PHY 252 Lab 6: The emission spectrum of hydrogen

Spring 2007

Introduction

In this experiment you will measure the visible part of the hydrogen spectrum, the Balmer series, and determine the Rydberg constant *Ry*.

If parallel rays of light are incident perpendicular to the plane of a diffraction grating, with uniform phase over the grating, one will observe a diffraction pattern. The pattern has a series of intensity maxima at angles ϑ satisfying the equation

$$n\lambda = d \sin\vartheta, \quad \text{with } n = 1, 2, 3, \ldots, \tag{1}$$

where λ is the wavelength of the incident light and d is the spacing between adjacent grooves on the grating. Thus the grating allows determination of an unknown wavelength of a light source by measuring the positions of the intensity maxima, provided d is known.

Apparatus

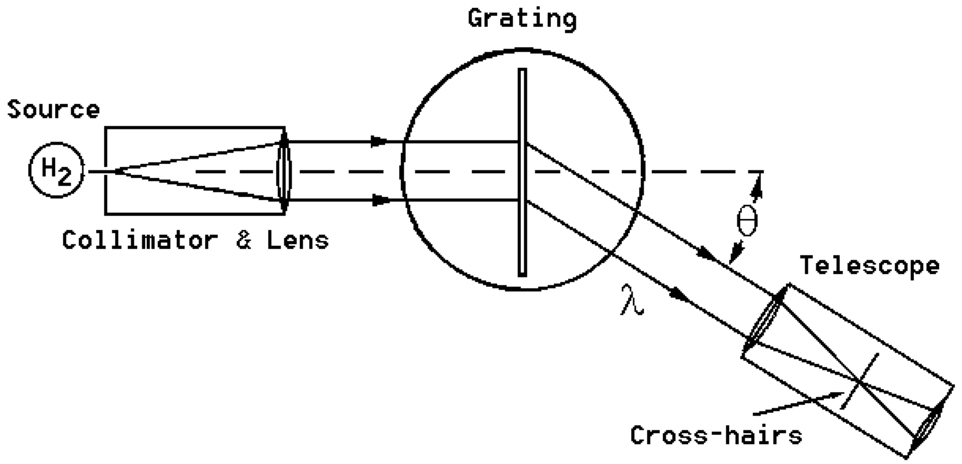

Figure 1. Spectrometer setup.

The means for mounting the grating, obtaining parallel rays and measuring the positions of the maxima are provided by the spectrometer table, indicated in the Figure. Light from the hydrogen source passes through a collimator which consists of a vertical slit and a lens. With proper adjustment, the collimator can be made to emit near parallel light rays. This light passes through the grating and is observed using a telescope, which has been focused to accept parallel refracted rays. The positions of the various intensity maxima are then determined by noting their position as angles on the vernier scale on the spectrometer table.

It is crucial for the success of this experiment that the spectrometer is properly aligned. Your TA will show how to do this. The steps are as follows:

1. Adjustment of telescope: Adjust the telescope so that it focuses parallel rays, by observing some distant object. This includes adjusting the eyepiece to ensure that the image of the distant object falls right on the cross hairs, with no evident parallax.

2. Adjustment of collimator to emit parallel rays: Move the telescope until you are looking directly into the collimator. Adjust the collimator until you obtain a sharp image of the collimator slit.

3. Adjustment of the grating table: Place a mirror in the position of grating. Introduce some light into the hole on the telescope barrel. You should see in addition to the cross hairs also a faint image of the cross hairs due to the light being reflected from the mirror. Adjust the grating table until the two images overlap.

Measurement

The spectrometer is now ready to measure wavelengths; make sure you understand how to read the vernier scale for the angles. Place the hydrogen source so that the light falls onto the collimator. Insert the grating in its slot, after reading off the spacing d. Now slowly change the angle setting of the telescope until you see a line. Read the angle. Continue until you see the next line (it will have a different color) and so on. You should see three different colors. The first one is actually the most difficult to see.

If you rotate the telescope still further, you will see the same sequence of lines appearing again, but much fainter. This is the second order pattern $n = 2$. Try to see as many lines and orders as possible. Repeat this sequence of measurements on the *opposite* side of the spectrometer. This gives better precision for the measured angles: for each line and order you now have a pair of measured angles ϑ_L and ϑ_R, so that $\vartheta = \frac{1}{2}(\vartheta_L + \vartheta_R)$.

Analysis

The hydrogen wavelengths can be fit to an empirical relationship of the form

$$1/\lambda = a + b/m^2, \quad \text{with } m = 3,4,5,\ldots.$$

Plot your values of $1/\lambda$ versus $1/m^2$. Hint: for the last line you see in a given order, the quantum number $m = 3$ (explain why!). Determine a and b from your graph and, by comparing equation (1) with the Rydberg formula for hydrogen, determine *Ry*. Include an analysis of errors.

Modified analysis of hydrogen spectrum data

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In the experiment, you have several diffraction angles θ for emission lines from an excited hydrogen gas. You know that the for the Bohr model photons should be emitted with energies corresponding to energy differences between intial i and final f states or

$$\Delta E = E_i - E_f = \frac{-E_0}{n_i^2} - \frac{-E_0}{n_f^2} = E_0\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \frac{hc}{\lambda}. \tag{1}$$

You also know that the diffraction angles should be given by

$$n_d \lambda = d \sin\theta \tag{2}$$

where n_d is the integer describing the diffraction order. Usually you can only see three spectral lines in both positive ($n_d = +1$) and negative ($n_d = -1$) first diffraction orders, and maybe also in second orders–but the angles at positive and negative orders, and at first order and second order, are all related to each other. Therefore if you can measure just three lines (perhaps red, blue-green, and violet), you have only three *independent* measurements. At the same time, ideally you'd like to use your data to determine E_0, d, n_i, and n_f for each of your lines but of course you can't determine four unknowns from three knowns!

What to do? Well, let's start by assuming that we know a rough value for $E_0 = 13.6$ eV and asking about what wavelengths we might have a chance of seeing:

$n_f = 1$			$n_f = 2$			$n_f = 3$		
n_i	ΔE (eV)	λ (nm)	n_i	ΔE (eV)	λ (nm)	n_i	ΔE (eV)	λ (nm)
4	12.75	97.3	5	2.86	434.2	6	1.13	1094
3	12.09	102.6	4	2.55	486.3	5	0.97	1282
2	10.20	121.6	3	1.89	656.5	4	0.66	1876

Since your eye can only see in the range $\lambda \simeq 400$–700 nm, we can pretty safely assume that you're seeing states with $n_f = 2$ and $n_i = 5$, 4, and 3. If we were to indeed assume that the Bohr model works and that we have these values for the principal quantum numbers n_i and n_f, then we can put Eqs. 1 and 2 together to obtain

$$\begin{aligned} \lambda = \frac{hc}{E_0}\frac{1}{1/n_f^2 - 1/n_i^2} &= \frac{d\sin\theta}{n_d} \\ y \equiv n_d \frac{hc}{E_0}\frac{1}{d} &= \sin\theta\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \end{aligned} \tag{3}$$

What varies between our different measuremnts? Just n_i and $\sin\theta$. Therefore in Eq. 3 we have put everything that remains constant on the left, and things that vary on the right. Therefore if we have three different values of θ, and three different values of n_i, and put those into the right side of Eq. 3, we should ideally get the same value (y) on the left side in each case. A logical strategy then is to obtain an average value $\bar{y}$ from the three calculations of $\sin\theta(1/n_f^2 - 1/n_i^2)$ for the three values of θ and the three values of n_i.

With that experimental value of $\bar{y}$, we can solve for only one of the variables n_d, h, c, E_0, and d out of the result of

$$\bar{y} = n_d \frac{hc}{E_0} \frac{1}{d}.$$

Now if we were Henry Rowland in the late 1800s at Harvard University, we might make a precision grating ruling engine based on measuring the thread pitch (such as in teeth per cm) of a screw against a precision meter stick, and use that plus measured gear ratios to cut scratches in glass at precise intervals, and thus have a grating with known d. In our case, I think it's more honest to say that we know $E_0 = 13.606$ eV and $hc = 1239.8$ eV·nm, and determine d from

$$d = n_d \frac{hc}{E_0} \frac{1}{\bar{y}}. \tag{4}$$

This is perhaps not as satisfying as determining E_0 from first principles, but it's well matched to an honest analysis of your experiment. Also, since you have three values of y, you can calculate the error on $\bar{y}$ from using what we did in class the other day:

$$\sigma_y = \sqrt{\frac{\sum(y_i - \bar{y})^2}{N-1}} \tag{5}$$

so that you can give a value of $y = \bar{y} \pm \sigma_y$ and thus get an error on your determination of d as well.