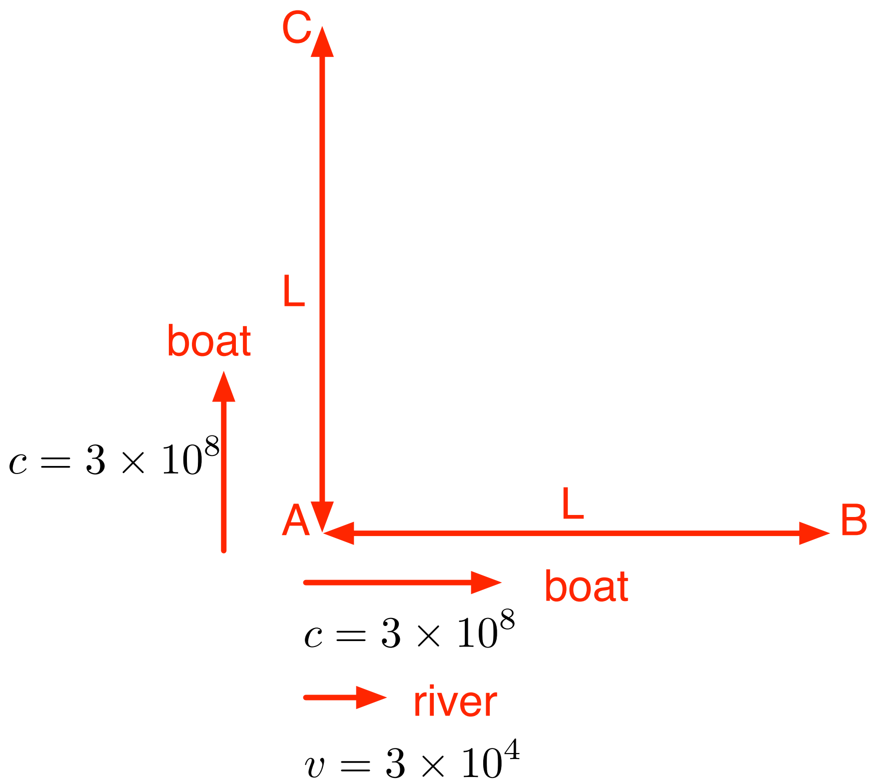
In this note we will show how to make estimates

I. THE MICHELSON MORLEY EXPERIMENT

For this example we will consider the classical treatment the setup of a *fast* boat moving at the speed of light $c = 3 \times 10^8$ in a river moving at the $v = 3 \times 10^4$ m/s. v in this example is close to the velocity of the earth relative to the sun. In the absence of the river the time it takes to go from A to C is

$$\Delta t_{AC} = \frac{L}{c}$$

We are interested in estimating (not calculating) the corrections to this result due to the river when the boat travels from A to B



In this case you can easily go calculate the correction (see homework). But in many of the things we will consider, you will only be able to estimate corrections rather than go through the difficult math to really calculate the effect. For instance, is the effect of venus on mercury a $\sim 1\%$ effect (1 degree per year), or is it $\sim 0.001\%$ effect (1 degree per 1000 years)?[1]

To estimate the correction due to the stream, we note that the river is moving much slower than the boat since

$$\frac{v}{c} \simeq 10^{-4}$$

[1] Answer 0.001% effect.

Thus the percent correction to Δt_o due to the stream really quite small. It seems intuitive (to me) that since v/c is of order $\sim 10^{-4}$ (0.01%), then the correction should also be of order $\sim 10^{-4}$ (0.01%). This is indeed correct logic. Following the section on the math below, we expect that

$$\frac{\Delta t_{AB} - \Delta t_{AC}}{\Delta t_{AC}} \simeq \text{const}\,\frac{v}{c} \tag{1.1}$$

Here const is a number (constant) such as $1, 4.3$, or $\pi/2$, which are all of order one, while $v/c \sim 10^{-4}$ is very small indeed. I don't care about the about the exact number and neither should you. I do care if the correction due to the stream is 10% or 0.01%.

To notate that we don't care about an overall constant we write

$$\frac{\Delta t_{AB} - \Delta t_{BC}}{\Delta t_o} \sim \frac{v}{c} \tag{1.2}$$

where $\sim$ means of order of magnitude of v/c. We can also take eq. (1.1) and write

$$\Delta t_{AB} \simeq \Delta t_{AC}\left(1 + \text{const}\,\frac{v}{c}\right) \tag{1.3}$$

Again, since it is tedious to write "const", I will use the notation

$$\Delta t_{AB} \simeq \Delta t_{AC}\left(1 + O\left(\frac{v}{c}\right)\right) \tag{1.4}$$

where $O(v/c)$ mean "of order of v/c"

1. There and back

Now lets calculate the time it takes to go from point A to point B and back to A, Δt_{ABA}. In the *absence* of the stream (travelling from ACA) the time is now

$$\Delta t_{ACA} = 2L/c$$

Now we will compute, Δt_{ABA}. Now its clear that the time going from A to B is shorter with the stream, while the time back (from B to A) is longer with the stream. Thus, in a linear approximation there is no correction. However, in a non-linear or quadratic approximation the two effects do not cancel. For instance, $(0.1) + (-0.1) = 0$ but $(0.1)^2 + (-0.1)^2 = 0.02$. We expect then that the percent correction is quadratic in v/c

$$\frac{\Delta t_{ABA} - \Delta t_{ACA}}{\Delta t_{ACA}} \sim \left(\frac{v}{c}\right)^2 \tag{1.5}$$

where we again write $\sim$ since we will not bother with the constant. Similarly we write

$$\Delta t_{ABA} = \Delta t_{ACA}\left(1 + O(v^2/c^2)\right) \tag{1.6}$$

A. Mathematical Justification

Consider any function $f(x)$ such as the shown in the figure below (Case A in figure). If x is very small, we can approximate the function by a line

$$f(x) \simeq ax + b\,, \tag{1.7}$$

Here b (the intercept) is the value of the function when x is switched off, and the slope of the line a is adjusted to match the slope of the function when x is set to zero. This can be written as

$$f(x) \simeq f(0)(1 + \text{constant} * x) \tag{1.8}$$

where $f(0) = b$ is the value of the function when x is switched off and constant $= a/b$. In the notation of the last section

$$f(x) \simeq f(0)\,(1 + O(x)) \tag{1.9}$$

Sometimes it can happen that the slope is zero (Case B–see figure). Then we can approximate the function by a parabola

$$f(x) = b + ax^2 \tag{1.10}$$

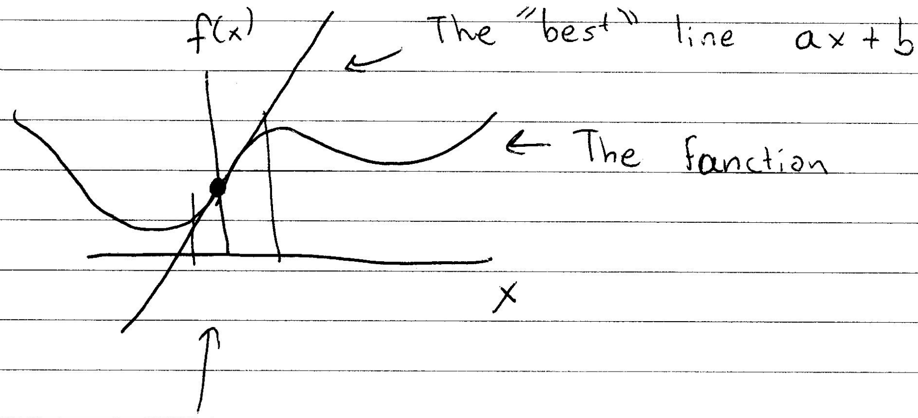
where the curvature of the parabola is adjusted to match the curvature of the function. This can be written as

$$f(x) = f(0)(1 + \text{constant} * x^2) \tag{1.11}$$

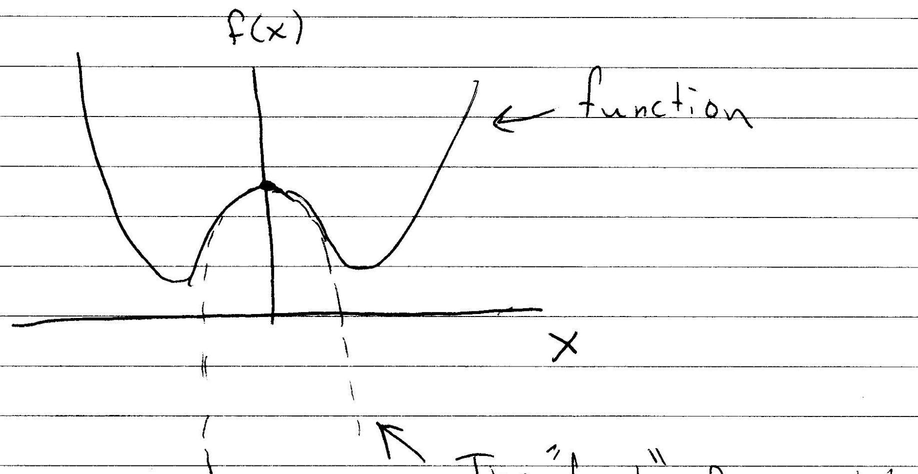
or

$$f(x) = f(0)(1 + O(x^2)) \tag{1.12}$$

Figure



CASE A



CASE B

II. PRECESSION OF MERCURY DUE TO VENUS

The number of degrees per day that mercury turns in the absence of the other planets is

$$D_o = \frac{360^o}{88\,\text{days}} \tag{2.1}$$

and is due to the force of sun. Then due to the force on mercury from Venus we expect a correction which causes the perihelion of Mercury to precess. (see slides)

Let D denote the number of degrees per day that mercury turns including the other planets. Since this is different from D_o the ellipse will precess and the rate of precession (degrees per day) is of order of the difference between these two rates

$$\text{precession rate} \sim D - D_o \tag{2.2}$$

The fractional correction to D_o is of order of the ratio between Venus' forces and the sun's forces

$$\frac{D - D_o}{D_o} \sim \frac{F_{\text{venus}}}{F_{\text{sun}}} \tag{2.3}$$

In this way

$$\text{precession rate} \sim D_o \frac{F_{\text{venus}}}{F_{\text{sun}}} \tag{2.4}$$

In lecture I carried out the rest of this estimate.

The force due to Venus is

$$F_{\text{venus}} \sim \frac{G M_v M_m}{\Delta r^2} \tag{2.5}$$

and the sun force is

$$F_{\text{sun}} \sim \frac{G M_{\text{sun}} M_m}{r_m^2} \tag{2.6}$$

So the ratio of forces is

$$\frac{F_{\text{venus}}}{F_{\text{sun}}} \sim \frac{M_v}{M_{\text{sun}}} \frac{r_m^2}{\Delta r^2} \tag{2.7}$$

Then we note that the radii are basically the $r_m \sim r_v \sim \Delta r \sim 0.4\,\text{AU}$ are all the same order of magnitue, so $r_m^2/\Delta r^2 \sim 1$. However the masses are *very* different. So

$$\frac{F_{\text{venus}}}{F_{\text{sun}}} \sim \frac{M_v}{M_{\text{sun}}} \sim 10^{-6} \tag{2.8}$$

With this we reach an estimate

$$\text{precessionrate} \sim D_o \frac{F_{\text{venus}}}{F_{\text{sun}}} \sim \frac{360^o}{88\,\text{days}} \times 10^{-6} \tag{2.9}$$

Or converting from degrees per day to arcseconds per century

$$\text{precessionrate} \sim 500 \frac{\text{arcsec}}{\text{century}} \tag{2.10}$$