Homework

1. **Black Body Radiation**.

   (a) The sun may be considered as a black body with a (surface) temperature of $5500^o K$. Determine the total power that the sun emits per unit time. Give your answer in watts.

   (b) Estimate the energy that arrives on the upper atmosphere of the earth per area per time.

   (c) Explain in your own words what we mean when we talk about the energy emitted per frequency interval Δf.

   (d) By googleling visual light, determine the range of photon energies (in electron volts) that can be seen.

   (e) The following two curves show the intensity per frequency interval at a temperature of 3000^o, K and 6000^o K respectively. The vertical lines for each temperature curve indicate a frequency region (or band) which carries 20% of the total energy at their respective temperatures. Each region also constitutes 20% of the total area under the curve. Thus, the shaded region for the 6000^o K case carries 20% of the energy (per area per time) and constitutes 20% of the total area under the blue curve. (For the 6000^oK case the % labels are shown, while in the 3000^o the % labels are not shown.) Note that the total area under the curve is the total intensity (*i.e.* energy per area per time).

       Based on the figure estimate the fraction of the energy that is emitted into the visual band (*i.e.* as light that you can see).

   (f) How much does the area under the curve change when going from the 3000^o K case (the red lines) to the 6000^o K case (the blue lines). I am looking for an exact answer here.

2. **Early cosmic electromagnetic plasmas**

   (a) Concisely describe the set of things that go on as the plasma in the early universe cools from $5000^o\,K$ downward to $\sim 2000^o\,K$. Be sure to define and explain the following terms: Compton scattering, photo-dissociation, and free-electron fraction.

   (b) What is the best experimental evidence for this early epic?

   (c) How could you estimate the scale factor of the universe at these high temperatures, given the experimental evidence described in (b).

3. **Hydrogen Lines.** For this problem give all energies in electron volts, give all wavelengths in nanometers, and give all speeds in m/s.

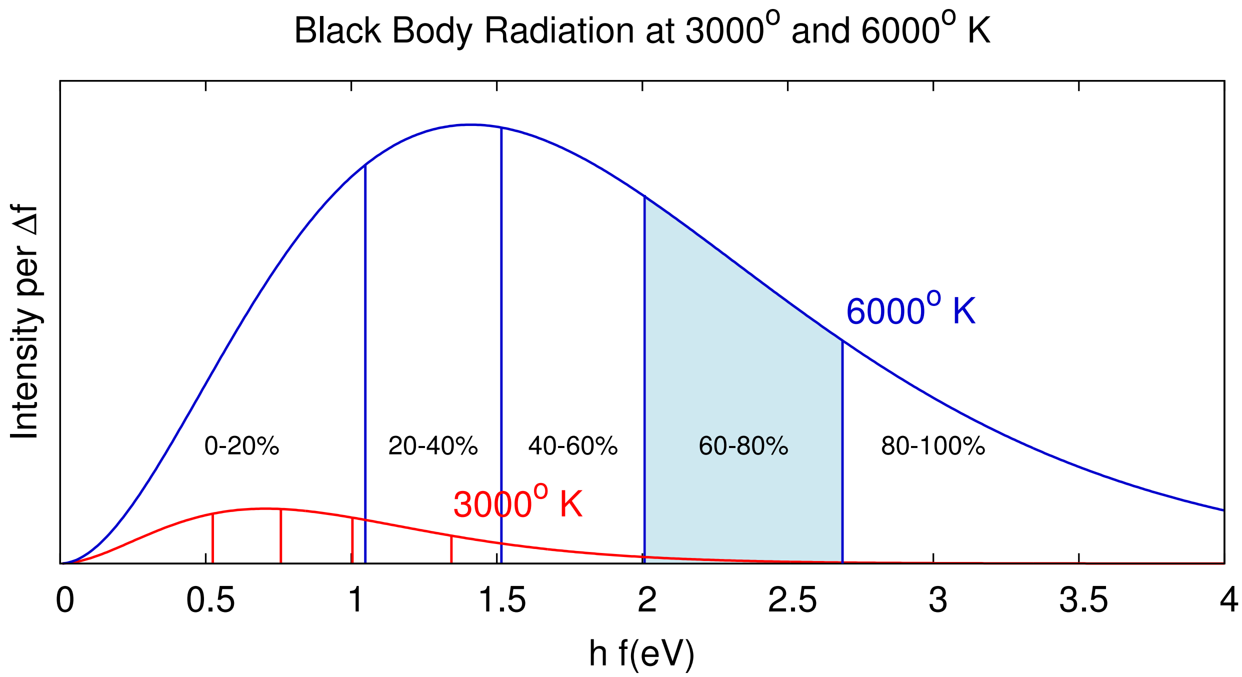

Figure 1: The black body spectrum at two temperatures

(a) What is the energy and wavelength of the photon that is emitted when a hydrogen atom decays from its $n = 2$ (electronic) vibrational state down to the $n = 1$ or ground state of hydrogen. This important transition is known as the Lyman-α transition.

(b) What is the energy and wavelength of the photon which comes from the $n = 3$ to $n = 1$ transition? This important transition is known as the Lyman-β transition.

(c) The energy required to rip and electron from a ground state hydrogen atom is $13.6\,\mathrm{eV}$. This energy is normally supplied by sending in an photon with sufficient energy to give the electron at least this much energy. If the photon has more energy than this, the remaining of the energy (after using up the $13.6\,\mathrm{eV}$ to ionize the hydrogen atom) will be used to give the electron kinetic energy (*i.e.* the energy associated with motion, K.E. $= \frac{1}{2}m_e v^2$).

If a $15.0\,\mathrm{eV}$ photon ionizes a ground state hydrogen atom, determine the velocity of the outgoing electron and compare this speed to the speed of light and the speed of sound.

(d) Estimate the typical speed of a hydrogen atom at room temperature. Hint, the kinetic energy is of order $k_B T$ (with T in Kelvin!) and the mass of the hydrogen atom is nearly the same as a proton, $m_p = 938 \times 10^6\,\mathrm{eV}/c^2 = 1.67 \times 10^{-27}\,\mathrm{kg}$.

A very similar problem was discussed in class (i.e. on-line) . You should find that the typical speed of atomic hydrogen is several times faster than the speed

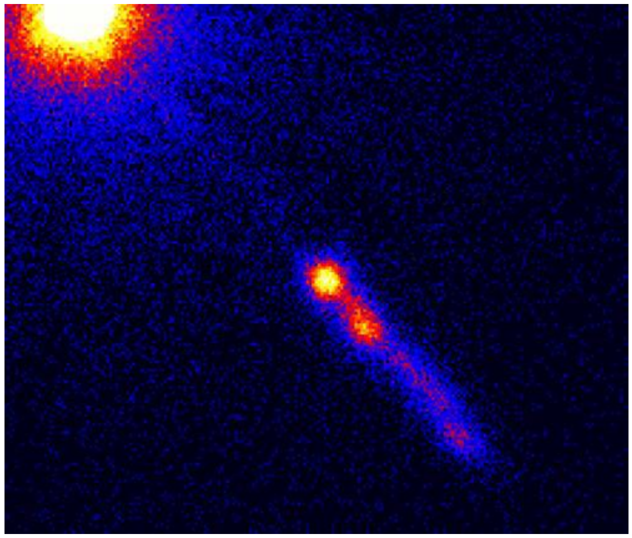

Figure 2: An X ray image of the M87 galaxy

of sound. For heavier molecules (such as molecular oxygen O_2 with 32 times the mass of atomic hydrogen) the speed is very close to the sound speed.

(e) For unexcited hydrogen (*i.e.* $n = 1$) moving away from you at the speed of sound, is the frequency of light that is absorbed by the hydrogen atom in the Lyman-α transition ($n = 1 \to n = 2$) higher or lower? Explain.

(f) The fractional change in the frequency of the light, $\Delta f/f$, which is absorbed by an atom moving with speed v is of order v/c. Estimate the percent change in the observed absorption and emission frequencies of hydrogen gas at room temperature. (Use part (d)).

4. **Quasars and the Gunn-Peterson Trough**.

5. (a) Describe briefly what is a quasar (Quasi-Stellar Object).

(b) Fig. 2 shows a real X-ray image of 3C2723 an active quasar. A jet of material is emitted from the active quasar in the Virgo cluster at $z = 0.158$. How far away is this quasar? Explain how you arrived at this answer using our $a(t)$. Is this close or far on galactic scales. The $a(t)$ curve needed is given in Fig. 3

(c) The Fig. 4 below shows the light from a distant quasar. The top figure shows prominent emission bands at $\lambda = 8300\,\AA$ and $\lambda = 7050\,\AA$. Give a quantitative explanation for these wavelengths. Note that $10\AA = 1\,\mathrm{nm}$.

(d) Describe what (besides the overall redshift) is qualitatively different about the farthest quasar spectra at $z \simeq 6.28$. Another set of figures showing the same basic trends as a function of redshift is given in in Fig. 5. (Note, higher z means the light was emitted at earlier times.)

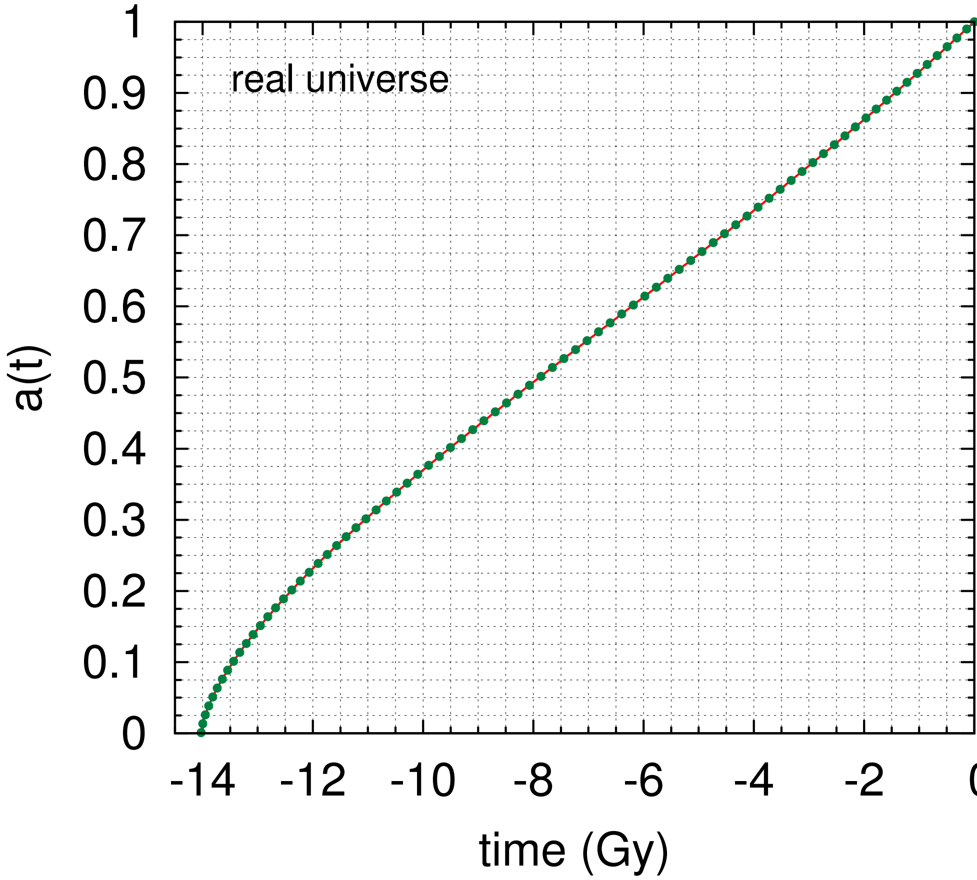

Figure 3: The scale factor as a function of time for the best model fit model cosmology.

(e) What is the physics that makes the spectra from these farthest quasars, *i.e.* $z = 6.28$ in Fig. 4 and $z = 6.42$ in Fig. 5, qualitatively different. Explain as precisely and concisely as you can. *Include a sketch* to clarify what you mean. For extra credit, give a guess why the observed changes in Fig. 5 in these spectra are not smooth functions of z

(f) What is happening between $z \simeq 5.5$ and $z \simeq 6.5$? Roughly how many years after the big bang does this z interval correspond to?

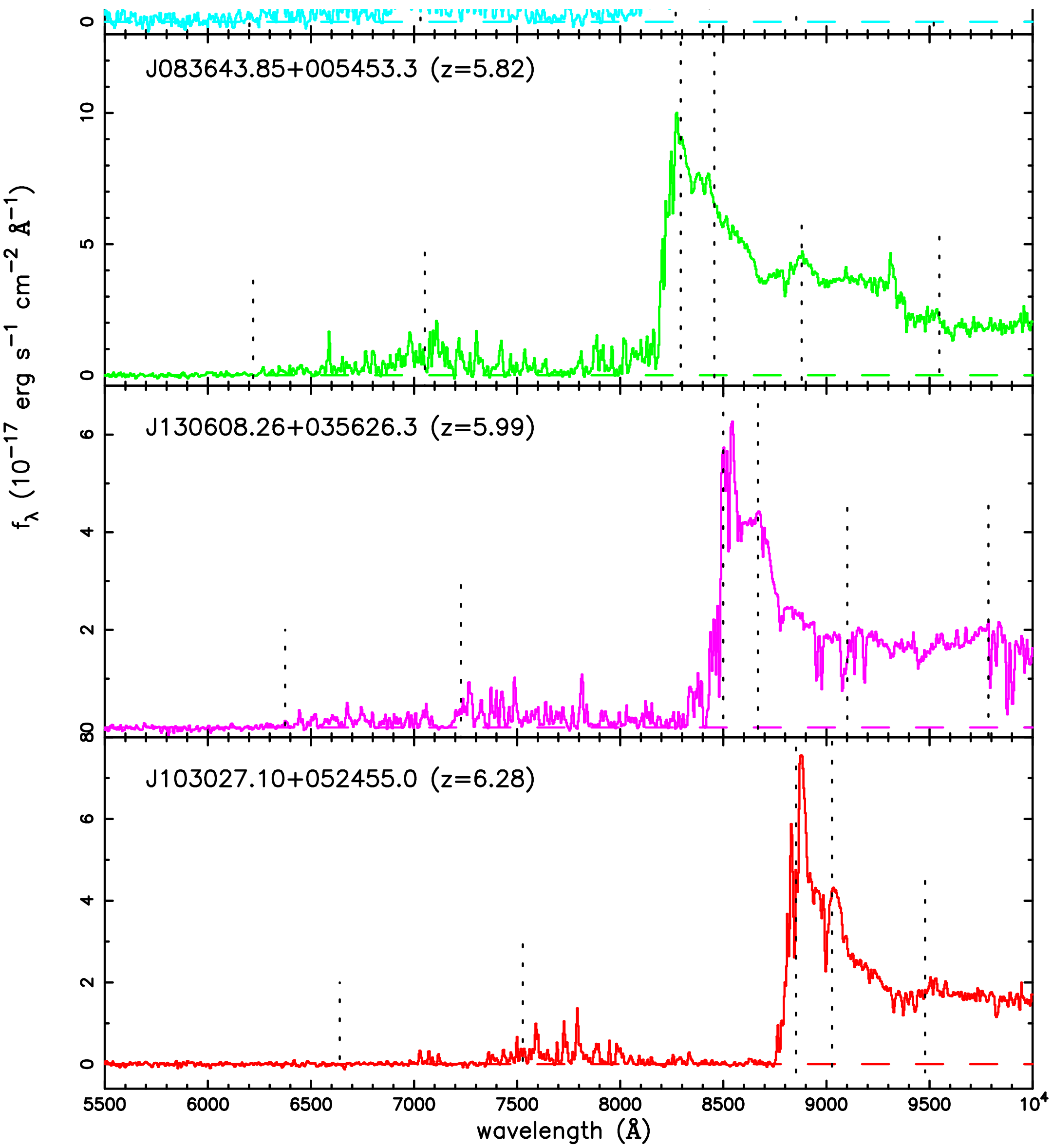

Figure 4: The light curves from three quasars at a variety of redshifts, z, from $z = 6.28$ (bottom) to $z = 5.82$ (top). What is shown is the energy emitted per area per time per wavelength interval $\Delta\lambda$ as a function of the wavelength. More precisely, by separating the light into its different colors with the help of a mirror or diffraction grating, a detector measures the intensity of the light for light with wavelengths between λ and $\lambda + \Delta\lambda$.

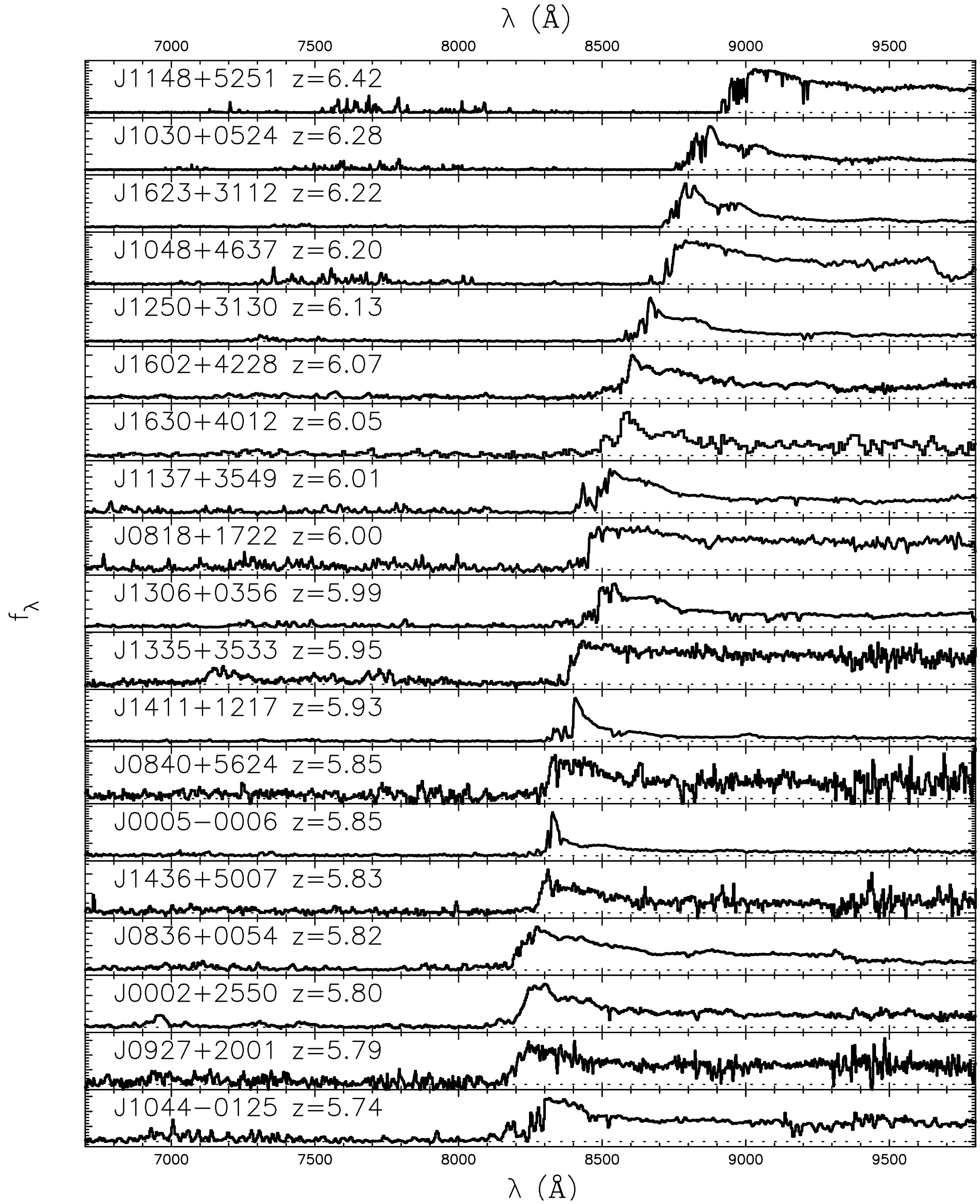

Figure 5: The light curves from several quasars at a variety of redshifts, z, from $z = 5.74$ (bottom) to $z = 6.4$ (top). What is shown is the energy per area per time per wavelength interval $\Delta\lambda$ that is observed, as a function of the wavelength. More precisely, by separating the light into its different colors with the help of a mirror or diffraction grating, a detector measures the intensity of the light for light with wavelengths between λ and $\lambda + \Delta\lambda$.