Last Time

- Briefly talked about photons

$$E = hf = \frac{hc}{\lambda}$$

energy of a single photon

$$hc = 1240 \text{ eV nm}$$

- Most important for us for a source with absolute Luminosity

$$L = \frac{\Delta E}{\Delta t} = hf \frac{\Delta N}{\Delta t}$$

energy per time

Energy of a photon

number of photons per time

Last Time (Continued)

- Supernova's -- Today's standard candle
  - know L = absolute luminosity (energy / time)
  - know ℓ = apparent luminosity (energy / area / time)
- Inferred distance

$$\ell = \frac{L}{4\pi D_L^2} \implies D_L = \left(\frac{L}{4\pi \ell}\right)^{1/2}$$

experimental D_L

- Review:

Hubble: $v = H_0 d$ $\qquad H_0 = 70 \frac{\text{km/s}}{\text{Megapc}}$

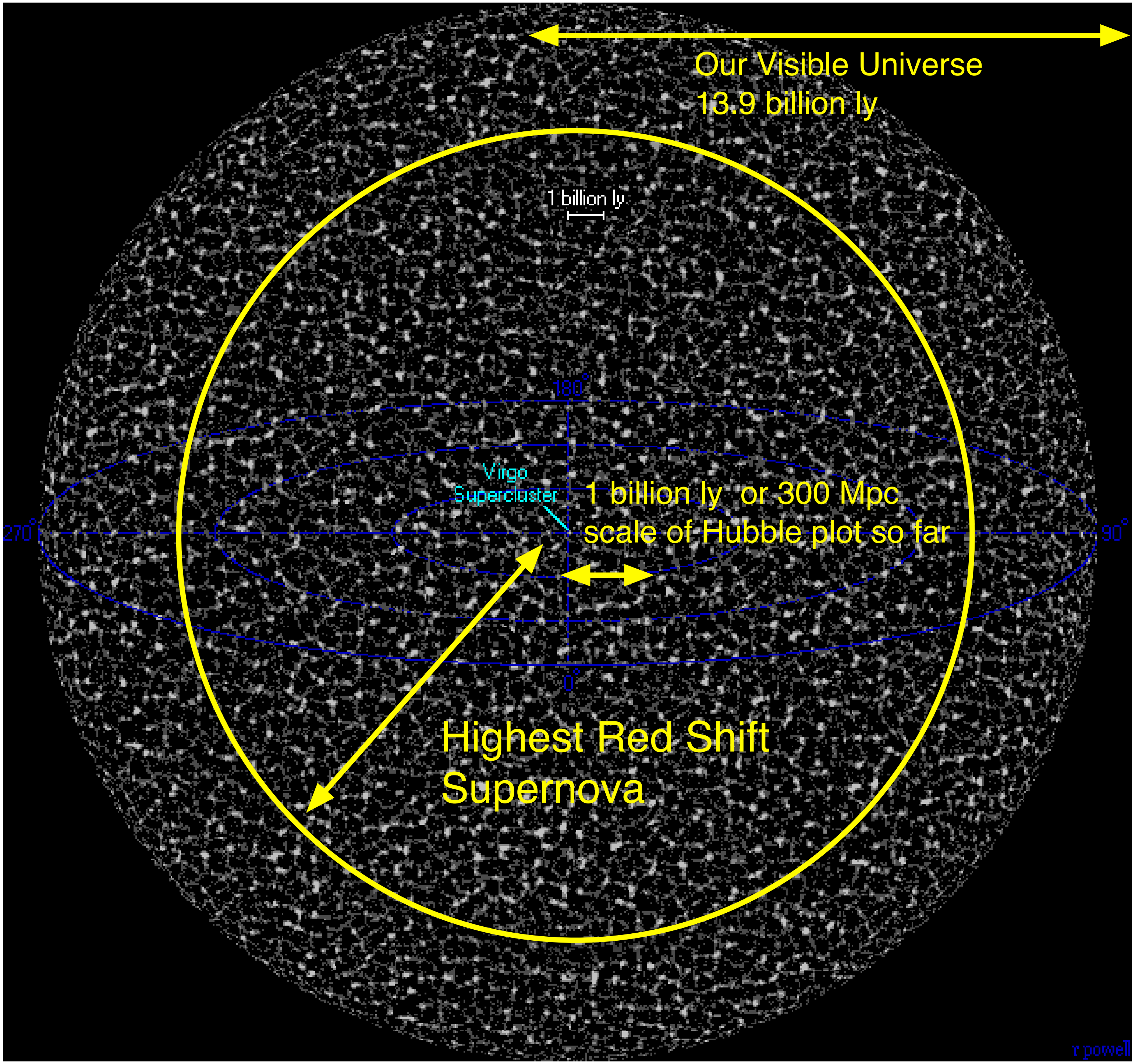
Red Shift: $\lambda_{obs} = (1+z)\lambda_{emit}$ $\qquad t_H = \frac{1}{H_0} = 13.9 \text{ Gly}$

giga light year

$$f_{obs} = \frac{f_{emit}}{(1+z)}$$

for $z \ll 1$:

$$\frac{v}{c} = z$$



Source www.atlasoftheuniverse.com

- Observe supernova at very large red-shifts $z \sim 1$

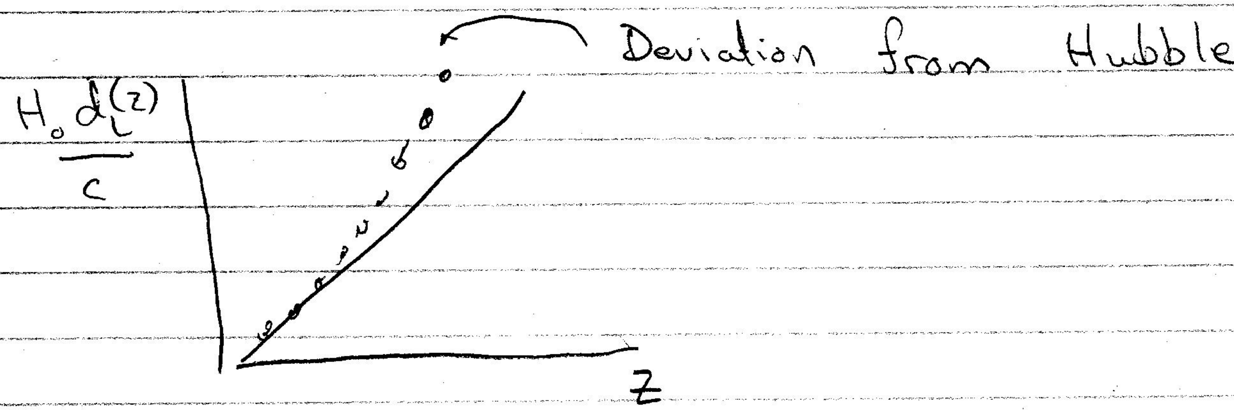
- Observe quasars out to $z = 5$

- For small z

$$\frac{v}{c} = H_0 \frac{d_L}{c}$$

$$\frac{H_0 d_L}{c} = z$$

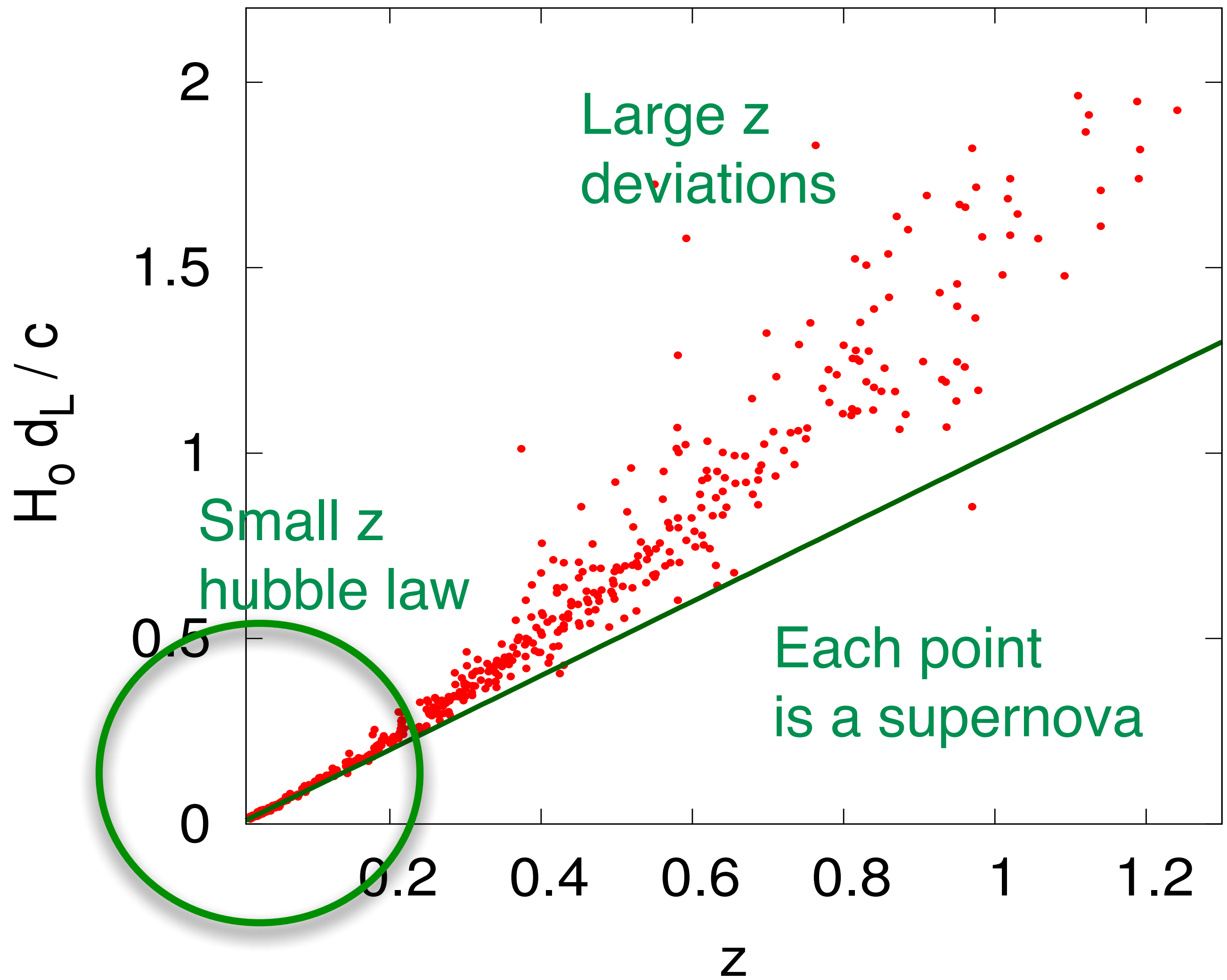
In general find (see slide)



Often plot

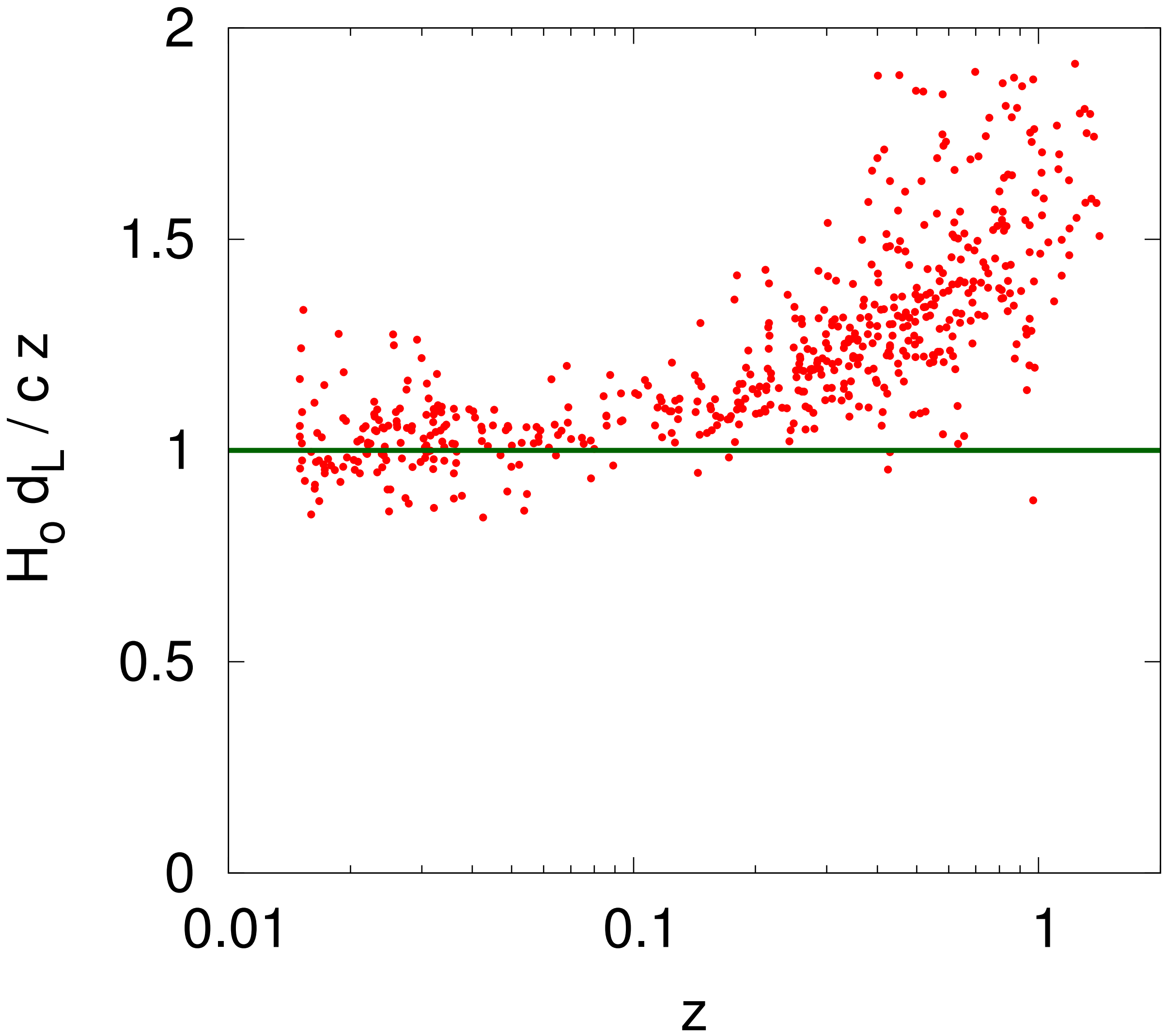
$$\frac{H_0 d_L(z)}{c\, z} \quad \text{vs.} \quad z$$

Supernova data:



The deviations from the hubble law can tell us about the past history of the universe!

Another plot of Supernova



Same Data. Deviations from one tell us about the past history of the universe!

The Expanding Universe:

- The universe is expanding. Thus the distance between objects at times past is smaller than today

$$d(t) = a(t)\ \Delta r$$

$d(t)$: distance between objects in times past

$a(t)$: Robertson-Walker scale factor

$a(t) < 1$

Δr: the distance between objects today

r - labels the lines seen in the slide

"coordinate distance"

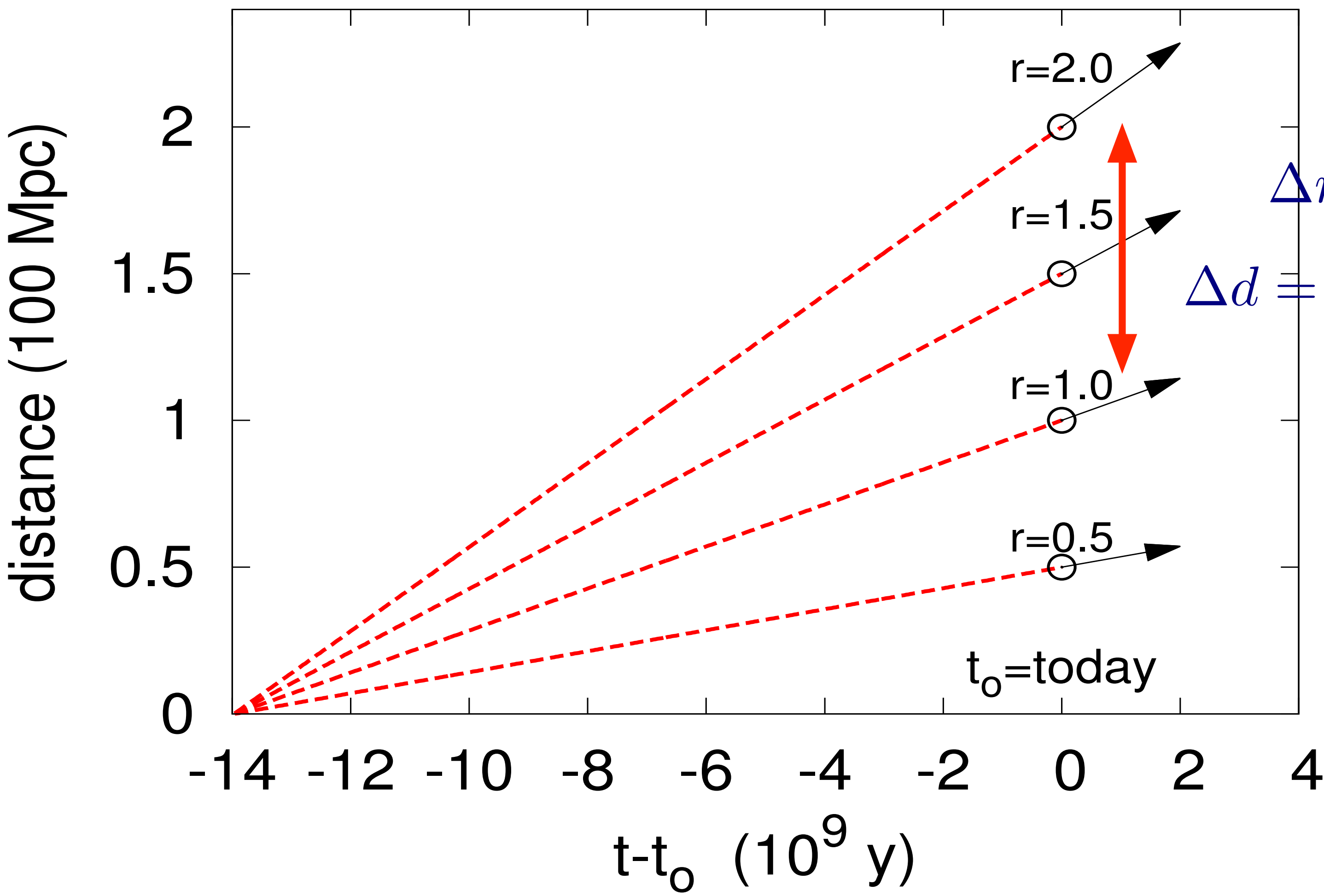
- Then $a(t)$ is defined so that

$$a_0 \equiv a(t_0) \equiv a(\text{today}) = 1$$

So

$$d(\text{today}) = \Delta r$$

Naive Backward Extrapolation



$\Delta r = 1$ and

$$\Delta d = \underbrace{a(t_o)}_{=1} \Delta r = 1$$

$$\Delta d(t) = a(t)\Delta r$$

• However, at an earlier time, $t_* - t_0 = -7 Gy$,
(Gy is short for Giga $= 10^9$ year)

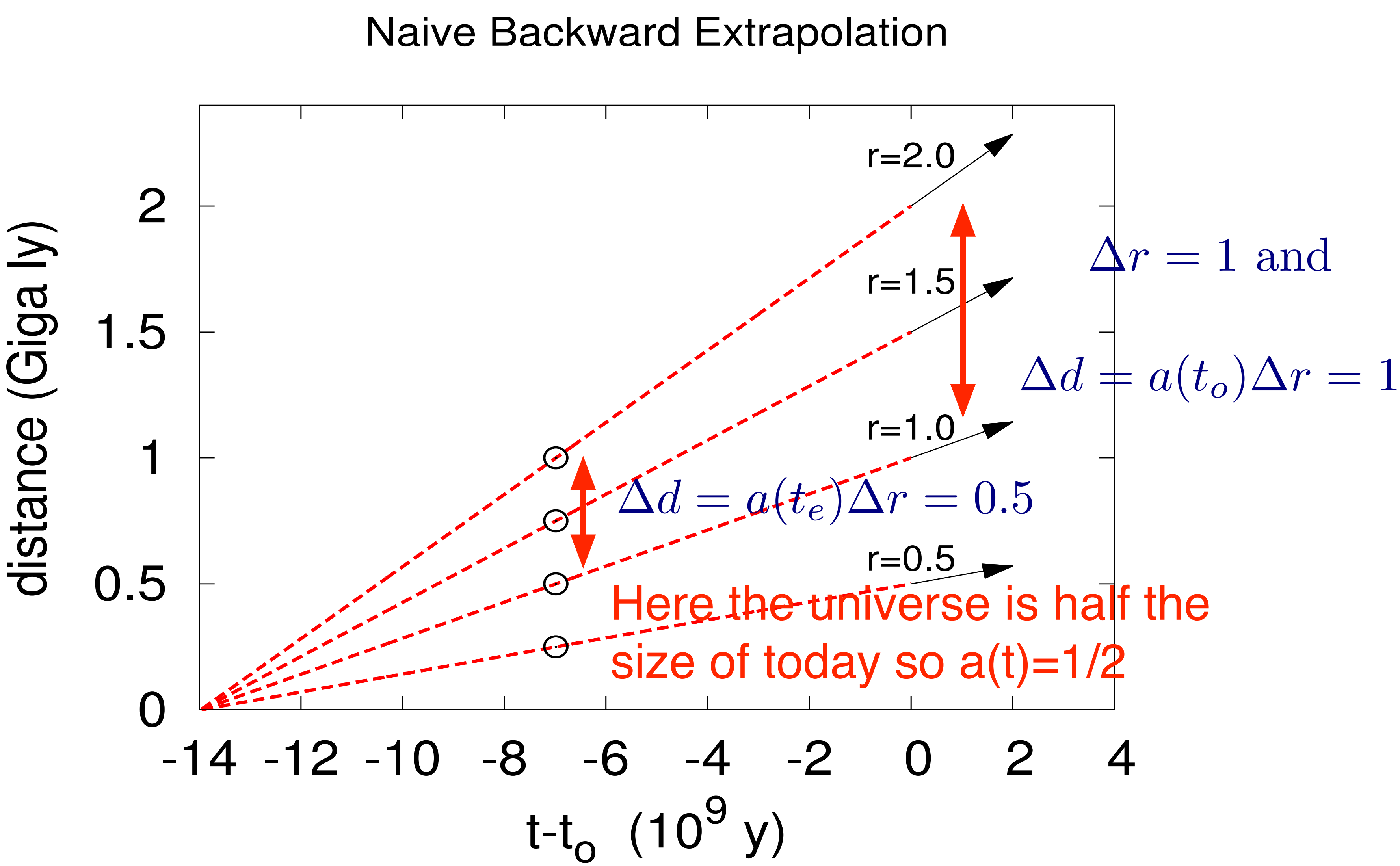
the distance between two objects
(at coordinates r_1 and r_2) is less.

Reading the graph on the next
lide we see that $t = -7 Gy$ ago

$$d \simeq \frac{1}{2} \times Gly$$

So

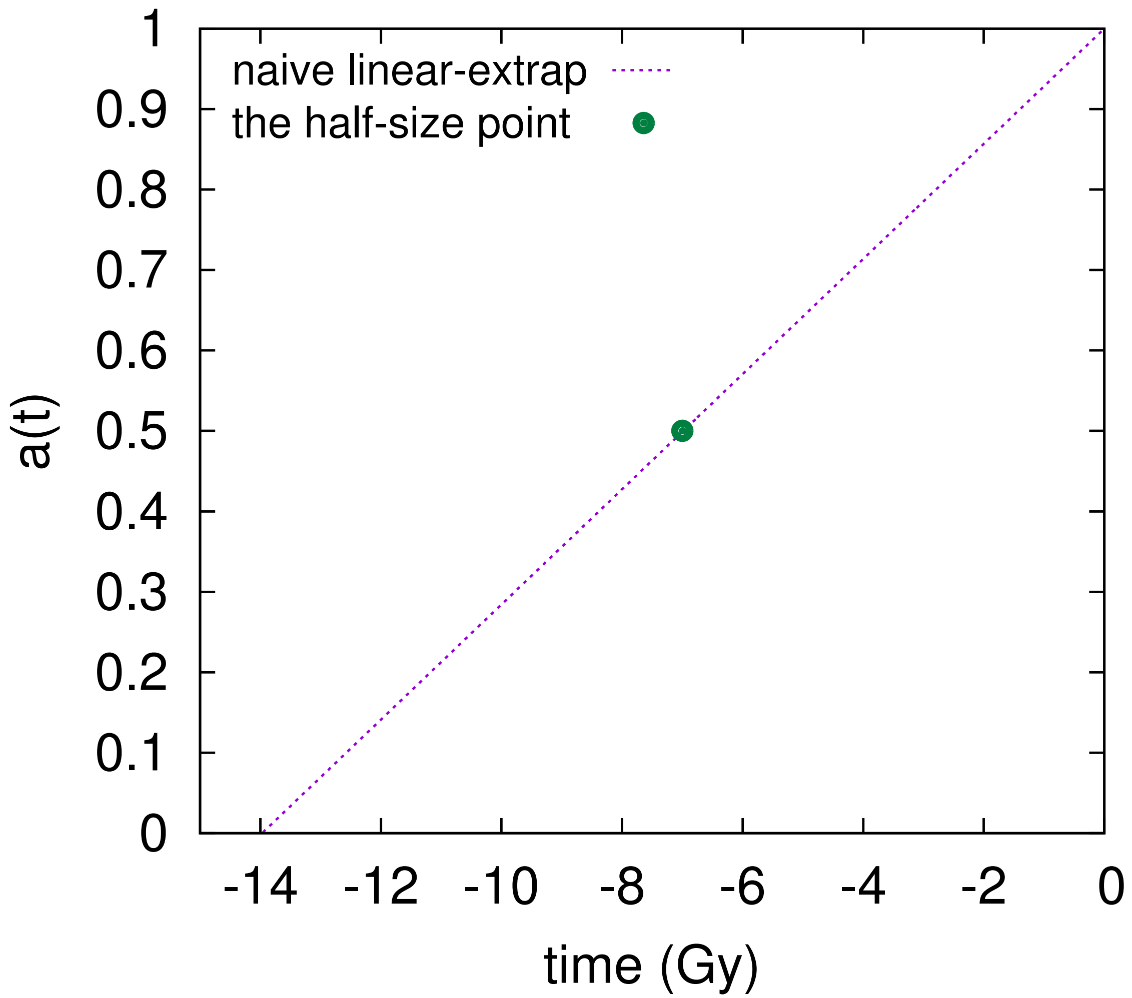
$$a(t_*) \simeq \frac{1}{2}$$



$$\Delta d(t) = a(t)\Delta r$$

$a(t)$ records how much the universe is smaller than today

$$\underbrace{\Delta d}_{\text{Distance at time } t} = a(t) \underbrace{\Delta r}_{\text{Distance today}}$$

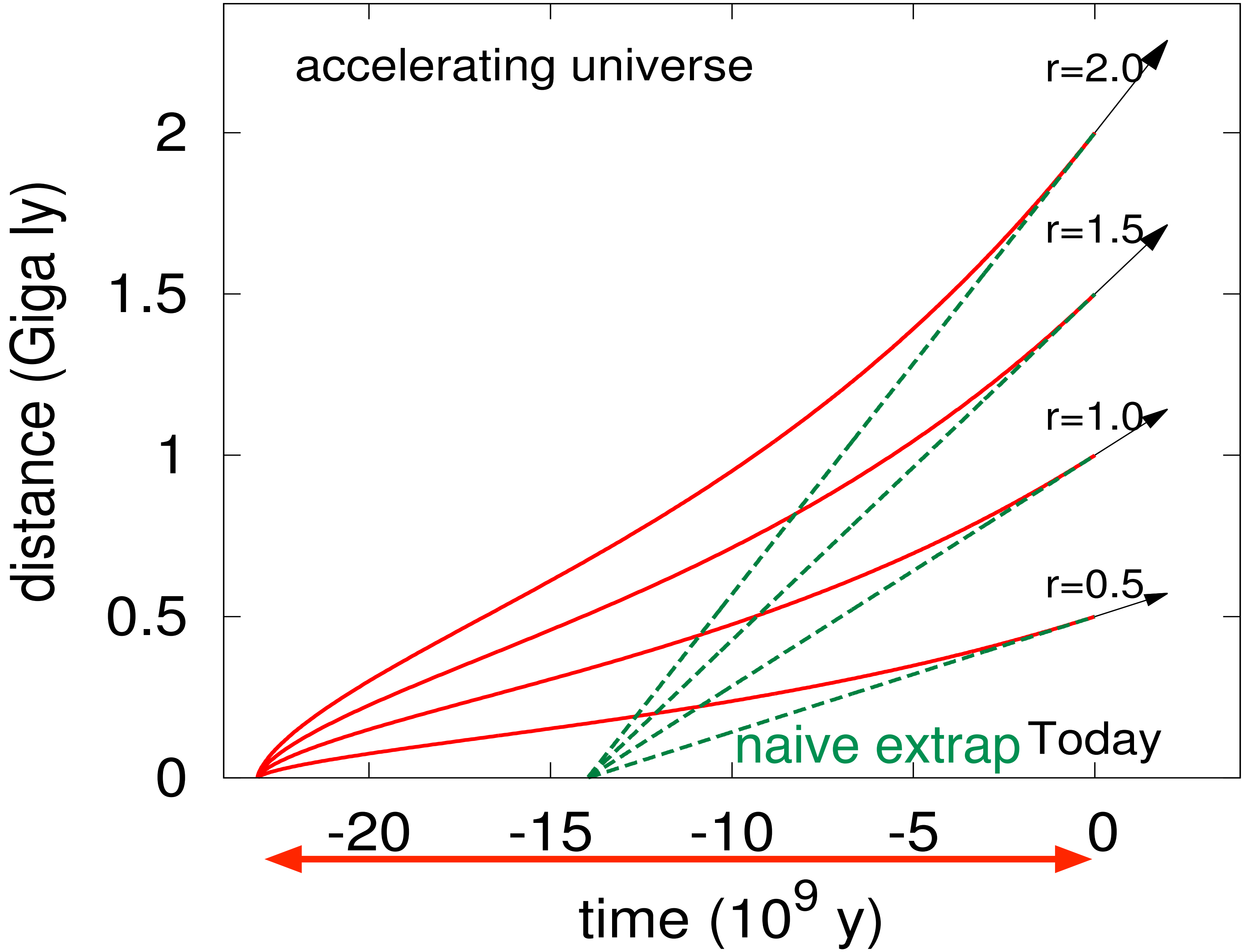


Three model universes which go beyond the Naive extrapolation

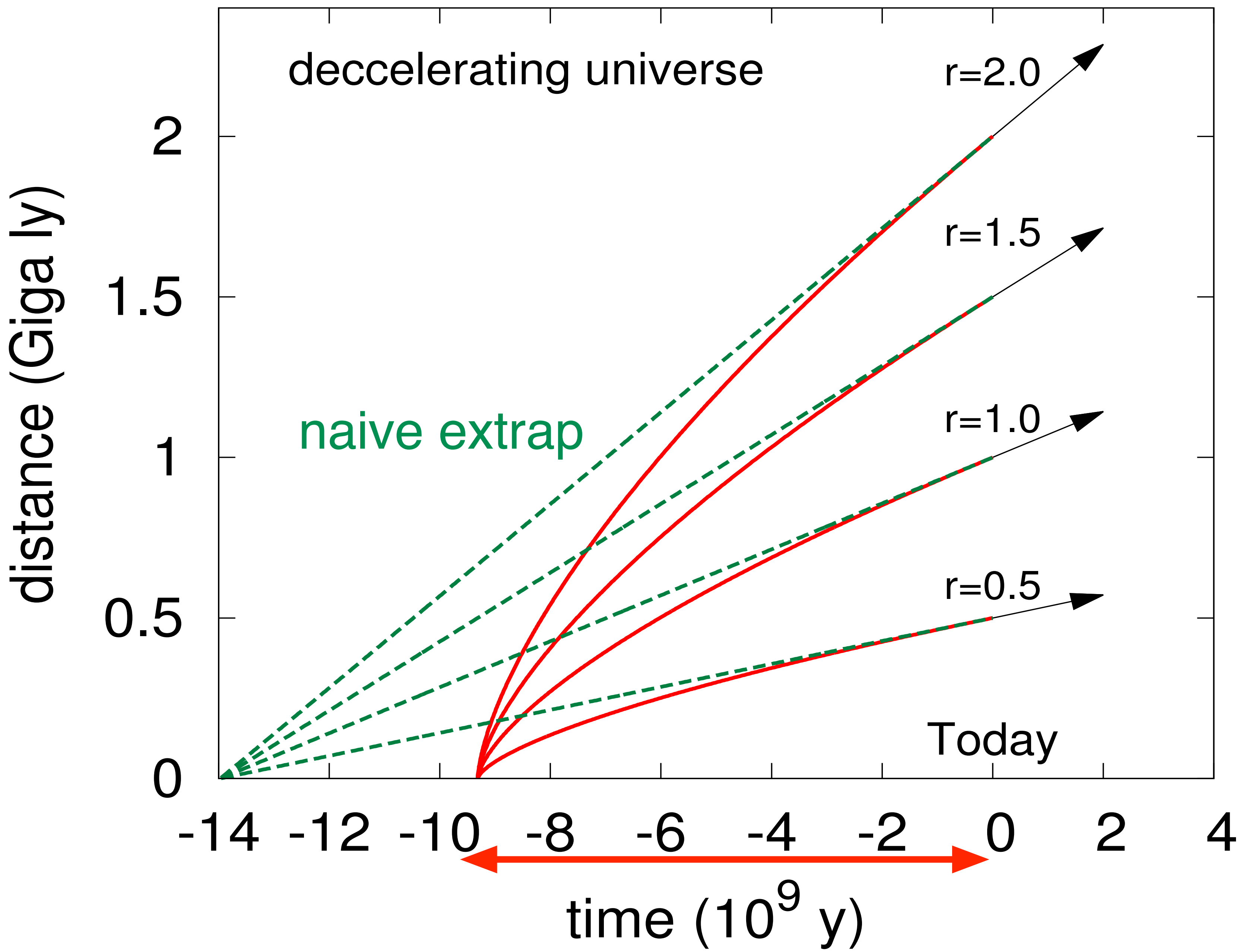
1. Accelerating

2. Decelerating

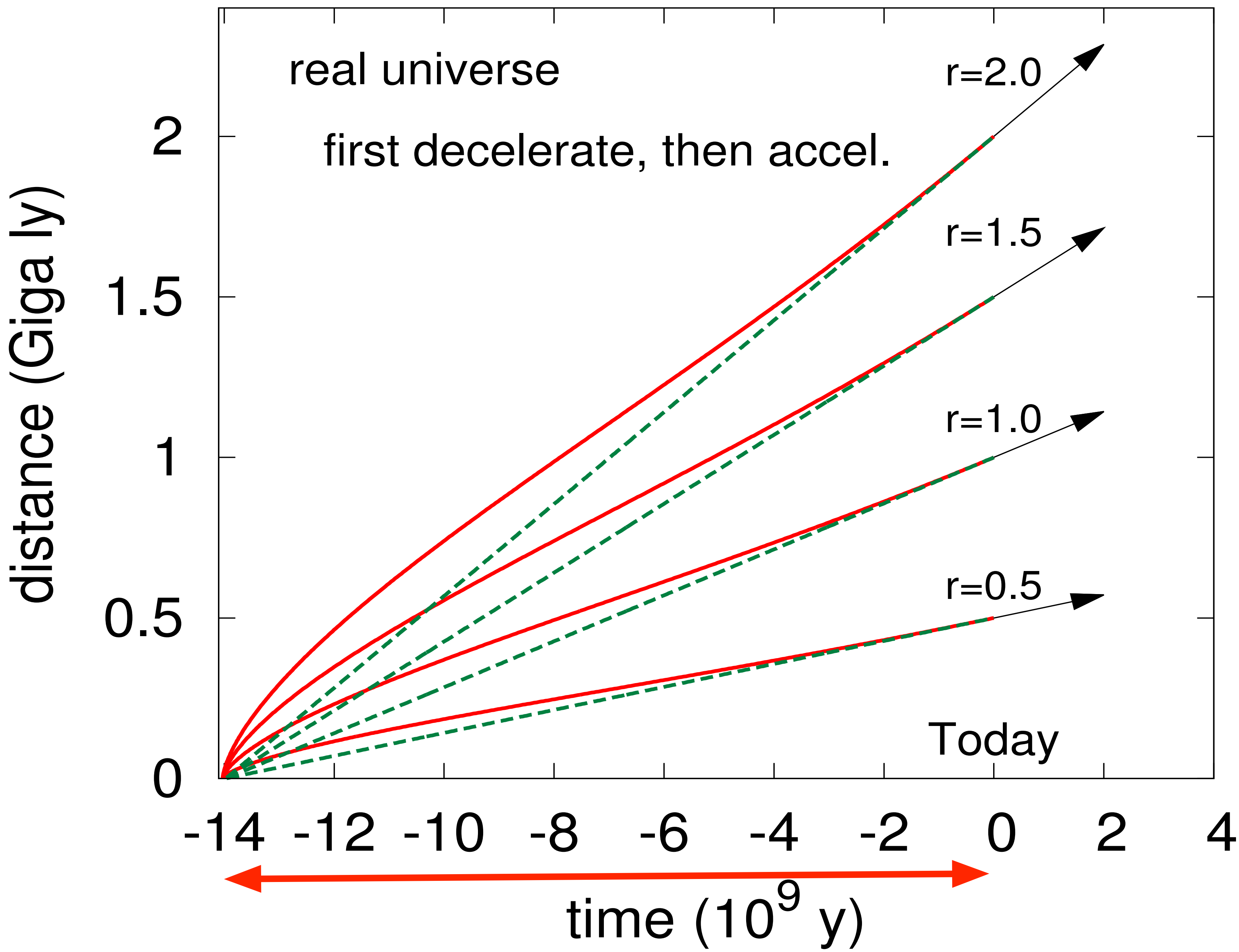
3. Real Case: Decelerating than Accelerating



Lifetime of Universe is (for this cosmology)
24 Billion Years

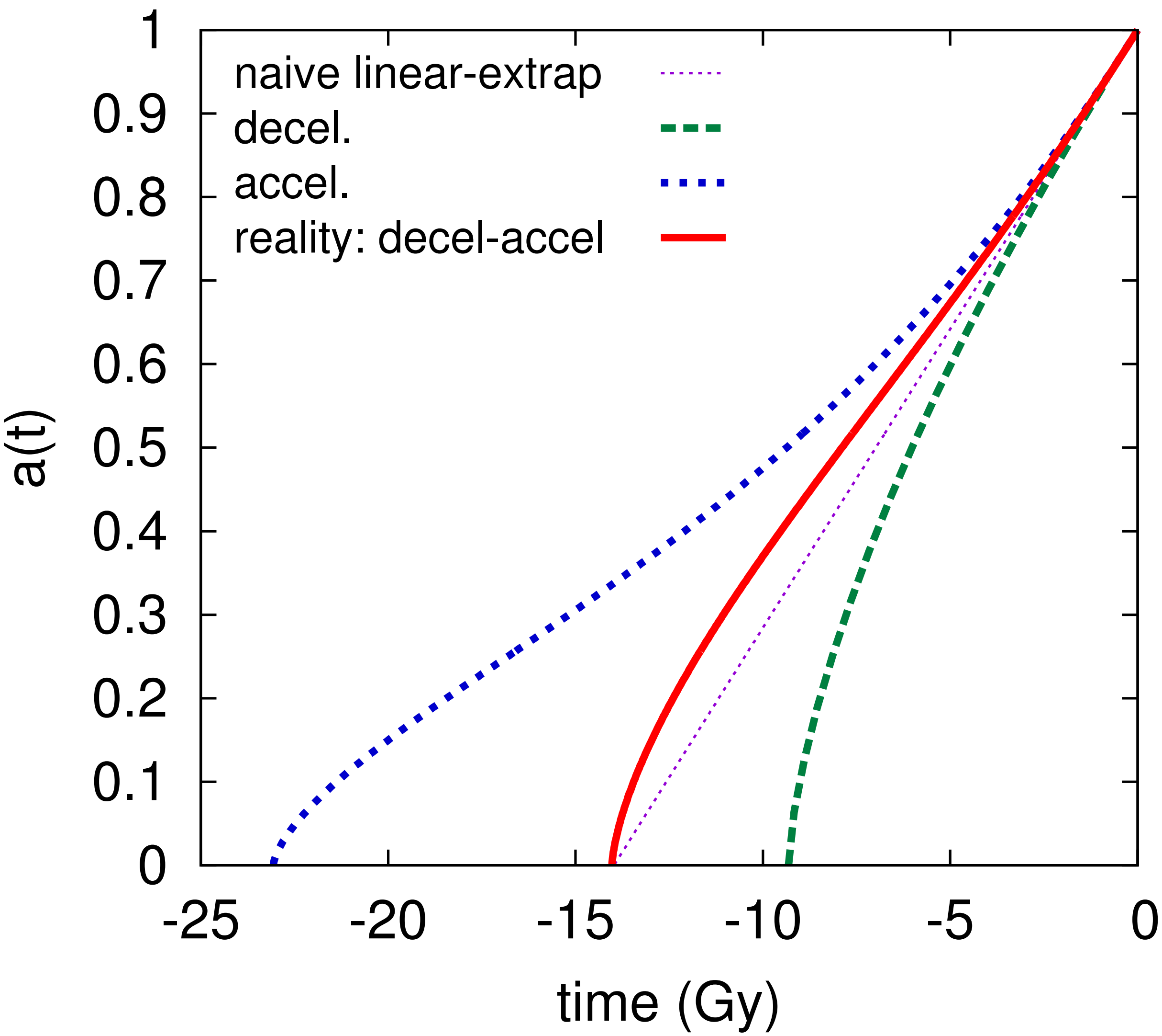


Lifetime of Universe is (for this cosmology)
~9 Billion Years



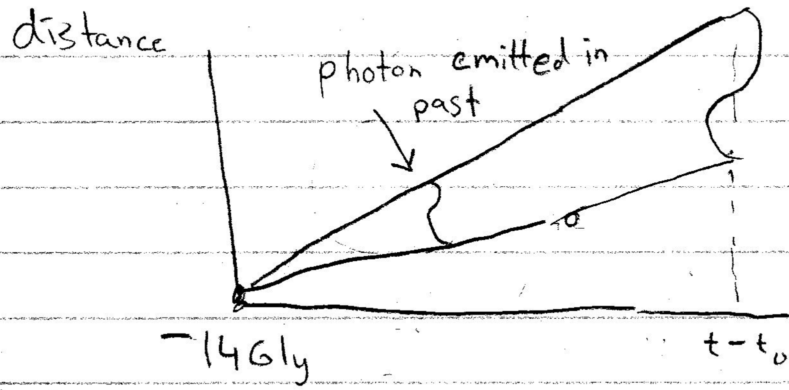
Lifetime of Universe is (for this cosmology)
14 Billion Years

Summarize all cosmologies (expansion histories) with $a(t)$



We will see that $a(t)$ records the red shift for light emitted at time t.

The Expanding universe i.e $a(t)$ and red Shift

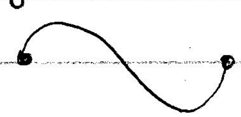


- Due to the expansion the photon today has a longer wavelength than when emitted, i.e. is red-shifted.

- All lengths increase by the ratio of scale factors: $\frac{1}{a(t_*)} = \frac{1}{a(t_*)}$

- Red Shifts work the same way. Consider a photon emitted 5 Gigaly ago:

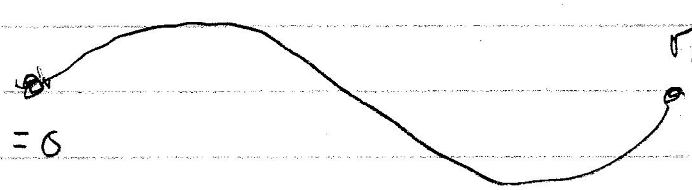
5 Gly Ago



$r=0$ $\quad r=\Delta r$

$$\lambda_{emit} = a(t_e)\,\Delta r$$

Today



$r=0$ $\quad r=\Delta r$

$$\lambda_{obs} = a(t_0)\,\Delta r$$

$$\lambda_{obs} = \frac{a(t_0)}{a(t_0)}\,\underbrace{a(t_e)\,\Delta r}_{\lambda_{emit}}$$

So with $a(t_0) = a_0 = 1$

$$\lambda_{obs} = \frac{1}{a(t_e)} \lambda_{emit}$$

Then comparison

$$\lambda_{obs} = (1+z)\ \lambda_{emit}$$

Says

$$\boxed{(1+z) = \frac{1}{a(t_e)}}$$

- Measured red shift is related to the scale factor at emission $a(t_e) < 1$
- Given a model cosmology, $a(t)$, can relate measured z to the time when the photon was emitted.

Problem

Problem three supernova are found at red-shifts

$$z_1 = 0.5 \qquad z = 1.0, \qquad z = 1.4$$

In the decelerating cosmology, determine when they exploded. First, determine the scale factors a_1, a_2, a_3 at emission:

Solution

$$1 + z = \frac{1}{a} \Rightarrow a = \frac{1}{1+z}$$

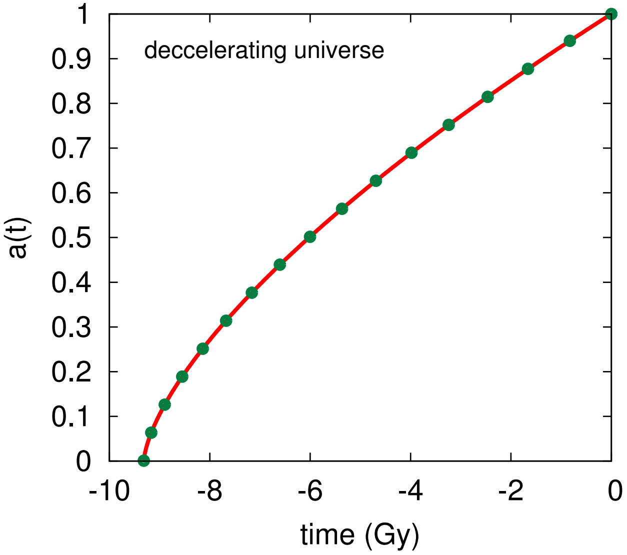
So

$$a_1 = \frac{1}{1+0.5} \qquad a_2 = \frac{1}{1.0+1} \qquad a_3 = \frac{1}{2.4}$$

$$a_1 = 0.66 \qquad a_2 = 0.5 \qquad a_3 = 0.41$$

From graph of a(t) find (next slide!)

$$t_1 = -4.2\,Gy \qquad t_2 \simeq -6.0\,G.y \qquad t_3 \simeq -6.8\,Gy$$



deccelerating universe
a(t)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
-10
-8
-6
-4
-2
0
time (Gy)

Giga

Light from Supernova's

10^9 light years = billion

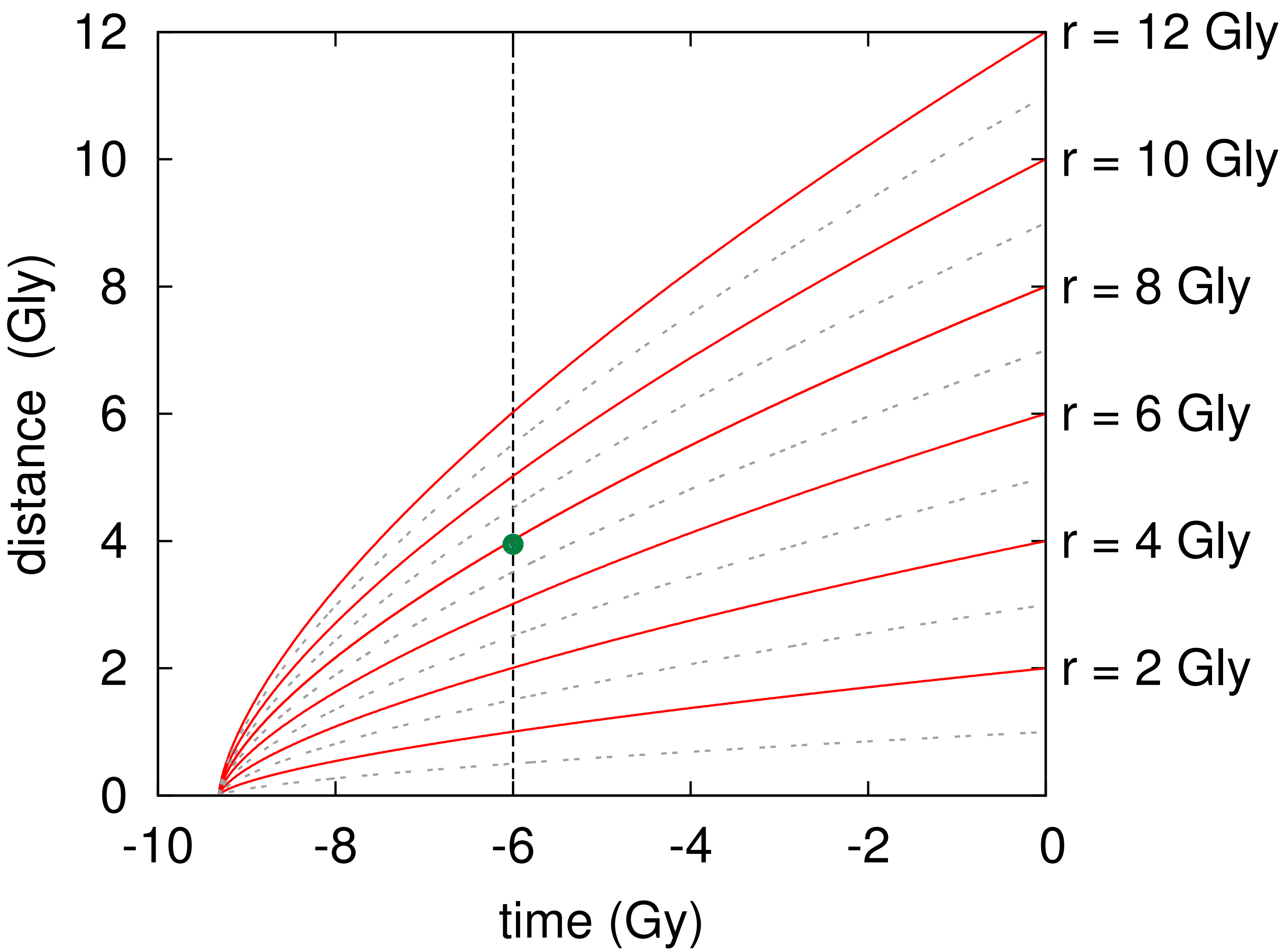
- If a supernova explodes today at distance of 5 Gly away. We wont see it till a time of order ~5 billion years later (i.e. we wont see it at all)

- Similary, if a supernova exploded 10 ly away 5 billion years ago, we want see the light today it will have passed us ~(5 billion − 10) years ago.

→ Thus for an explosion at time t_e there is only one distance $\underline{d_*(t_e)}$ which we will see.

- One might think that if and explosion happened five billion ly away, we will see it in 5 billion years, This assumes that the universe is a fixed size. But it is not fixed it is expanding.

What does $r_*(t_e)$ mean?



r_* records where a supernova must explode at time t_e

to be observed today

How to Calculate $d_*(t_e)$? Given $a(t)$

- Start today and trace the light backward in time. In a time Δt, the light moved:

small

$$\Delta d_0 = c\,\Delta t_0$$

$$a(t)\,\Delta r_0 = c\,\Delta t$$

$$\Delta r_0 = \frac{c\,\Delta t}{a(t_0)}$$

change in radius at $t = t_0$

change in radius at $t = t_0 - \Delta t$

- Now repeat:

$$\Delta r_1 = \frac{c\,\Delta t}{a(t_1)}$$

$$\Delta r_2 = c\,\frac{\Delta t}{a(t_2)}$$

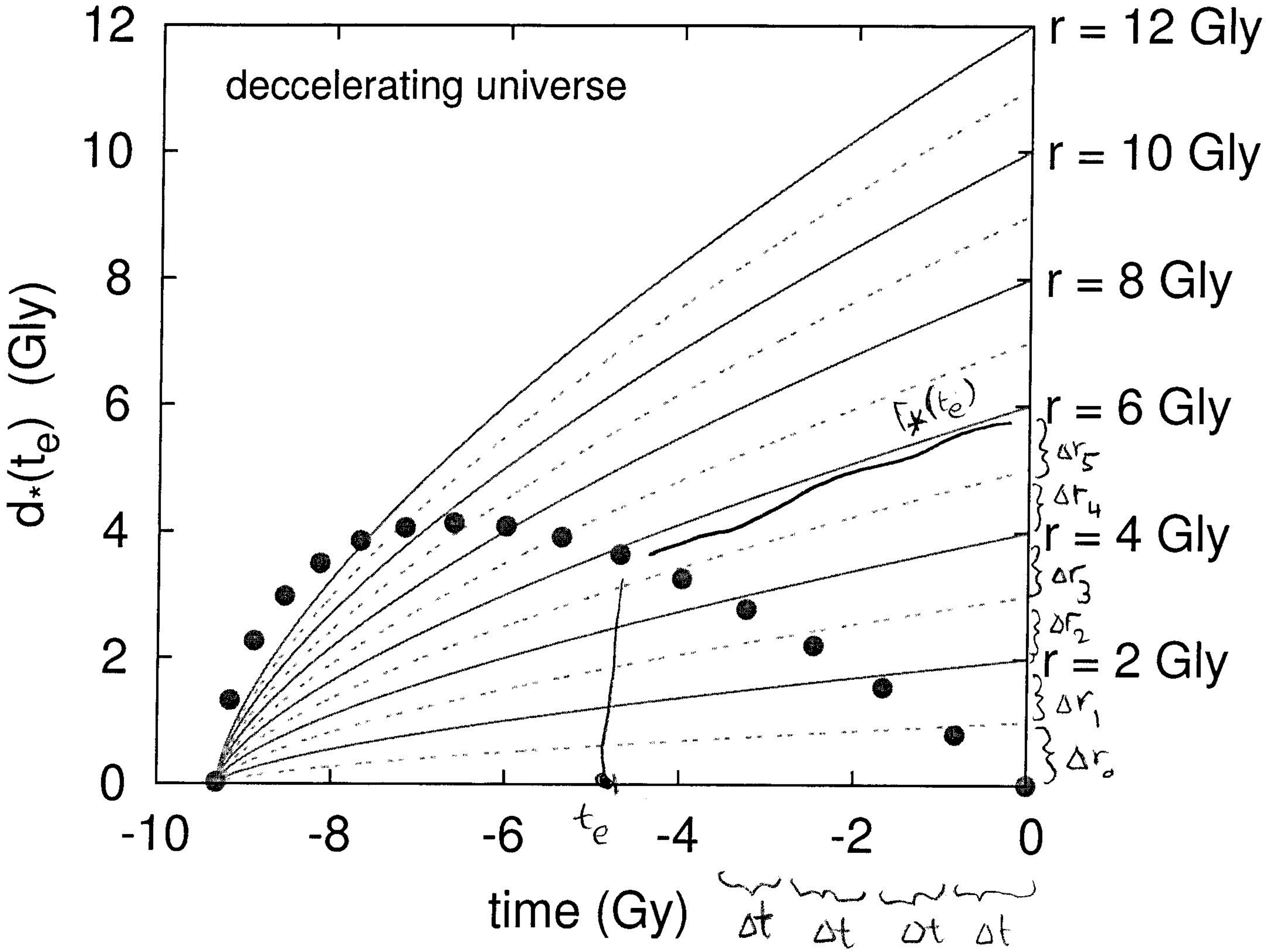
Change in radius at $t = t_0 - 2\Delta t$

$$\Delta r_3 = \frac{c\,\Delta t}{a(t_3)}$$

Change in radius at $t = t_0 - 3\Delta t$

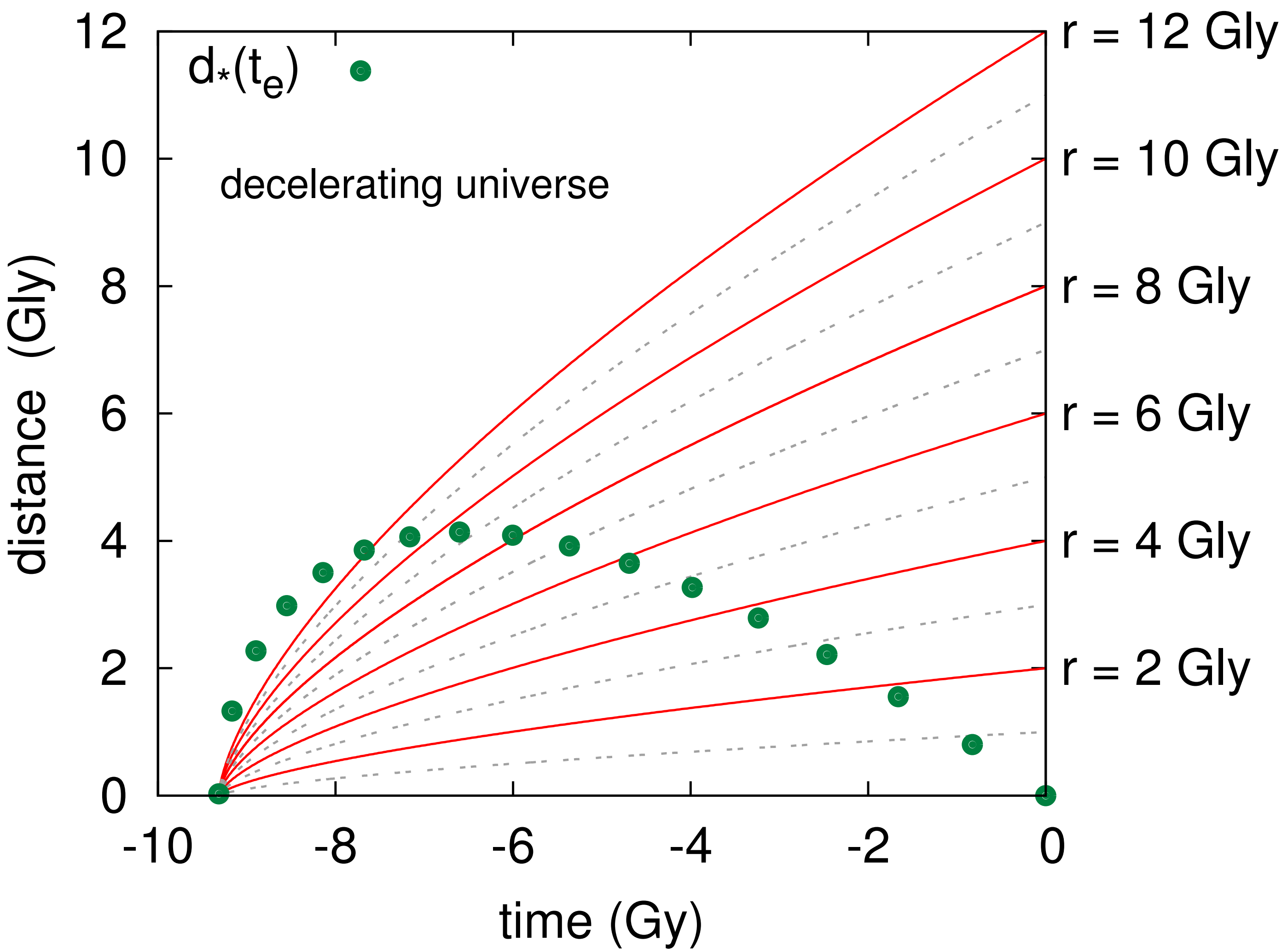
See slides

Computing $r_*(t_e)$ and $d_*(t_e)$:



$r_*(t_e)$ (and $d_*(t)$) record where a supernova must explode at time t_e to be observed today

Computing $r_*(t_e)$ and $d_*(t_e)$:



r_* records where a supernova must explode at time t_e to be observed today. Note $d_*(t) = a(t)r_*(t)$.

Thus we determine the radius of emission:

$$r_*(t_e)$$

Then the distance is

$$d_*(t_e) = a(t_e)\, r_*(t_e)$$

Important: $r_*(t_e)$ depends on the whole history of expansion

- for those who know calculus

$$r_*(t_e) = \int_{t_e}^{\text{today}} \frac{c\,dt}{a(t)}$$

Thus we determine the radius of emission:

$$r_*(t_e)$$

Then the distance is

$$d_*(t_e) = a(t_e)\, r_*(t_e)$$

Important: $r_*(t_e)$ depends on the whole history of expansion

- for those who know calculus

$$r_*(t_e) = \int_{t_e}^{\text{today}} \frac{c\,dt}{a(t)}$$

Trajectory of Light and Remains of SN:

So if a supernova explodes 6 Billion years ago (in the decelerating cosmology)

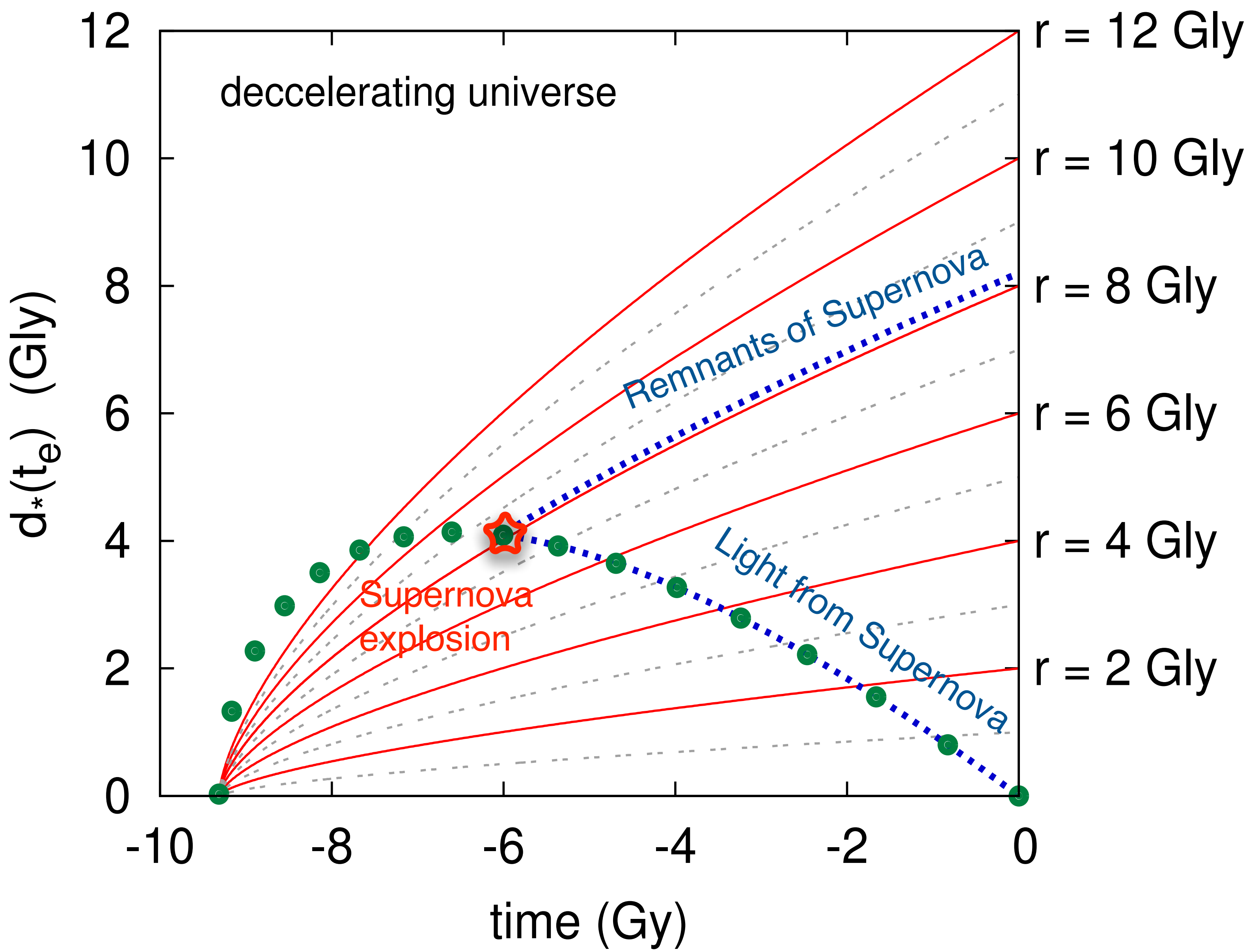
- From graph (see slide) we see that the

$d_*(t) \simeq 3.9$ Gly ← distance from "earth" at explosion

- And

$r_* = 8.2$ Gly ← distance of remnants today

Light and remnants from a supernova



Problem for the three supernova studied previously,

Determine $r_*(t_e)$ for the decelerating cosmology

Solution:

– The three emission times are

$t_1 = -4.2$ $\quad t_2 = -6.0$ Gy $\quad t_3 = -6.8$ Gy

$r_*(t_1) = 5.1$ Gly $\quad r_*(t_2) \simeq 8.2$ Gly

and $r_*(t_3) \simeq 9.9$ Gly

• These numbers are read from graph

(See table for exact #'s)

• $r_*(t_1)$, $r_*(t_2)$, $r_*(t_3)$ are the distances from us that supernova remnants are today.

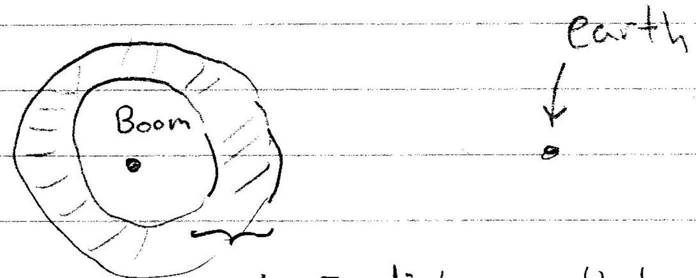
Relation Between Absolute and apparent luminosity for an expanding universe:

Formula: $\ell = \frac{1}{4\pi r_*^2} \left(a(t_e)\right)^2 L_*$

- ℓ: apparent luminosity today.
- r_*: the radius of the source at emission.
- $a(t_e)$: scale factor at emission
- L_*: absolute luminos at source.

Derivation:

5 Gy ago:

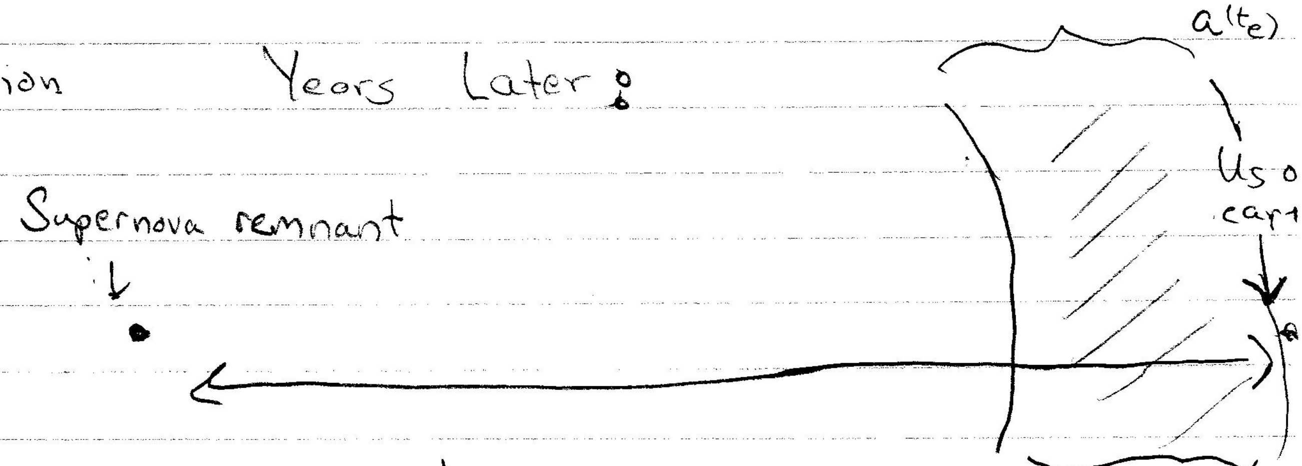


$c\Delta t_*$ = distance that the shockwave takes up

$d_* = a(t_e)\, r_*$ = distance between SN and earth.

$c\Delta t = \frac{c\Delta t_*}{a(t_e)}$

5 Billion Years Later:



$d = a_0\, r_* = r_*$

So

Energy of source is spread out over $4\pi r_*^2$

$$l = \frac{L}{4\pi r_*^2} \times \left(\frac{a(t_e)}{a_0}\right) \times \left(\frac{a(t_e)}{a_0}\right)$$

- the shell has expanded, so the number per time is less. $\frac{\Delta N}{\Delta t} = a_e \frac{\Delta N}{\Delta t_*}$
- Each photon is red shifted and hence carries less energy

$$\boxed{l = \frac{1}{4\pi r_*^2(t_e)} a(t_e)^2 L}$$

Or:

$$\boxed{D_L^{theory} = \left(\frac{r_*(t_e)}{a(t_e)}\right) = \left(\frac{L}{4\pi l}\right)^{1/2} = D_L^{experiment}}$$

D_L^{theory} for a given z, and ~~a~~ an assumption about $a(t)$, determine $r_*(t_e)/a(t_e)$ which can be compared to the measured D_L to see if $a(t)$ is right.

Problem:

Determmite the luminosity distance D_L^{theory} for the three red-shifts we had

Also compute $\frac{c}{H_0 z} D_L^{theory}$ for the

Three red shifts we had

Solution

$$D_L^{theory} = \frac{r_*}{a(t_e)}$$

with r_* and $a(t_e)$ in table
find

CHRIS DOWD AARON ANKUDAVICH

Decelerating Cosmology

inputs →

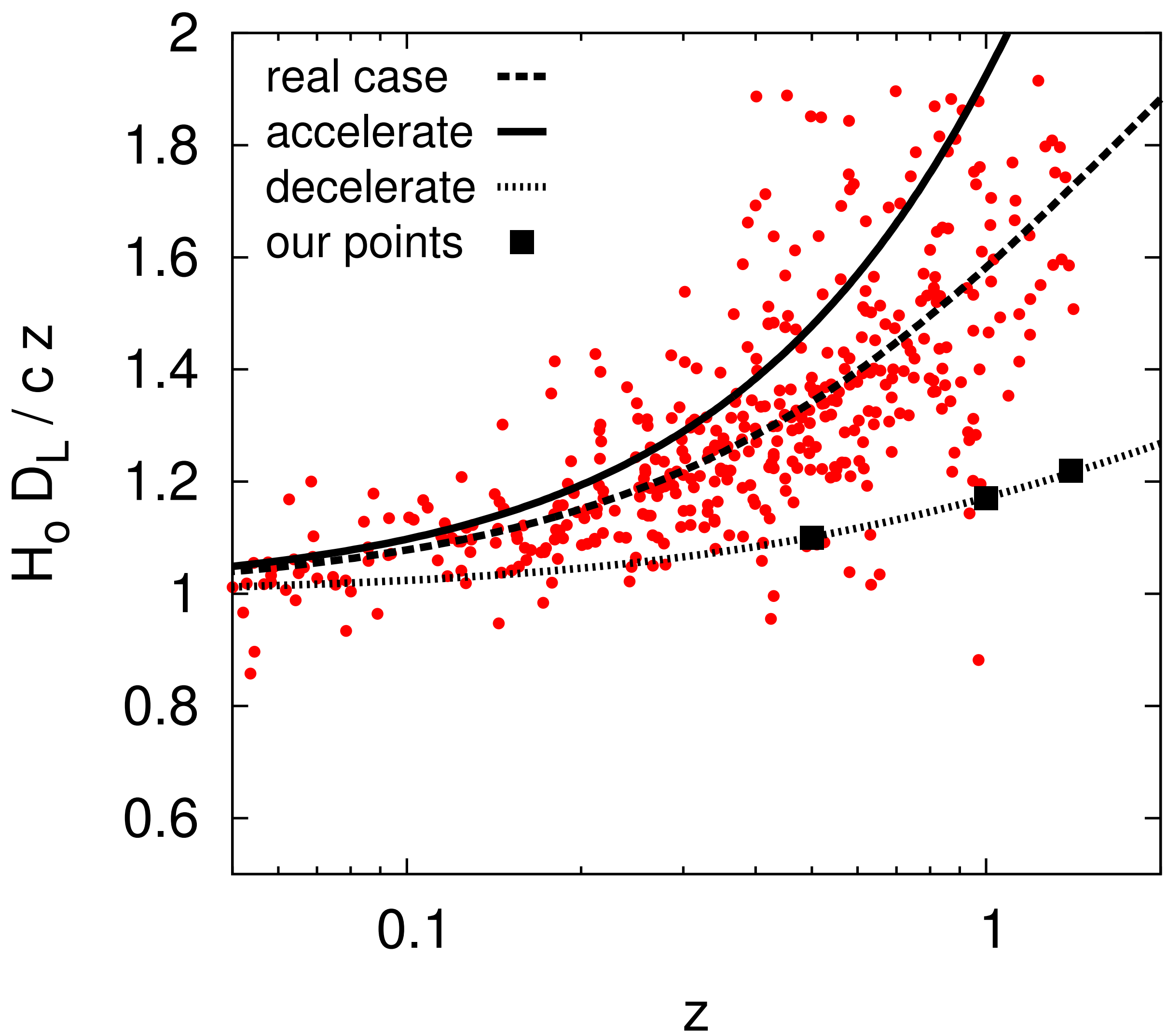
	z	0.5	1	1.4
	$a(t_e)$	0.66	0.5	0.42
(Gy)	time	-4.24	-6.02	-6.81
(Gly)	$r_*(t_e)$	5.12	8.18	9.91
(Gly)	$D_L^{th}(t_e)$	7.69	16.36	23.77
	$\frac{H_0 D_L^{th}}{c\, z}$	1.10	1.17	1.22

a(t) is also input

output

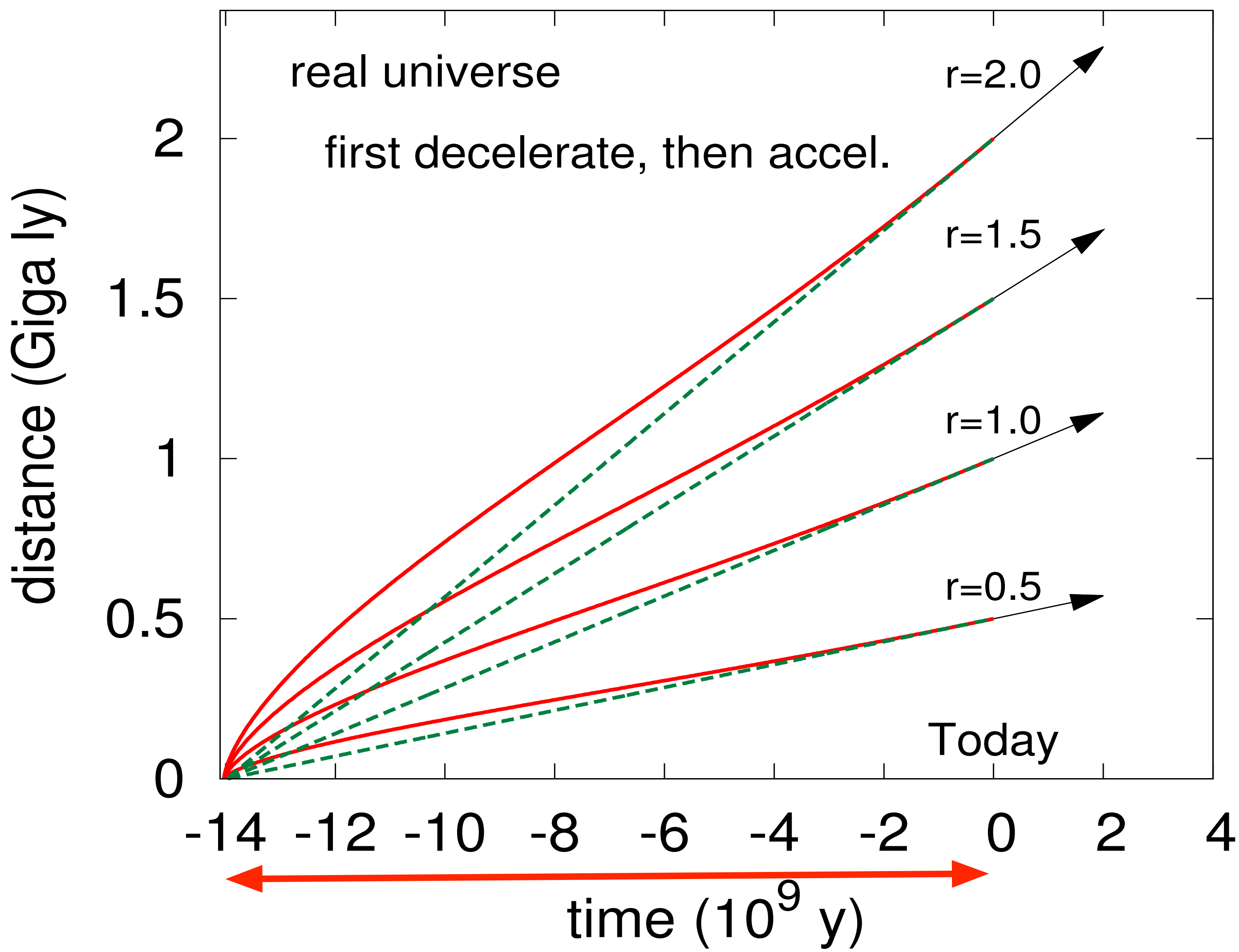
$$\frac{c}{H_0} = \frac{3\times10^8 \text{ m/s}}{70 \text{ km/s/mega pc}} = 13.97 \text{ Gly}$$

What cosmology does the data prefer?



The data want first a period of deceleration then a period of acceleration!

Preferred acceleration history:

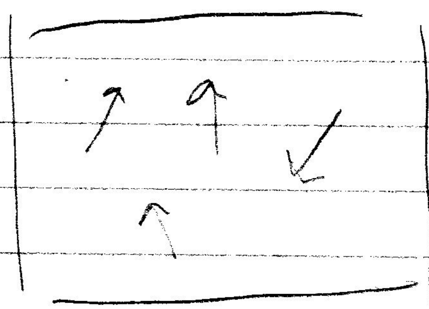


Lifetime of Universe is (for this cosmology)
14 Billion Years

Black Body Radiation

~1880 Mankind Understands what is temperature

E and N



If you think of temperature (times boltzman constant) as the energy per particle you are <u>almost</u> righ

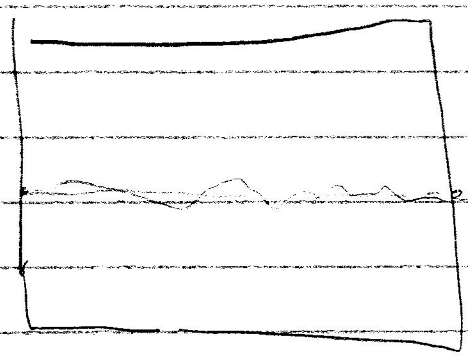
$$k_B T \sim \frac{E}{N}$$

Boltzman Constant

$$k_B = \frac{\frac{1}{40}\, eV}{300^\circ K} = \frac{1.38 \times 10^{-}}{K}$$

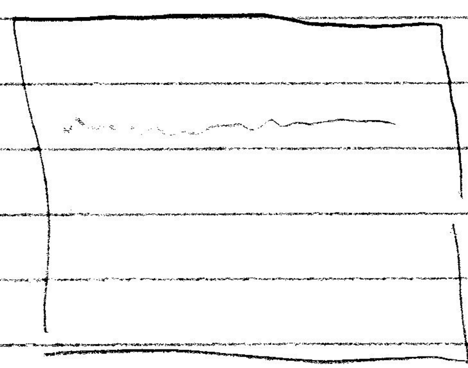
More correctly temperature records how energy is shared between the particles

Then imagine stretching a thin light wire across the box:



The wire would start to vibrate, sharin part of the available ener

Similarly, the electromagnetic field starts to vibrate due to interaction with charged partices in walls.



- The electric-magnetic field shares the energy in the box.

It is for this reason that things glow when they are hot:

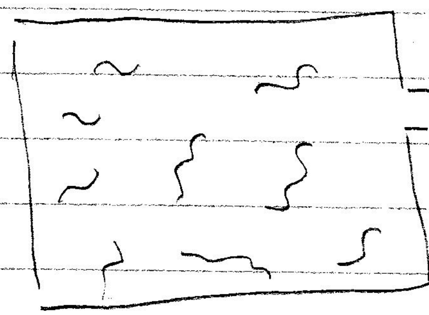
(1) The glow different colors at different temperatures
Higher Temperatures ⟷ Shorter wavelengt
(2) Higher Temperature ⟷ more light

The hot walls in equilibrium with the radiation in an oven

Basics of Black Body Radiation:

- All wavelengths are allowed
  but the average wavelength / frequency
  is set by the temperature:

  $$hf \sim k_B T$$

- 

  Most Probable frequency:

  $$\boxed{hf_{max} = 2.82\, k_B T}$$

  exact formul

The number of photons in box
is set by the size of box:

$$\frac{\text{volume}}{\text{photon}} \sim \lambda^3$$

So $$\frac{V}{N} \sim \lambda^3 \sim$$

Or $$\frac{N}{V} \sim \frac{1}{\lambda^3} \sim \left(\frac{k_B T}{hc}\right)^3$$

For 1000°K

$$hf_{max} = 2.82 \frac{\frac{1}{40}eV \times 3000°k}{300°k}$$

$$hf_{max} = 0.705 eV \leftarrow \text{infrared light}$$

Since you can't see infrared, the light appears red, there is very little blu

But For 10,000°K:

$$hf_{max} = 2.35 eV \leftarrow \text{yellow}$$

For this wavelength this is the most likely wavelength. In general see a lot of different wavelengths.

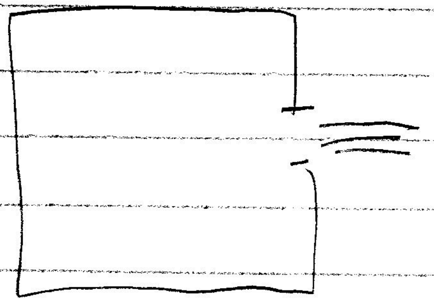
So the energy density of light in the box is:

$$e = \frac{\text{energy}}{\text{volume}}$$

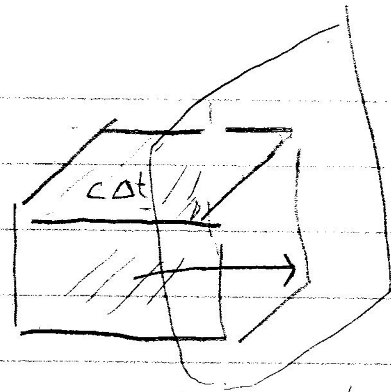
$$\sim \underbrace{hf}_{\substack{\text{energy of a photon} \\ \sim k_B T}} \; \underbrace{\frac{N}{V}}_{\text{Number per volume}}$$

$$e \sim (k_B T) \left(\frac{k_B T}{hc}\right)^3 \propto \frac{k_B^4}{(hc)^3} T^4$$

Finally how much light does the box emit per time?



want to know the energy emitted per area per time



So in a given time Δt, a length $c\Delta t$ moves

$$\Delta E = \frac{\text{energy}}{\text{volume}} A \cdot c\Delta t$$

So the energy per area per time:

$$\frac{1}{A}\frac{\Delta E}{\Delta t} = \frac{\text{energy}}{\text{volume}} \cdot c$$

$$\frac{1}{A}\frac{\Delta E}{\Delta t} \sim \frac{c}{(\hbar c)^3}(k_B T)^4$$

More generally the emitted light due to radiation

$$\boxed{\frac{1}{A}\frac{\Delta E}{\Delta t} = \sigma T^4}$$

up to a $\# = \frac{2\pi^5}{15}$

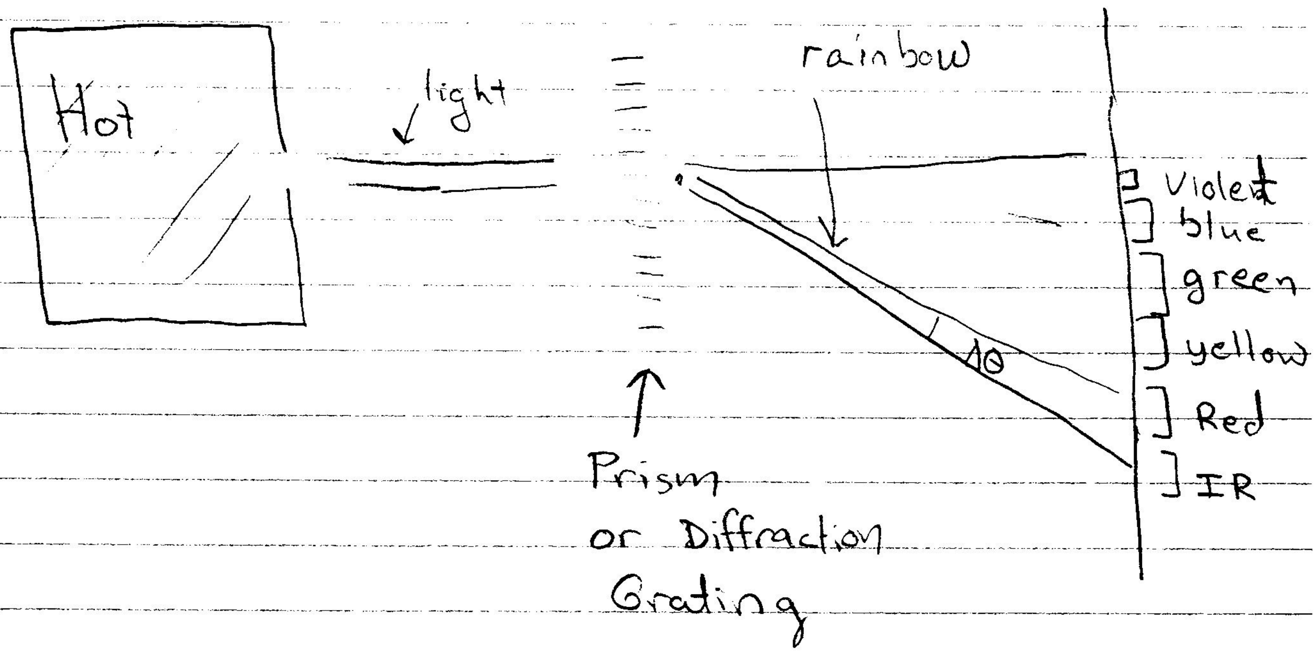
these are the same.

where

$$\sigma = \frac{2\pi^5}{15} \times \frac{k_B^4 c}{(\hbar c)^3}$$

$$\sigma = 5.6 \times 10^{-8} \frac{W}{m^2 \, {}^\circ K^4}$$

The Spectrum of the Black Body



- Then want to know how much of the light goes to red green blue etc

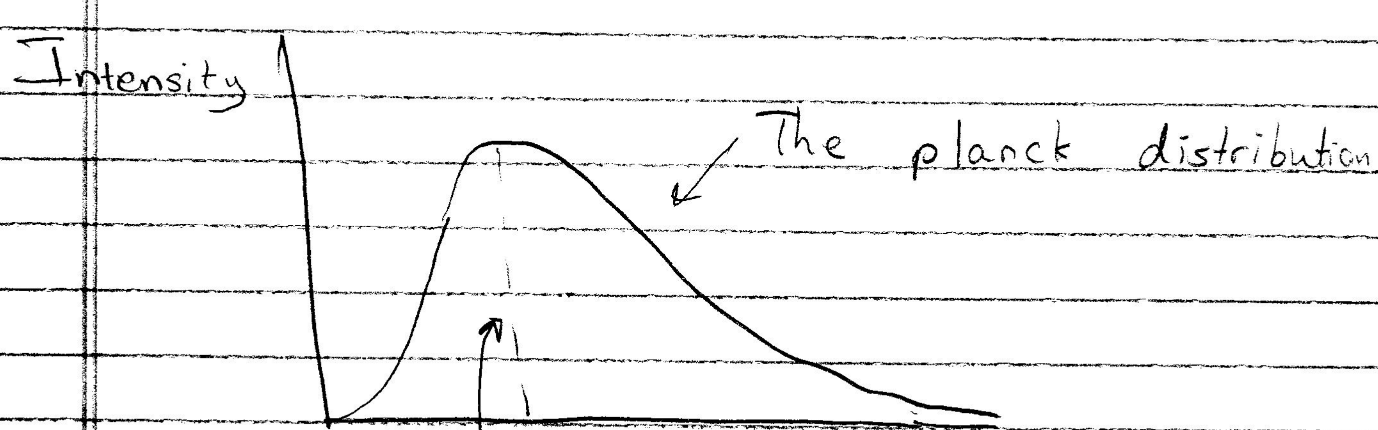
- For a detector of angular width $\Delta\theta$ measure light in a range of frequencies Δf

$$f - \frac{\Delta f}{2} < f < f + \Delta f/_2$$

in reality this is ang

$$\frac{1}{A}\frac{\Delta E}{\Delta t} = \underbrace{2\frac{hf^3}{c^2}\frac{1}{e^{hf/k_BT}-1}}_{\text{energy per area per time per frequ}} \Delta f$$

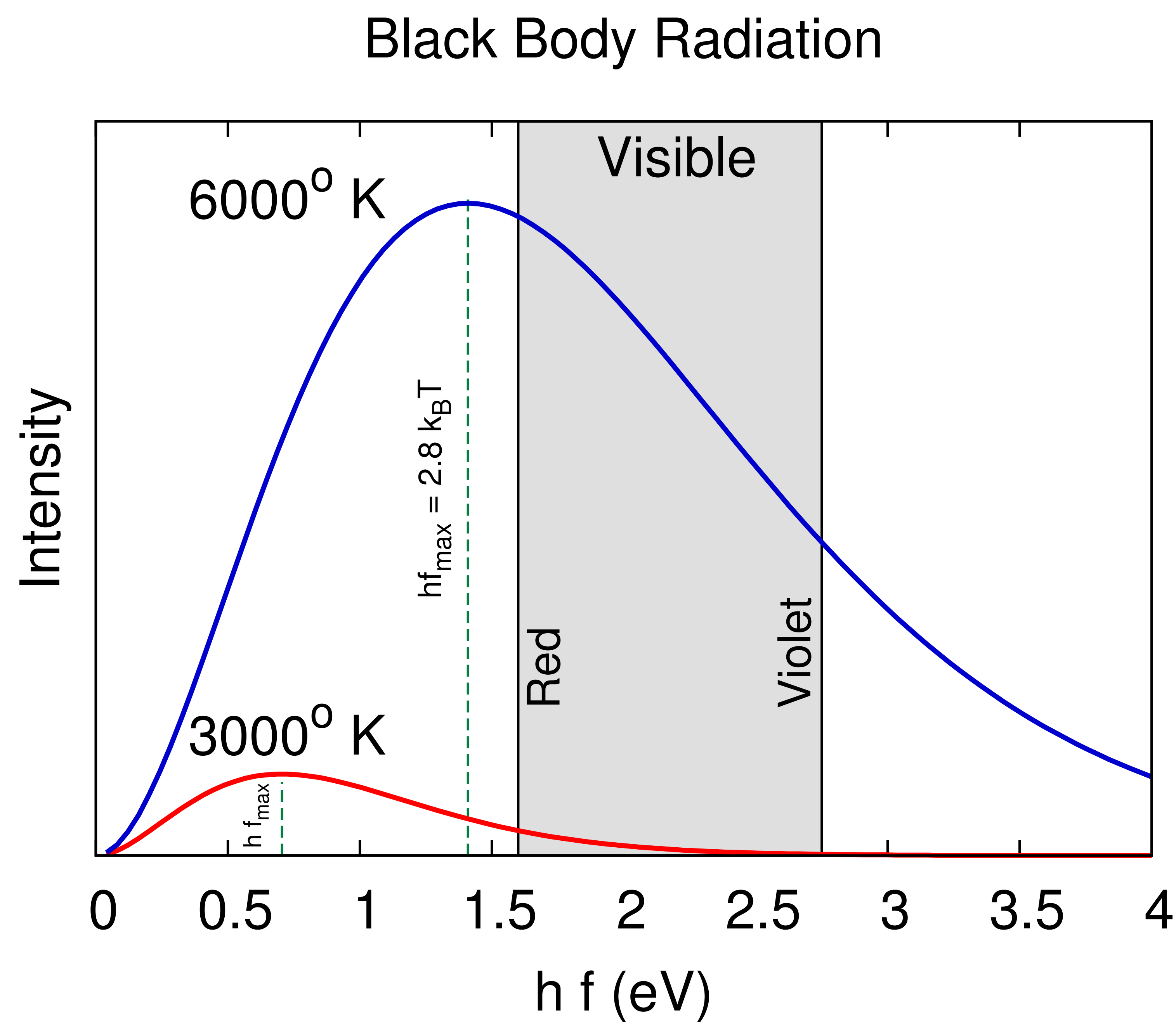
It is not important to know this formula. It is important to understand that it has a characteristic shape.



$hf_{max} = 2.82\ k_B T$

$= 1.4\,eV$ for $6000^\circ K$ (see slide

- Find a rage of frequencies
- The most probable is $2.82\, k_B T$
- Not very likely to find a photon with energy much higher than $\sim 10 k_B T$
- The greatest prediction ever of quantum mechanics



Black Body Radiation
Visible
6000° K
3000° K
hf_max = 2.8 k_B T
h f_max
Red
Violet
Intensity
0
0.5
1
1.5
2
2.5
3
3.5
4
h f (eV)