter wavelength by 2. $p$ is twice larger, $E \propto \frac{p^2}{2m}$ is 4x larger.

5.12

$$\psi = \underbrace{A \sin\left(\frac{2\pi x}{a}\right)}_{\psi_n(x)} e^{-iEt/\hbar}$$

$$\overline{p^2} = \int_{-a/2}^{a/2} \psi^* \, -\hbar^2 \frac{\partial}{\partial x^2} \psi$$

$$= \int_{-a/2}^{a/2} A^2 \psi_n(x) \left(-\hbar^2 \frac{\partial^2}{\partial x^2} \psi_n(x)\right) \cancel{e^{+iEt/\hbar}} \, \cancel{e^{-iEt/\hbar}}$$

Now

$$-\hbar^2 \frac{\partial^2}{\partial x^2} A\sin\left(\frac{2\pi x}{a}\right) = +\hbar^2\left(\frac{2\pi}{a}\right)^2 \sin\left(\frac{2\pi x}{a}\right)$$

So

$$\overline{p^2} = \hbar^2\left(\frac{2\pi}{a}\right)^2 \int_{-a/2}^{a/2} dx\ A^2 \sin^2\left(\frac{2\pi x}{a}\right) \qquad A^2 = \frac{2}{a}$$

$$= \hbar^2\left(\frac{2\pi}{a}\right)^2 \cdot A^2\, a \,\langle \sin^2 \rangle$$

$$= \hbar^2\left(\frac{2\pi}{a}\right)^2 \cdot \frac{2}{a} \cdot a \cdot \frac{1}{2}$$

$$\boxed{\overline{p^2} = \hbar^2\left(\frac{2\pi}{a}\right)^2}$$

$p$

Now 5.13

$$\Delta x^2 = \overline{x^2} - \cancel{\bar{x}^2}^{\,0}$$

$$\Delta x = \sqrt{\overline{x^2}} = a\,(0.265)$$

$$(\Delta p)^2 = \overline{p^2} - \cancel{\bar{p}^2}^{\,0} \Rightarrow \Delta p = \sqrt{\overline{p^2}} = \hbar\,\frac{2\pi}{a}$$

$$\Delta x\, \Delta p = a(0.265)\ \hbar\ \frac{2\pi}{a}$$

$$\Delta x\, \Delta p = \hbar\ (1.66)$$

(b) Compared to The ground state

$$\Delta x\, \Delta p = 0.57\, \hbar \longleftarrow \text{ground}$$

The product

$\Delta x \Delta p$ is 3x larger

This makes sense, as $n \to \infty$ the product $\Delta x \Delta p$ becomes many many $\hbar$

5.14 $\psi(x) = \frac{1}{\sqrt{\sqrt{\pi} L}} e^{-x^2/2L^2}$ with $L = \left(\frac{\hbar^2}{mk}\right)^{1/4}$

$$PE = \int_{-\infty}^{\infty} dx\, \psi^*(x) \frac{1}{2} k x^2\, \psi(x)$$

$$PE = \int_{-\infty}^{\infty} dx\, \frac{1}{\sqrt{\sqrt{\pi} L}} e^{-x^2/2L^2} \left(\frac{1}{2} k x^2\right) \frac{1}{\sqrt{\sqrt{\pi} L}} e^{-x^2/2L^2}$$

Problem 1

$$\psi_n(x,t) = \psi_n(x)\, e^{-iE_n t/\hbar}$$

$$\overline{E} = \int_{-\infty}^{\infty} dx\, \psi^*(x,t)\, +i\hbar \frac{\partial}{\partial t} \psi(x,t)$$

$$+i\hbar \frac{\partial}{\partial t} \psi_n(x,t) = \psi_n(x)\, +i\hbar \frac{\partial}{\partial t} e^{-iE_n t/\hbar}$$

$$= \psi_n(x)\, E_n\, e^{-iE_n t/\hbar}$$

So

$$\overline{E} = \int_{-\infty}^{\infty} dx\, \psi_n^*(x)\, e^{+iE_n t/\hbar}\, \psi_n(x)\, E_n\, e^{-iE_n t/\hbar}$$

$$\overline{E} = E_n \underbrace{\int_{-\infty}^{\infty} dx\, \psi_n^*(x)\psi_n(x)}_{=1} = E_n$$

$$\overline{E^2} = \int_{-\infty}^{\infty} dx\, \psi_n^*(x)\, e^{+iE_n t/\hbar} \left(+i\hbar \frac{\partial}{\partial t}\right)^2 \psi_n(x)\, e^{-iE_n t/\hbar}$$

$$= \int dx\, \psi_n^*(x)\, e^{+iE_n t/\hbar}\, E_n^2\, \psi_n(x)\, e^{-iE_n t/\hbar}$$

$$= E_n^2 \underbrace{\int_{-\infty}^{\infty} \psi_n^*(x)\, \psi_n(x)}_{=1} = E_n^2$$

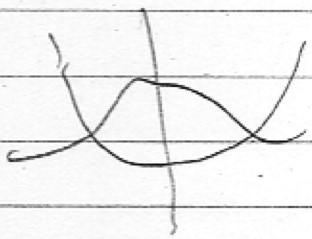
Now

$$\Delta E^2 = \overline{E^2} - \overline{E}^2 = E_n^2 - (E_n)^2 = 0$$

$$\Delta E = 0$$

Problem 2

1) The typical size $L$, occurs when $KE \sim PE$:



$$KE \sim \frac{\hbar^2}{2mL^2} \qquad PE \sim \frac{1}{2} k L^2$$

So $KE \sim PE$

$$\frac{\hbar^2}{2mL^2} \sim \frac{1}{2} k L^2$$

$$\frac{\hbar^2}{mk} = L^4 \quad \text{or} \quad L = \left(\frac{\hbar^2}{mk}\right)^{1/4}$$

So with $k = m\omega_0^2$ we have

$$L = \left(\frac{\hbar^2}{m^2\omega_0^2}\right)^{1/4} = \sqrt{\frac{\hbar}{m\omega_0}}$$

2) Then $KE \sim \frac{\hbar^2}{2mL^2} \sim \frac{\hbar^2}{2m\frac{\hbar}{m\omega_0}} \sim \hbar\omega_0$

Since $KE \sim PE$

$$PE \sim \hbar\omega_0$$

3) Now $\psi_0 = \left(\frac{1}{\sqrt{\pi} L}\right)^{1/2} e^{-x^2/2L^2}$

Want to show

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + \frac{1}{2} k x^2 \psi = E \psi$$

First we note that the overall factor

$\frac{1}{\sqrt{\sqrt{\pi} L}}$ is irrelevant since if $e^{-x^2/2L^2}$

satisfies the Schrödinger Eq. So does

$$\left(\frac{1}{\sqrt{\pi} L^2}\right)^{1/2} e^{-x^2/2L^2}$$

Then

$$\frac{\partial^2}{\partial x^2} e^{-x^2/2L^2} = \frac{1}{L^2} \frac{\partial^2}{\partial (x/L)^2} e^{-x^2/2L^2} \qquad u = x/L$$

$$= \frac{1}{L^2} \frac{\partial^2}{\partial u^2} e^{-u^2/2}$$

$$= \frac{1}{L^2} e^{-u^2/2} (1 - u^2)$$

So

$$\underbrace{-\frac{\hbar^2}{2m}\frac{1}{L^2}\, e^{-u^2/2}(1-u^2)}_{=\, -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}} + \underbrace{\frac{1}{2}kL^2\,(u^2 e^{-u^2/2})}_{=\,\frac{1}{2}kx^2\psi} = E\psi$$

$$= -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + = \frac{1}{2}kx^2\psi = E\psi$$

Note $\quad \dfrac{\hbar^2}{2mL^2} = \dfrac{1}{2}kL^2 = \dfrac{\hbar\omega_0}{4}$

So

$$\frac{\hbar\omega_0}{2}\left[e^{-u^2/2}(1-u^2) + u^2 e^{-u^2/2}\right] = E\, e^{-u^2/2}$$

$$\frac{\hbar\omega_0}{2}\left[e^{-u^2/2}\right] = E\, e^{-u^2/2}$$

So if $E = \dfrac{\hbar\omega_0}{2}$ $\quad \psi(x) = e^{-x^2/2L^2} = e^{-u^2/2}$

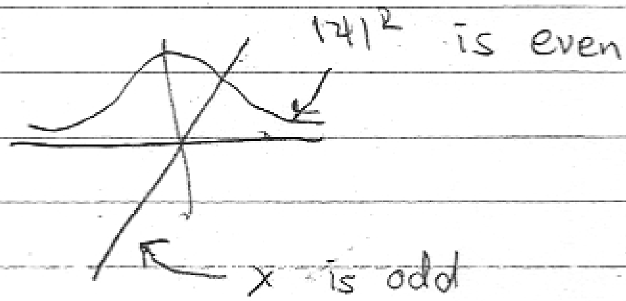
Is a solution to the time indep shródiner

d) Note:

$$\bar{x} = \int_{-\infty}^{\infty} \psi^*(x)\; x\; \psi(x) \qquad\qquad \psi = \frac{1}{\sqrt{\sqrt{\pi} L}} e^{-x^2/2L^2}$$

$$\bar{x} = \int_{-\infty}^{\infty} dx \;\underbrace{x}_{\substack{\uparrow \\ \text{odd}}}\; \underbrace{\frac{1}{\sqrt{\pi} L} e^{-x^2/L^2}}_{\psi^* \psi \;\leftarrow\; \text{even}}$$

$$\boxed{\bar{x} = 0}$$




$$\bar{p} = \int_{-\infty}^{\infty} dx \;\psi^*(x)\; -i\hbar \frac{\partial}{\partial x} \;\frac{1}{\sqrt{\sqrt{\pi} L}} e^{-x^2/2L^2}$$

$$\bar{p} = -\frac{i\hbar}{L} \frac{\partial}{\partial \left(\frac{x}{L}\right)} e^{-x^2/2L^2}$$

$$\bar{p} = -i\hbar \frac{1}{L} e^{-x^2/2L^2} \left(-\frac{x}{L}\right)$$

$$\boxed{\bar{p} = 0}$$

So to compute $\overline{x^2}$ and $\overline{p^2}$ we note

$$\overline{PE} = \overline{\frac{1}{2}kx^2} = \frac{1}{2}k\,\overline{x^2}$$

$$\overline{KE} = \overline{\frac{p^2}{2m}} = \frac{1}{2m}\,\overline{p^2}$$

So I will compute $\overline{KE}$ and $\overline{PE}$ : (Because I did it before last year and have a sol written)

$$\overline{PE} = \int_{-\infty}^{\infty} dx\, \psi^*(x)\, \frac{1}{2}kx^2\, \psi(x)$$

So with $\psi(x) = \frac{1}{\sqrt{\sqrt{\pi}L}} e^{-x^2/2L^2}$ we have

$$PE = \int_{-\infty}^{\infty} dx\, \frac{1}{\sqrt{\sqrt{\pi}L}} e^{-x^2/2L^2} \cdot \frac{1}{2}kx^2 \cdot \frac{e^{-x^2/2L^2}}{\sqrt{\sqrt{\pi}L}}$$

$$\overline{PE} = \int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi}\,L}\; e^{-x^2/L^2}\; \frac{1}{2} k x^2$$

$$= \int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi}\,L}\; e^{-x^2/L^2}\; \frac{1}{2} k \frac{x^2}{L^2} \cdot L^2$$

$$\overline{PE} = \left(\frac{1}{2} k L^2\right) \underbrace{\int_{-\infty}^{\infty} \frac{du}{\sqrt{\pi}}\; e^{-u^2}\; u^2}_{\text{from handout}}$$

$$\overline{PE} = \left(\frac{1}{2} k L^2\right) \cdot \frac{1}{2} = \frac{1}{2}\left(\frac{1}{2} k L^2\right) = \frac{1}{4}\left(\frac{k^2 \hbar^2}{mk}\right)^{1/2}$$

$$= \frac{1}{4}\left(\hbar^2 \frac{k}{m}\right)^{1/2}$$

$$= \frac{1}{4} \hbar \omega_0 \qquad \omega_0 = \sqrt{\frac{k}{m}}$$

$$\overline{KE} = \int_{-\infty}^{\infty} dx \; \psi^* \; \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} \psi$$

$$\psi = A\, e^{-x^2/2L^2} \qquad A = \frac{1}{\sqrt{\sqrt{\pi} L}} \qquad L = \left(\frac{\hbar^2}{mk}\right)^{1/4}$$

So

$$\overline{KE} = \int_{-\infty}^{\infty} dx \; \psi^* \; \frac{-\hbar^2}{2m} \frac{1}{L^2} \frac{d^2}{d\left(\frac{x}{L}\right)^2} \psi$$

let $u = \frac{x}{L}$

$$-\frac{d^2}{du^2}\left(e^{-u^2/2}\right) = -\frac{d}{du}\left(e^{-u^2/2}(-u)\right)$$

$$= -\left(e^{-u^2/2}(-u)(-u) - e^{-u^2/2}\right)$$

$$= e^{-u^2/2}\left(1 - u^2\right)$$

$$KE = \int_{-\infty}^{\infty} dx \; \frac{1}{\sqrt{\sqrt{\pi} L}} e^{-u^2/2} \; \frac{\hbar^2}{2mL^2} \; e^{-u^2/2} (1-u^2) \frac{1}{\sqrt{\sqrt{\pi} L}}$$

$$\overline{KE} = \frac{\hbar^2}{2mL^2} \int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi} L} \; e^{-u^2} (1-u^2)$$

So

$$\overline{KE} = \frac{\hbar^2}{2mL^2} \int \frac{du}{\sqrt{\pi}}\, e^{-u^2} (1 - u^2)$$

$$\overline{KE} = \frac{\hbar^2}{2mL^2} \left(1 - \frac{1}{2}\right)$$

$$\overline{KE} = \frac{\hbar^2}{2mL^2} \frac{1}{2}$$

Putting $\frac{1}{L^2} = \sqrt{\frac{mk}{\hbar^2}}$ we have

$$\overline{KE} = \frac{1}{2} \frac{\hbar^2}{2m} \sqrt{\frac{mk}{\hbar^2}} = \frac{1}{4} \sqrt{\frac{k}{m}}\ \hbar = \frac{1}{4} \hbar\omega_0$$

So comparing we see that

$$\overline{PE} = \overline{KE} = \frac{1}{4} \hbar\omega_0$$

Classically the time averged PE and KE of a spring are equal, When the PE is low the KE is high

now

$$\overline{PE} = \frac{1}{2} k \overline{x^2} = \frac{1}{4}\hbar\omega_0$$

$$\frac{1}{2} m\omega_0^2 \, \overline{x^2} = \frac{1}{4}\hbar\cancel{\omega_0} \quad \text{since } \omega_0 = \sqrt{\frac{k}{m}}$$

$$\overline{x^2} = \frac{1}{2}\frac{\hbar}{m\omega_0}$$

$$\overline{KE} = \frac{\overline{p^2}}{2m} = \frac{1}{4}\hbar\omega_0$$

$$\overline{p^2} = \frac{1}{2} m\hbar\omega_0$$

So

e) $$\Delta x^2 = \overline{x^2} - \cancelto{0}{\bar{x}^2} = \frac{\hbar^2}{2m\omega_0}$$

$$\Delta p^2 = \overline{p^2} - \cancelto{0}{\bar{p}^2} = \frac{1}{2} m\hbar\omega_0$$

So

$$\boxed{\Delta x^2 \Delta p^2 = \left(\frac{\hbar}{2m\omega_0}\right)\left(\frac{1}{2} m\hbar\omega_0\right) = \frac{\hbar^2}{4}} \Rightarrow \boxed{\Delta x \Delta p = \frac{\hbar}{2}}$$

f) see above

g) $L = \left(\frac{\hbar^2}{mC}\right)^{1/4} = \left(\frac{(\hbar c)^2}{mc^2\, C}\right)^{1/4}$

$$L = \left[\frac{(1970\ \text{eV Å})^2}{(938 \times 10^6\ \text{eV})(29.6\ \text{eV}/\text{Å}^2)}\right]^{1/4}$$

$L = 0.1\ \text{Å}$ $\quad \sqrt{\overline{x^2}} = \sqrt{\frac{L^2}{2}} = \frac{L}{\sqrt{2}} \sim 0.05\ \text{Å}$

$$\Delta E = \hbar\omega_0 = \hbar\sqrt{\frac{C}{m}} = \sqrt{\frac{(\hbar c)^2\, C}{mc^2}}$$

$$\Delta E = \left(\frac{(1970\ \text{eV Å})^2}{938 \times 10^6\ \text{eV}}\ \frac{29.6\ \text{eV}}{\text{Å}^2}\right)^{1/2}$$

Small compared to spacing

$\Delta E = 0.35\ \text{eV} \sim$ 6-7 times smaller than as 2eV

h) $\overline{KE} = \overline{\frac{1}{2} m v^2} \Rightarrow \sqrt{\overline{V^2}} = \sqrt{\frac{2\,\overline{KE}}{m}} = \sqrt{\frac{2\,\hbar\omega_0/4}{m}}$

$$\sqrt{\overline{V^2}} = 4097\ \text{m/s}$$

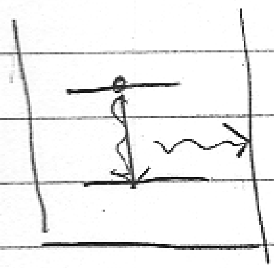
Compaire

$$\frac{\sqrt{\overline{V^2}}}{V_{\text{bohr electron}}} \simeq 0.002$$

Problem 3

a) $L \sim 2a_0 \sim$ Diameter $\sim 1\,\mathring{A}$

b)



$$\Delta E = E_2 - E_1 = \frac{\hbar^2 \pi^2 (2^2 - 1^2)}{2mL^2}$$

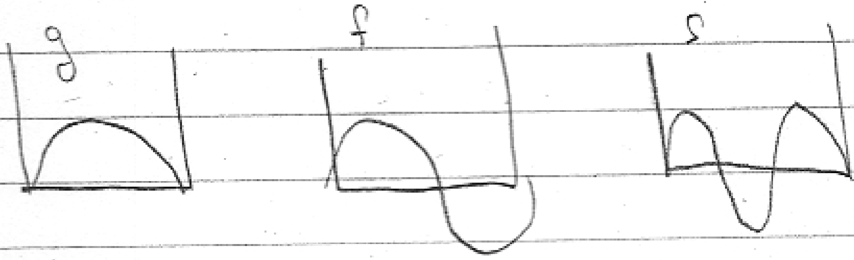
$$= \frac{\hbar^2}{2m} \frac{\pi^2 (3)}{4a_0^2}$$

$$\Delta E = \frac{\hbar^2}{2ma_0^2} \left( \frac{3\pi^2}{4} \right)$$

$$\Delta E = 13.6\ \text{eV} \left( \frac{3}{4} \pi^2 \right) \simeq 100\ \text{eV}$$

c)

$\psi \rightarrow$




$|\psi|^2 \rightarrow$