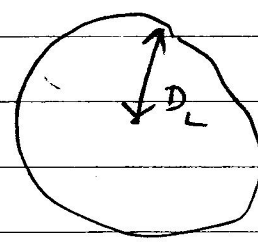
Last Time

- Discussed how observations of Supernova at various red shifts reveal the acceleration history of the universe

$$L = \frac{\text{Energy}}{\text{Time}} = \text{Abs. Luminosity}$$

$$\ell = \frac{\text{Energy}}{\text{Area (time)}} = \text{Apparant Luminos}$$

$$\ell = \frac{L}{4\pi D_L^2}$$



$$\Rightarrow \quad D_L = \left(\frac{L}{4\pi \ell}\right)^{1/2}$$

$$= \text{Luminosity distance}$$

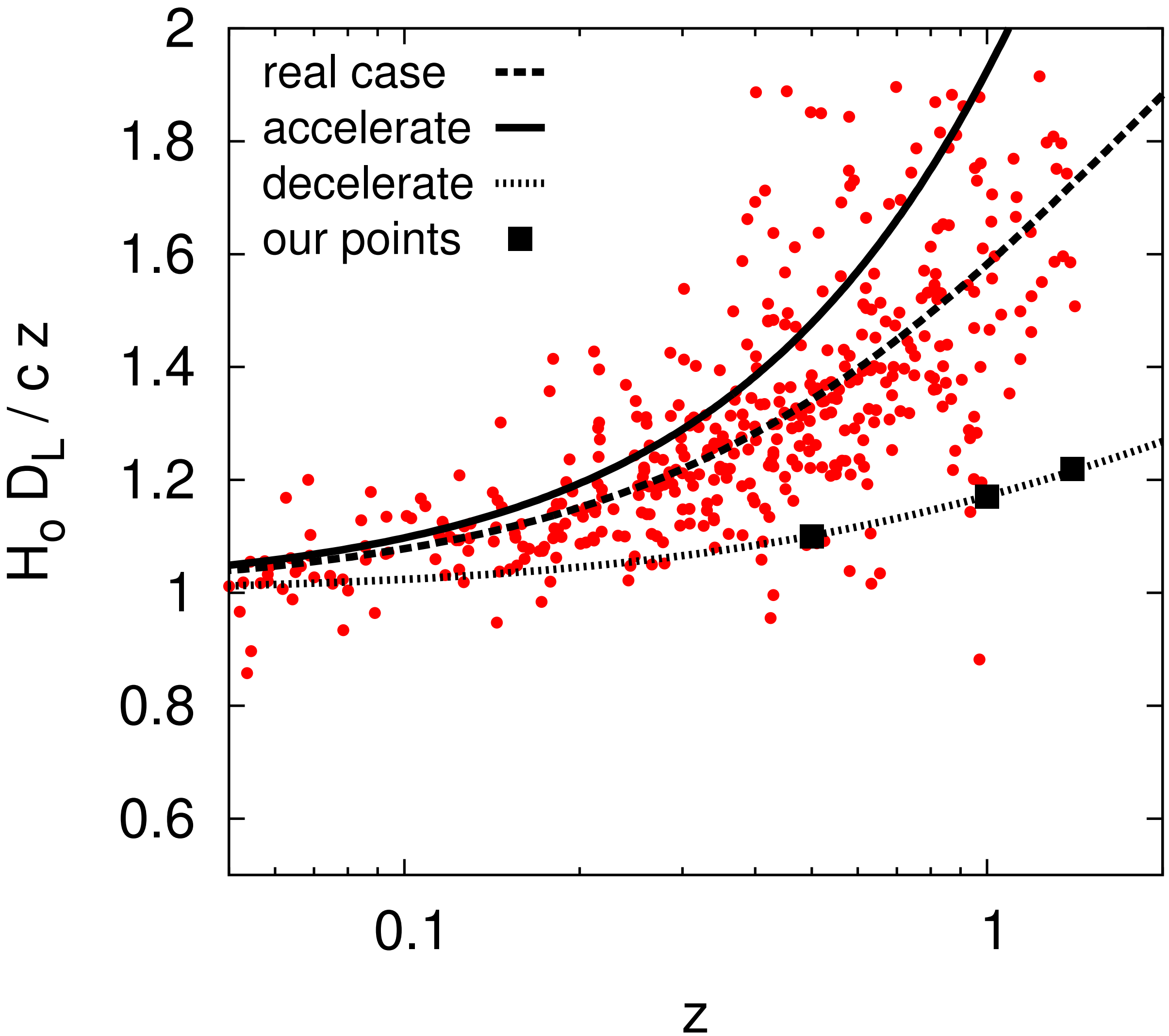
$\Rightarrow$ Red Shift

$$\lambda_{obs} = (1 + z)\, \lambda_{emit}$$

Then D_L vs. z, allows one to determine $a(t)$ vs. t. D_L vs z is like dist vs time.

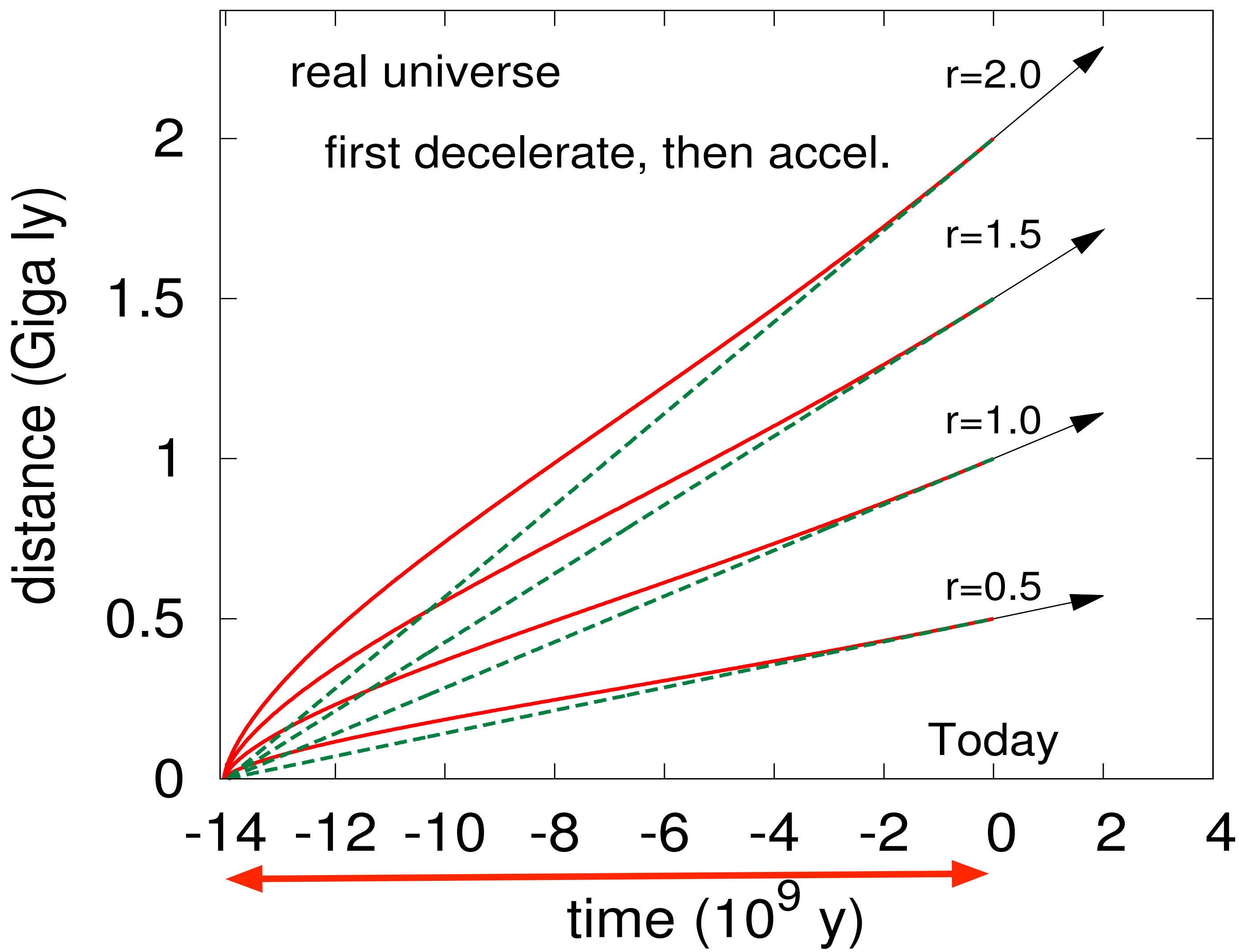
scale factor = % smaller the universe was at time = t relative to today

What cosmology does the data prefer?



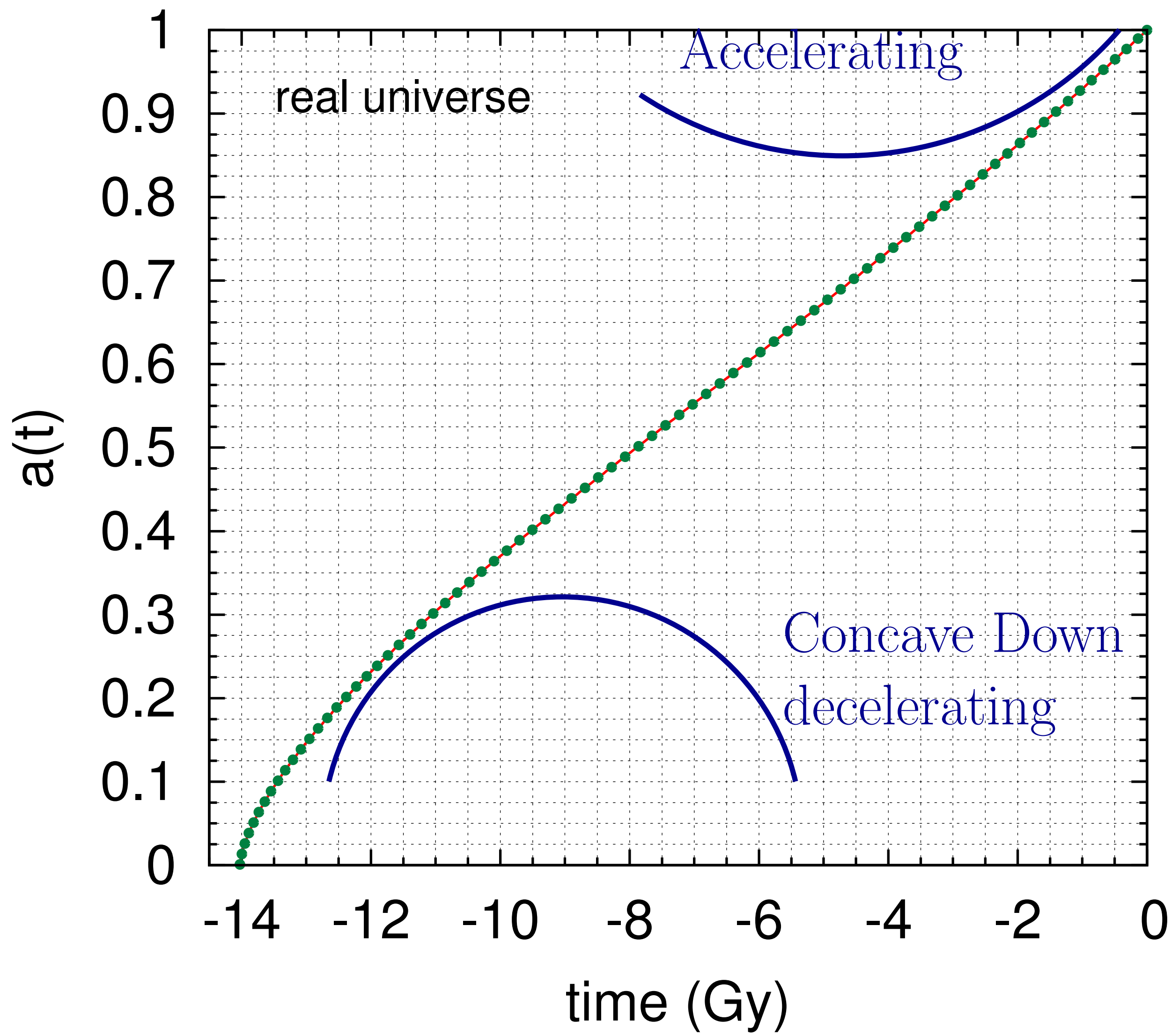
The data want first a period of deceleration then a period of acceleration!

Preferred acceleration history:



Lifetime of Universe is (for this cosmology)
14 Billion Years

The expansion of the universe is summarized by: $a(t)$=fraction of universe-size today

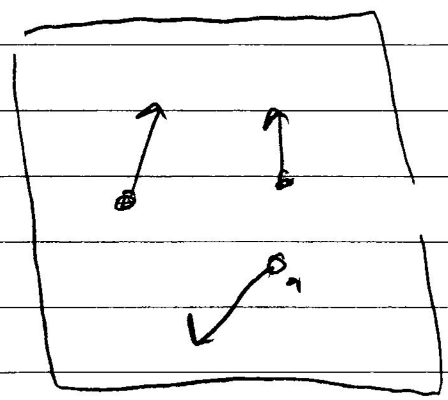


The data want first a period of deceleration, then a period of acceleration

$8\ \mathrm{Gy}$ after big bang or $6\ \mathrm{Gy}$ ago!

<u>Review of Temperature</u>

Then we started talking about Black Body radiation and temperature



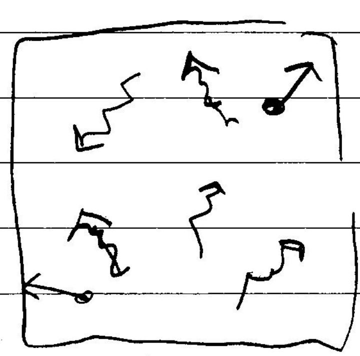
$$\frac{E}{N} \sim k_B T$$

energy per particle

Where k_B is a constant:

$$k_B = \frac{1/40\ eV}{300°K} = \frac{\text{energy}}{°K}$$

Then why do things glow when they get hot:



Atoms bounce around giving some of there energy to creatings the light particles -- photons

① The higher the temperature the higher the frequency, the shorter the wavelength.

② Also see more photons the higher the temperature (since the wavelength is shorter more fit in box)

typical Temperature of Black Body

The typical energy of a photon is:

$$E_{typ} = h f_{typ} \sim k_B T$$

In general dont just find photons at one frequency but find photons at a range of frequencies

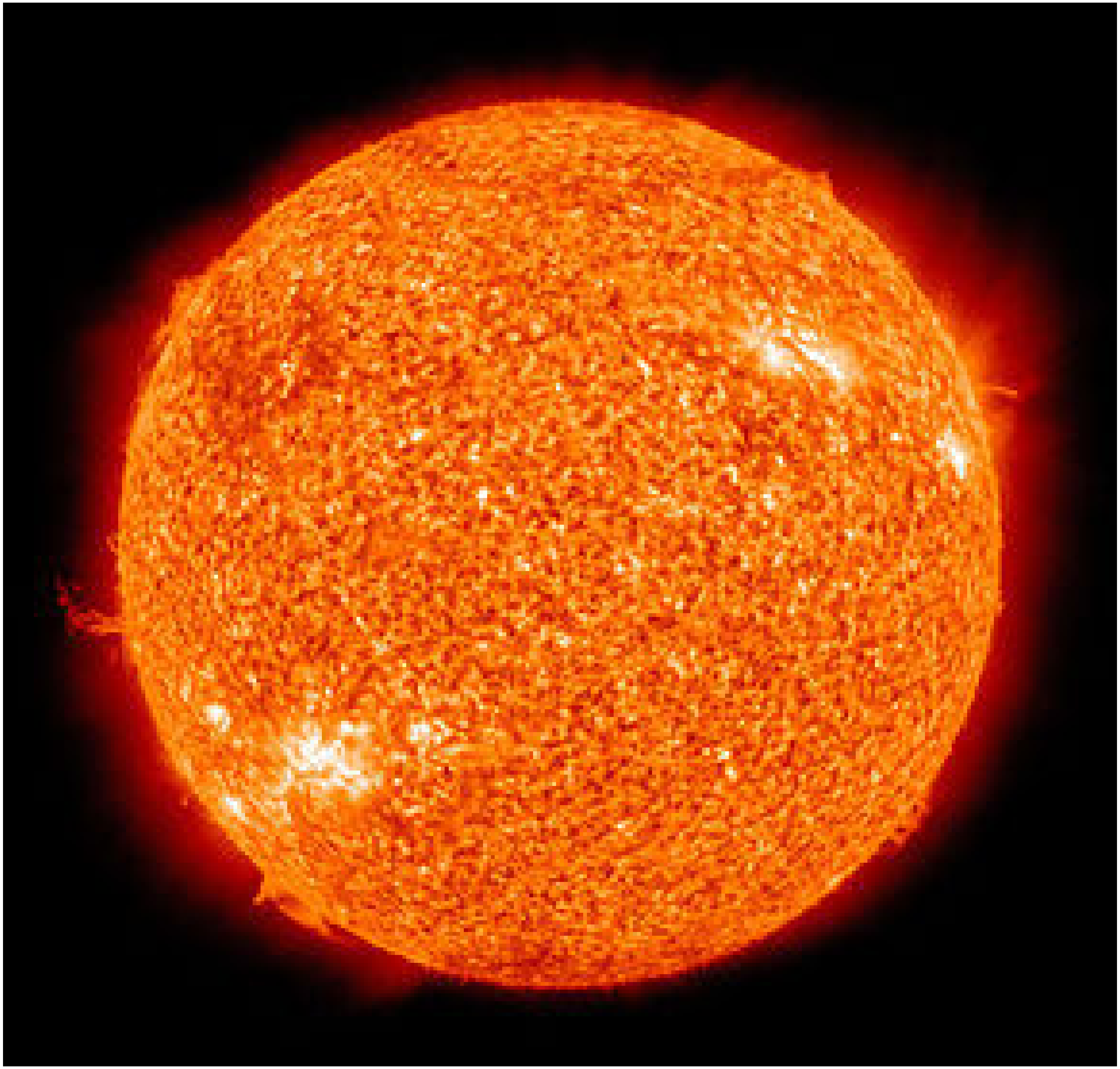
$$h f_{mp} = E_{mp} = 2.82\, k_B T$$

Energy and frequency where it is most likely to find a photon

* Do Problem

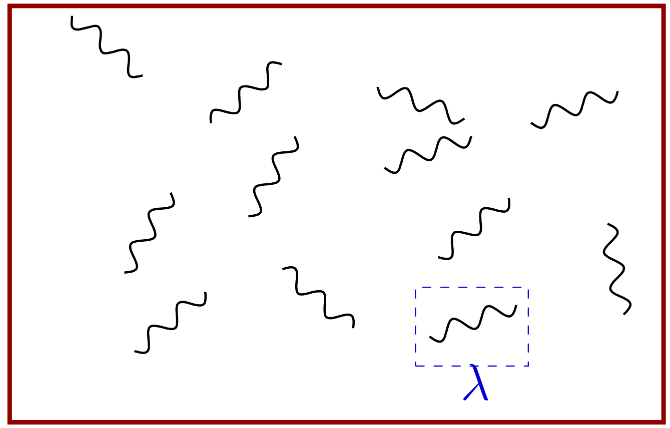
The hot walls in equilibrium with the radiation in an oven

The light from the sun which is in equlibrium with the free electrons and protons of the sun



Real Life – Black Body

Theoretical Physicists Black Body



Photon Gas in Box

Problem on BB

- Estimate the energy and velocity of an electron in a 5000°K plasma like the sun:

Hint: Useful formulas $E \simeq \frac{1}{2} m v^2$

Ignore the electrons potential energy

1 • Estimate the typical wavelength of a photon in a 5000°K plasma

Useful stuff you should know:

$E = \frac{hc}{\lambda}$ $m_e = 0.5 \frac{\text{MeV}}{c^2} = \frac{0.5 \times 10^6 \text{ eV}}{(3 \times 10^8 \text{ m/s})^2}$

a unit of mass (mega electron volts) / (speed of light)^2

$k_B = \frac{1/40 \text{ eV}}{300° K}$

$hc = 1240 \text{ eV nm}$

Answers:

1a $E \sim 0.41 \text{ eV}$ 1b $v \sim \frac{1}{1000} c$ (c: 3×10^8 m/s) 2a $E_{phot} \sim 1.15 \text{ eV}$ 2b $\lambda \sim 1000$

infrared

Solution

(1) Using

$$E_e \sim k_B T$$

$$\sim \frac{1/40\,eV}{300^\circ K} \times 5000^\circ K$$

a) $$\sim 0.41\ eV$$

Now

$$E = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{\frac{2E}{m}} = \left(\frac{2 \times 0.41\,eV}{\frac{0.5 \times 10^6\,eV}{(3 \times 10^8\,m/s)^2}} \right)^{1/2}$$

Oops, I messed up here plugging the numbers into the calculator, replace 2.7 with 3.8

b) $$= 2.7 \times 10^5\ m/s$$

$$\sim \frac{1}{1000} \text{ of Light speed}$$

(2) For a photon in a 5000°K black body we have

$$E \simeq 2.8\, k_B T \simeq 1.15\,eV$$

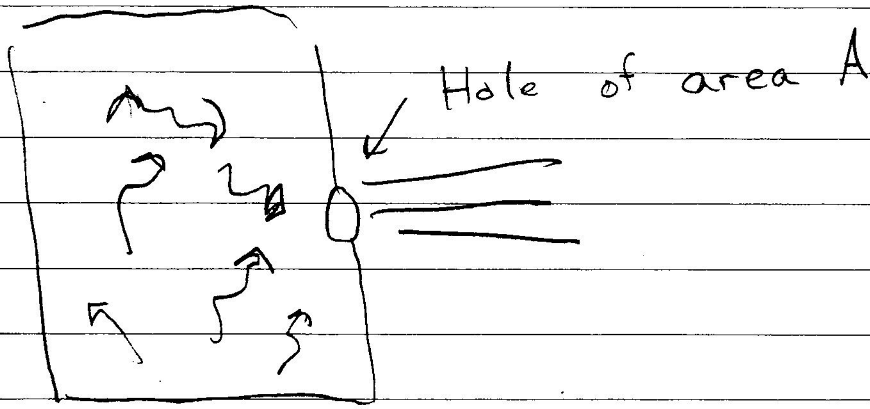
So

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{1240\,eV\,nm}{1.15\,eV} = 1078\ nm$$

infrared

Radiation from Black Body

Now how much light energy does the box emit :



$$\frac{\Delta E}{A\,\Delta t} = \sigma T^4 \quad \text{or} \quad \frac{\Delta E}{\Delta t} = \sigma T^4 (\text{Area})$$

↑ the energy per area per time emitted by the photon gas

$$\sigma = 5.6 \times 10^{-8} \frac{W}{m^2 \, {}^{\circ}K^4}$$

$$\sigma = \frac{2\pi^5}{15} c \frac{k_B^4}{(hc)^3}$$ ← an exact number

Problem

- Estimate the energy per time (The absolute luminosity) emitted by the sun assume the sun is a 5800°K black body

- Estimate the apparent luminosity of the sun just outside the earth's atmosphere. (If you have time)

Answers:

① 3.9×10^{26} W ② $\ell = 1380 \frac{W}{m^2}$

agrees w observation

~1 kW per meter^2

Useful

$R_\odot \simeq 700{,}000$ km

Solution

(1) $$\frac{\Delta E}{\Delta t} = \sigma T^4 \times (\text{Area of Sun})$$

$$= 5.6 \times 10^{-8} \frac{W}{m^2} (5800\,°K)^4 \times (4\pi (7 \times 10^8 m)^2)$$

$$\frac{\Delta E}{\Delta t} = 3.9 \times 10^{26}\ W$$

(2) To find the energy per area on earth (outside atmosphere)

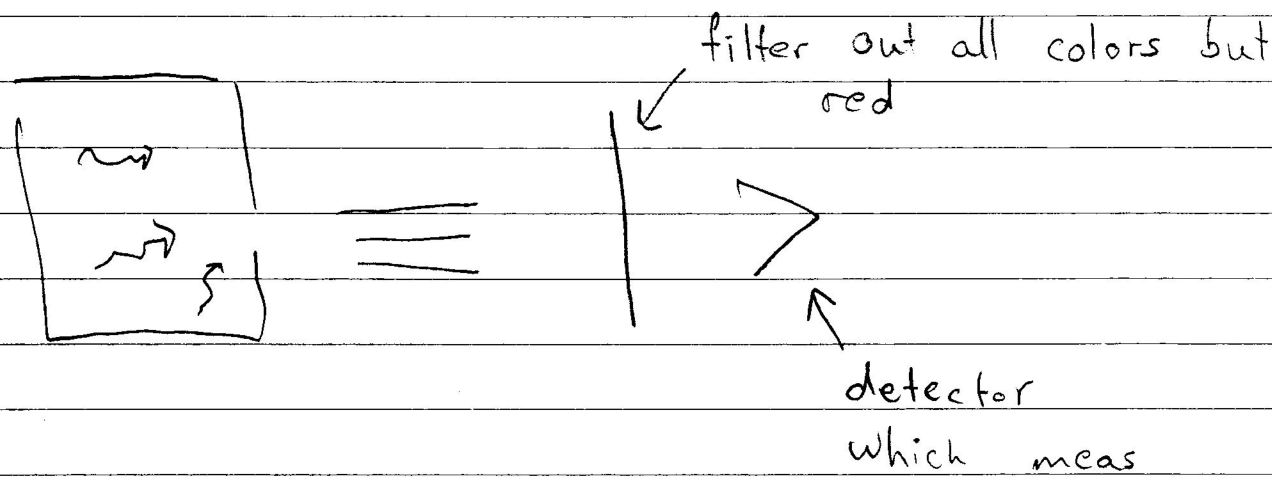
$$l = \frac{L}{4\pi R^2} = \frac{3.9 \times 10^{26}\ W}{4\pi (1.5 \times 10^8 km)^2}$$

distance to sun

$$l = 1380 \frac{W}{m^2}$$

Frequency dependence of BB radiation

Then we started talking about the color of the Light



- In reality a filter will filter in a certain range, for example:

$620\text{nm} \lesssim \lambda \lesssim 650$

$2.0 \text{ eV} \lesssim hf \lesssim 2.1 \text{ eV}$

$\Delta f \simeq 0.1 \text{ eV}$ for this case

Then

$$\frac{\Delta E}{A\,\Delta t} = I_f(T)\,\Delta f$$

frequency of detector

power per area

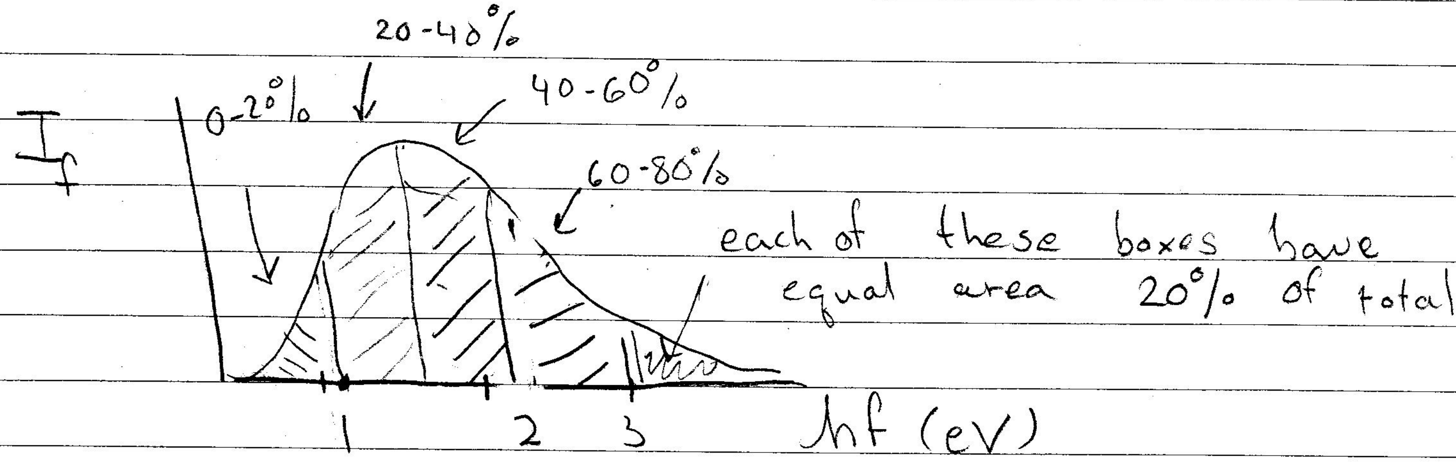
power per area per frequency

Frequency Dependence of Black Body Spectrum (continued)

Last time we stated w/out proof:

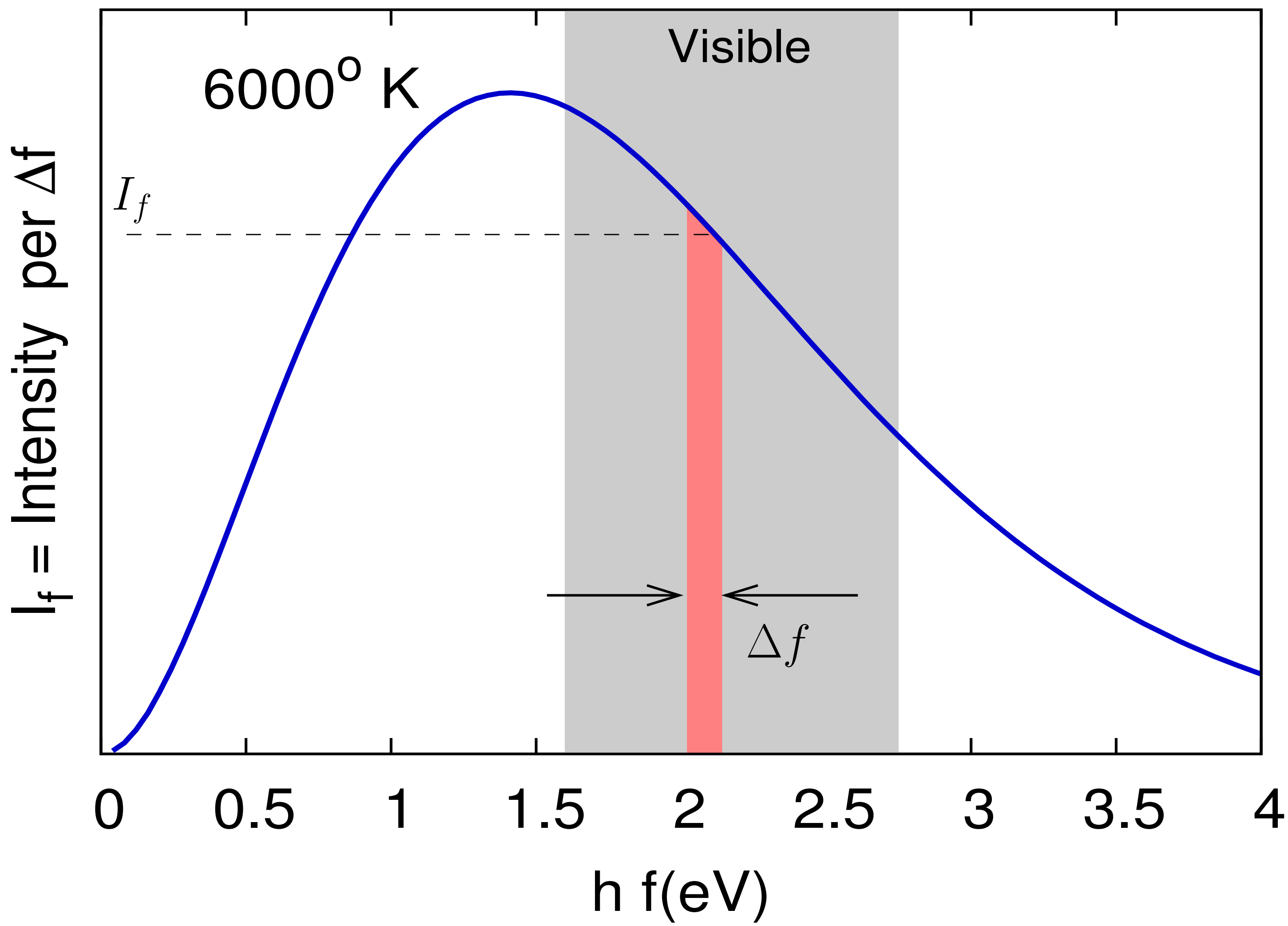
$$I_f \propto \frac{f^3}{e^{hf/k_BT} - 1}$$

- See slide:
- See slide



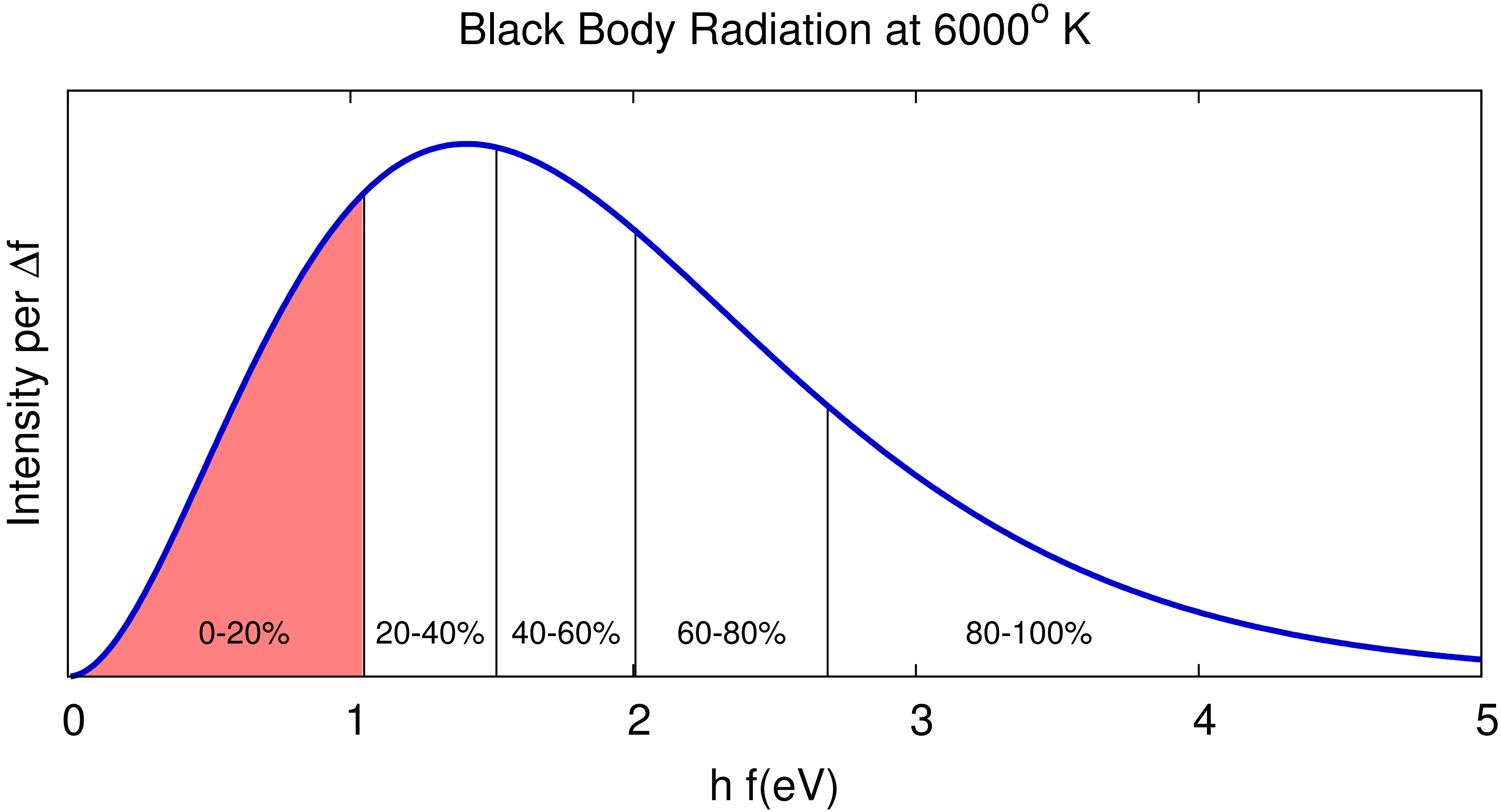
- Thus 20% of the energy is carried by photons with energy less than 1eV
- And 20% of the energy is carried by photons with E > 2.7 eV.

Black Body Radiation

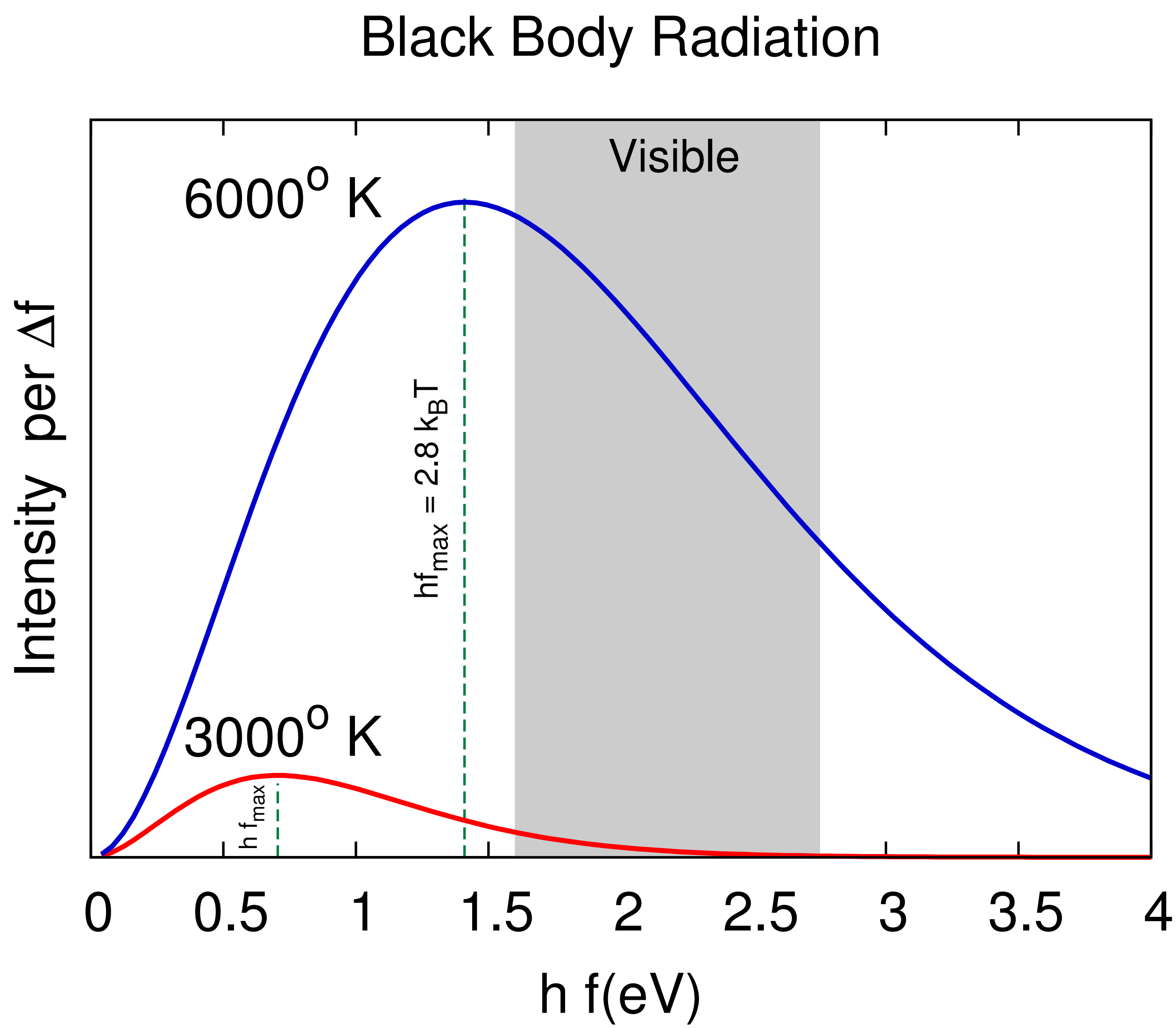


$$\frac{\Delta E}{A \Delta t} = I_f \times \Delta f$$

The black black body spectrum records how much light is carried by a given frequency



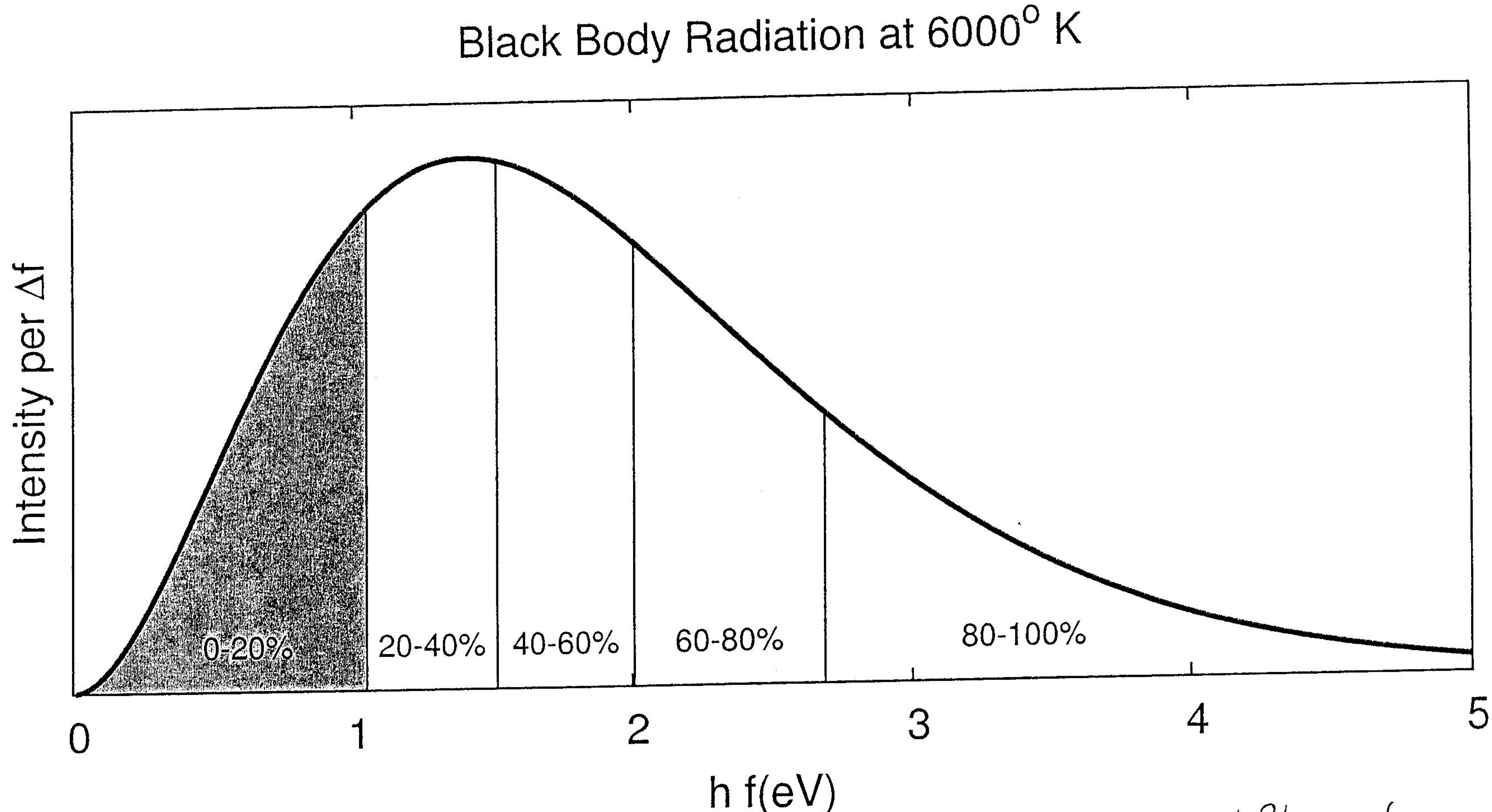
Each of these frequency ranges have equal area under the curve and carry 20% of the energy



1. The height increases rapidly with with temperature (as T^3).

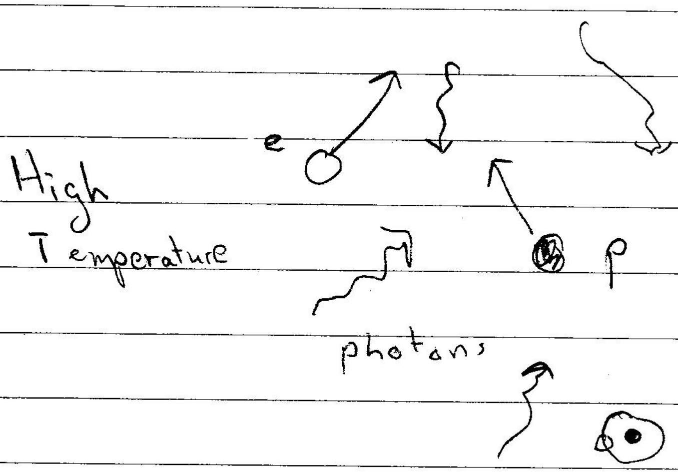
2. The typical frequency (the base) increases linearly T.

Problem on the frequency-dep of BB radiation



Problem: Photons with energy $> E_*$ carry only 10% of the energy. Estimate E_*:

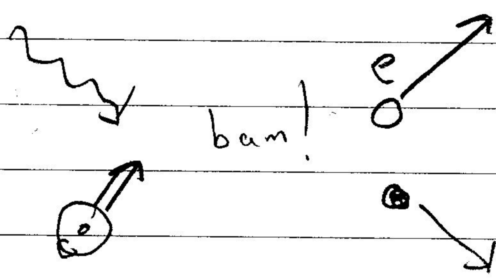
High Temperature Plasma & Recombination $\quad k_B T \sim 10\,eV$



- The number of electrons and protons are equal

← Very occasionally make Hydrogen

- It takes a certain amount of energy to rip an electron from hydrogen $\Delta E = 13.6\,eV$



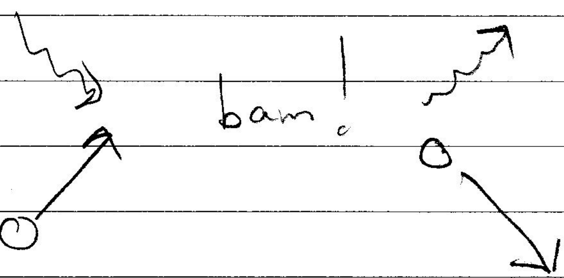
This is known as "photo-dissociation."

$$\gamma + H \longleftrightarrow p + e$$

In high temperature plasma there are plenty of photons @ energy greater than 13.6 eV

Recombination Continued

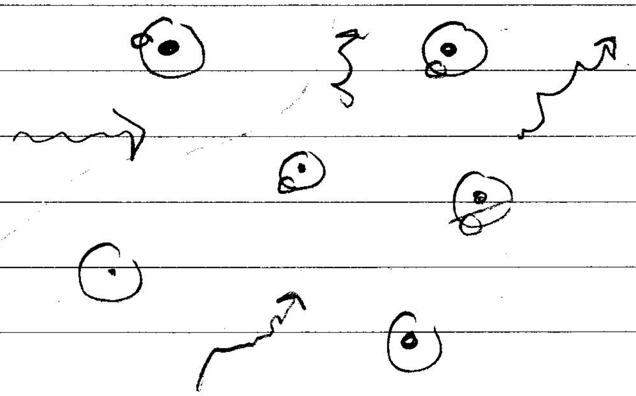
- There is also photon – electron (known as Compton scattering) scattering which happens a-lot, keeping the plasma in equilibrium.



As the plasma cools to $k_B T \sim 0.2\,eV$ there are almost no photons @ energy $> 13.6\,eV$

→ less than one in a million have $E > 13.6\,eV$

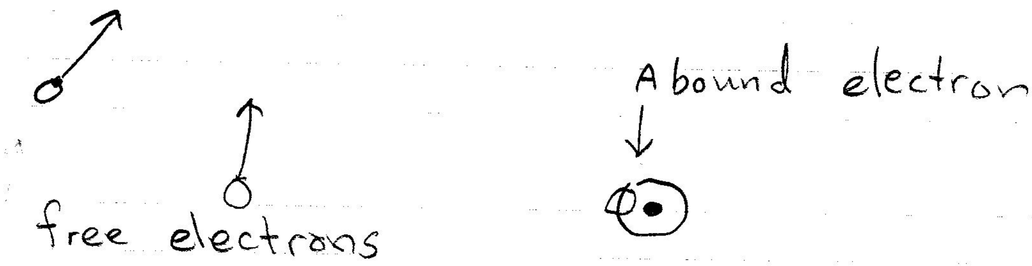
- As a result the photons can no longer ionize the hydrogen gas



- This is known as recombination

Recombination Continued

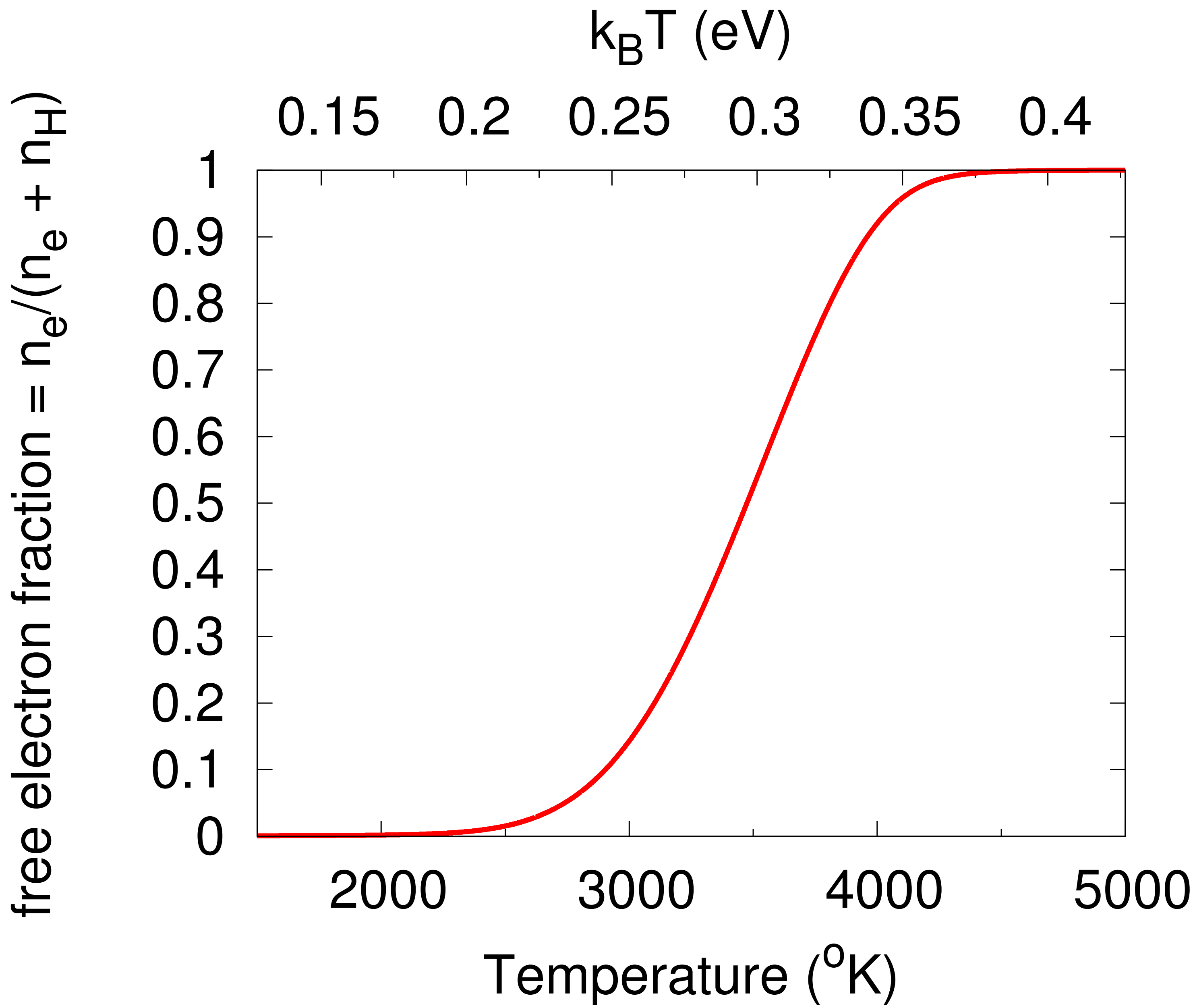
The free electron fraction



$$X_e = \begin{array}{l}\text{fraction}\\ \text{of electrons}\\ \text{free}\end{array} = \frac{n_e}{n_e + n_H}$$

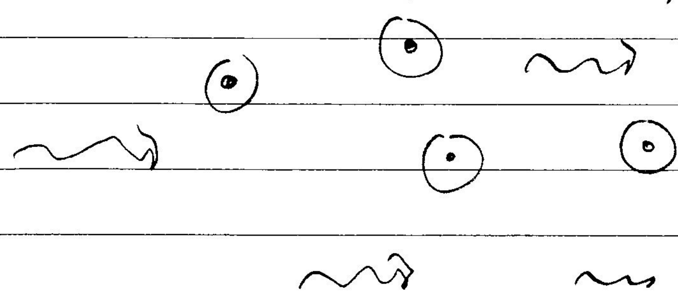
See slide for plot of X_e vs Temperature

The free electron fraction in high temperature plasma:



Recombination Continued

- At this point the photons are effectively free, since electron-photon (or compton) scattering has stopped.



- The light scatters only very weakly off neutral hydrogen (at least if the energy is much less than 13.6 eV as is the case here)
- Then the light flys freely until today!!
- What is the spectrum of Light that is seen today?

Computing The Sepectum Today

- Definitions:

  T_r = the temperature at recombination

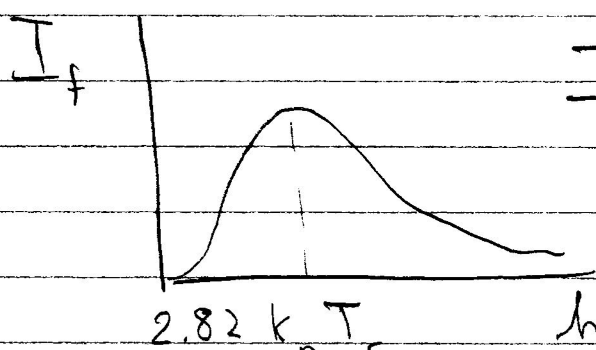
  a_r = the scale factor at reco.

  t_r = the time at reco

  f_r and λ_r = the frequency and wavelength at reco

Computing The Spectrum or Temperature Today

At recombination, the spectrum (i.e. the probability to find a photon with frequency f) is given by a black body radiation spectrum @ $T_r \simeq 3000°K$. T_r is the temperature at recombination.



$$I_f \Big|_{reco} \propto \frac{f_{reco}^3}{e^{hf_{reco}/k_B T_r} - 1}$$

- Now the universe continues to expand. The wave lengths <u>all</u> increase by a uniform factor

wavelength today

wavelength at recombination

$$\lambda_0 = \frac{\lambda_{reco}}{a_{reco}}$$

frequency today

$$f_0 = a_{reco} f_{reco}$$

Computing the spectrum today continued:

$$I_f \Big|_{\text{today}} \propto \frac{(f_0/a)^3}{e^{h f_0/k_B a T_r} - 1}$$

$$\propto \frac{f_0^3}{e^{h f_0/k_B T_0} - 1}$$

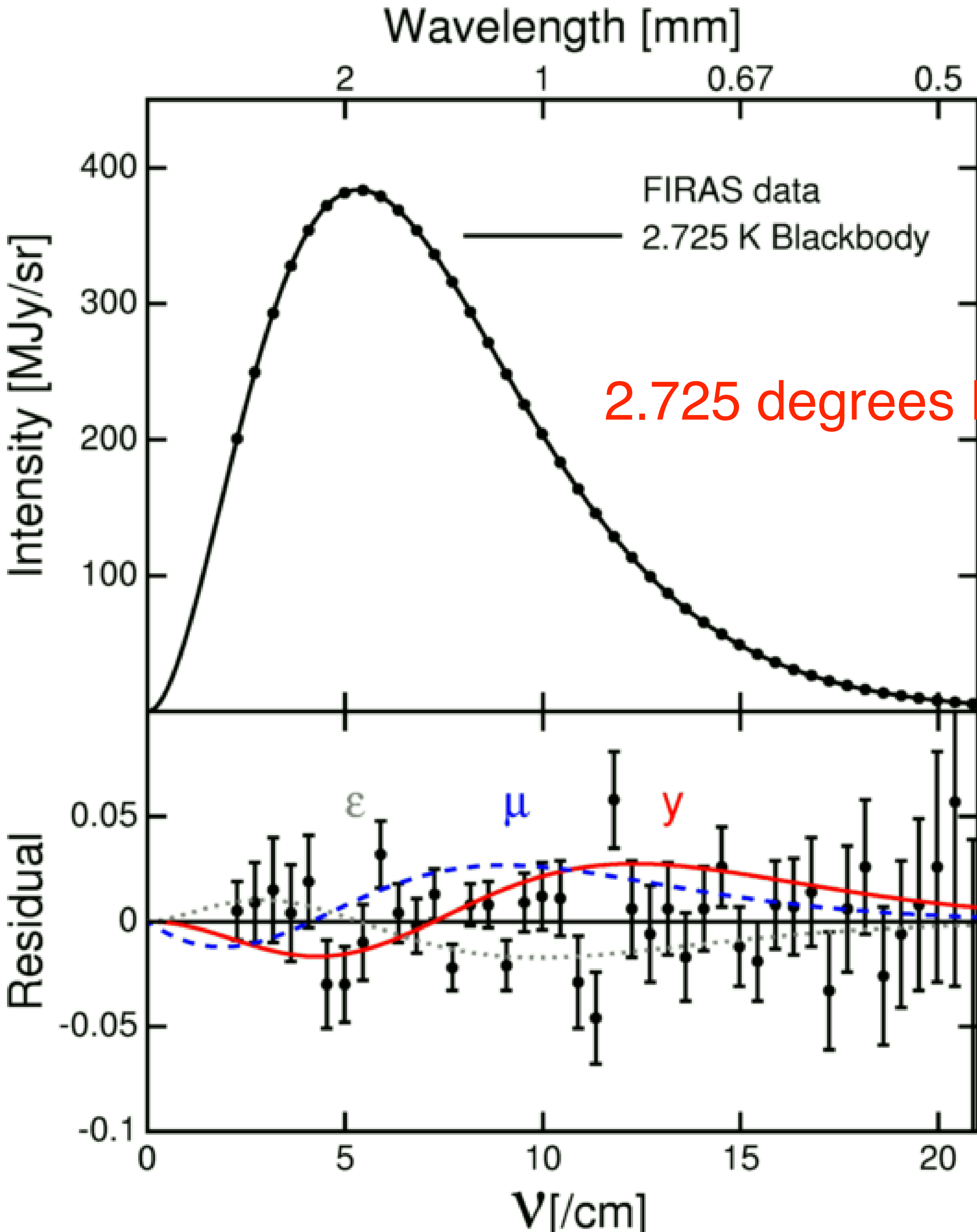
Where $\boxed{T_0 = a_r T_r}$

Thus we expect a blackbody spectrum <u>today</u> with a <u>lower</u> temperature

$$T_0 = a_r T_r$$

- The temperature today is observed to be

$$T_0 \simeq 2.728^\circ K.$$



2.725 degrees Kelvin BB

Deviation from BB

Frequency (units speed of light/cm)

History :

- The basic description of recombination was Gamov, Alpher, and Hermam ~ 1940's

→ They estimated that the scale factor at recombination was

$$1/a_r \sim 100 \leftrightarrow 1000$$

This leads to a prediction that with $T_r \simeq 3000^\circ K$, the temperature today would be

$$T_0 \sim 3^\circ \longleftrightarrow 30^\circ K$$

History Continued

- The original prediction of the microwave background lay dormant for 20yrs

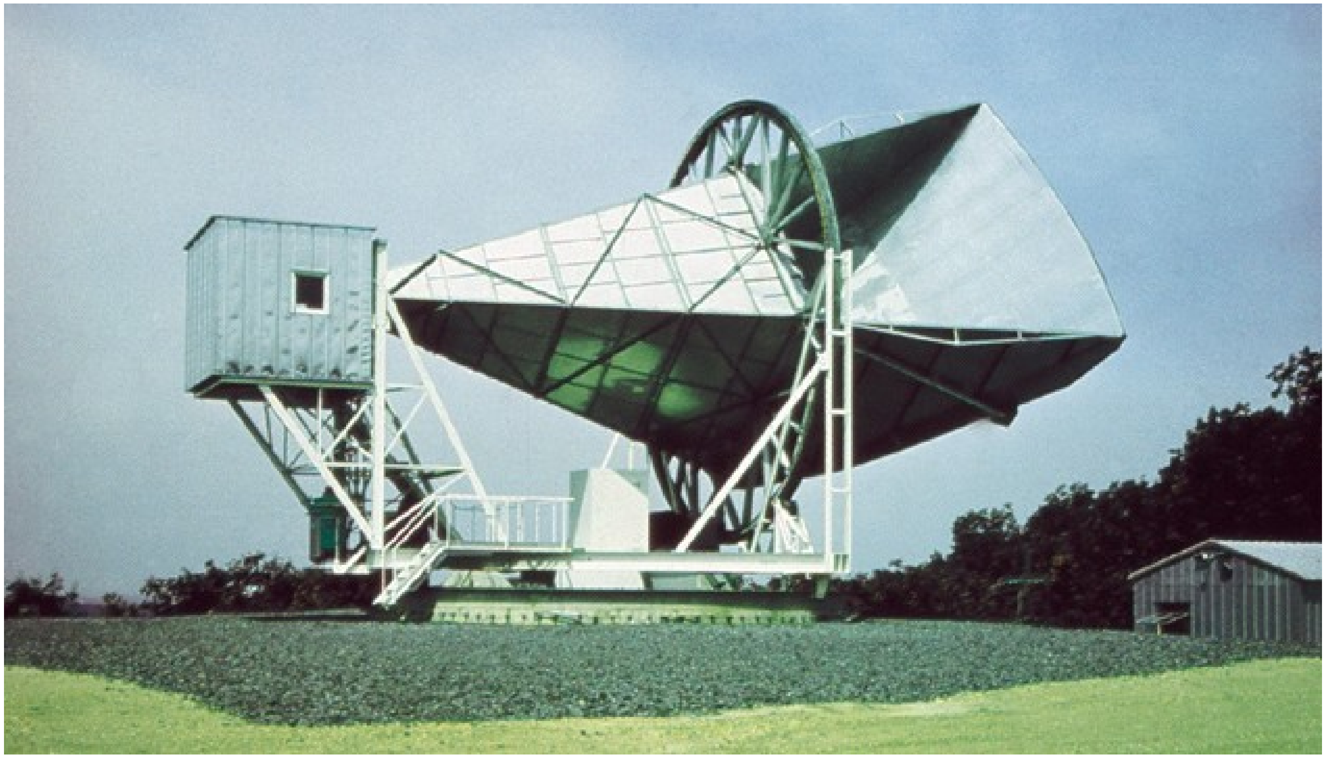
- An important paper ~ 1960 by Jim Peebles revived interest.

- Dicke, Peebles, + Wilkenson set out to measure the micro waves (Princeton)

- At the same time, "Penzias and Wilson discovered" the microwave background by accident 1965

→ Penzias and Wilson were awarded the Nobel Prize.

Penzias and Wilson discover the Black Body Radiation in Crawford Hill, NJ. 1965



These bell Labs astronomers found a mysterious microwave signal

Princeton astronomers Dicke, Peebles, and Wilkenson describe the origin of the signal

The time and Scale factor at recombination:

Then, we also can estimate the Scale factor at recombination:

$$T_0 = a_r T_r$$

$2.72^\circ K$ $\qquad$ $3400^\circ K$

So $\quad 1/a_r = (1+z) = \dfrac{T_r}{T_0} \simeq 1250$

(See next slide)

Then we can also estimate the time at recombination from $a(t)$

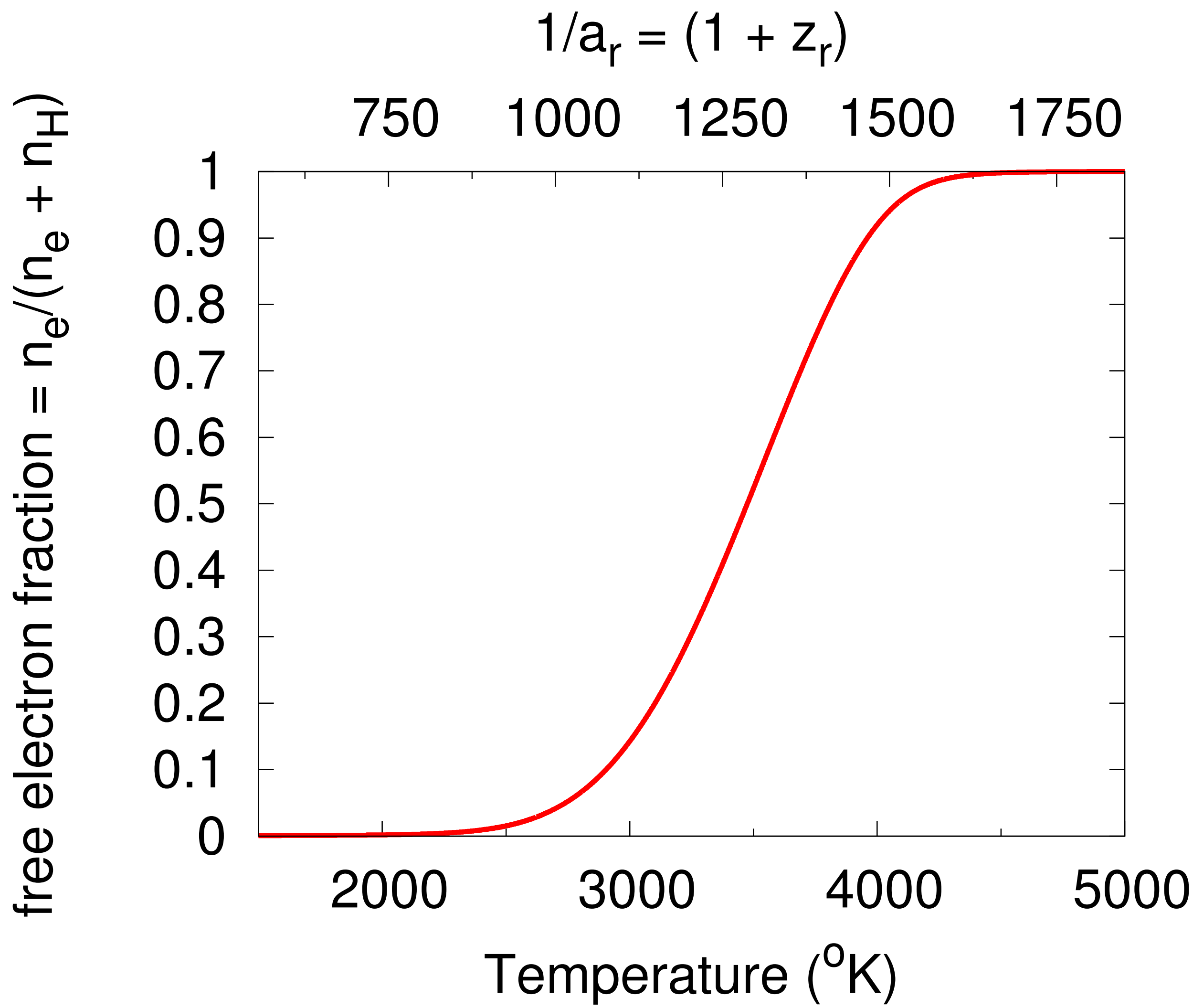
$$a_r(t_r) = \frac{1}{1250}$$

So from the graph of $a(t)$ find

$$t_r \simeq 400{,}000 \text{ y}$$

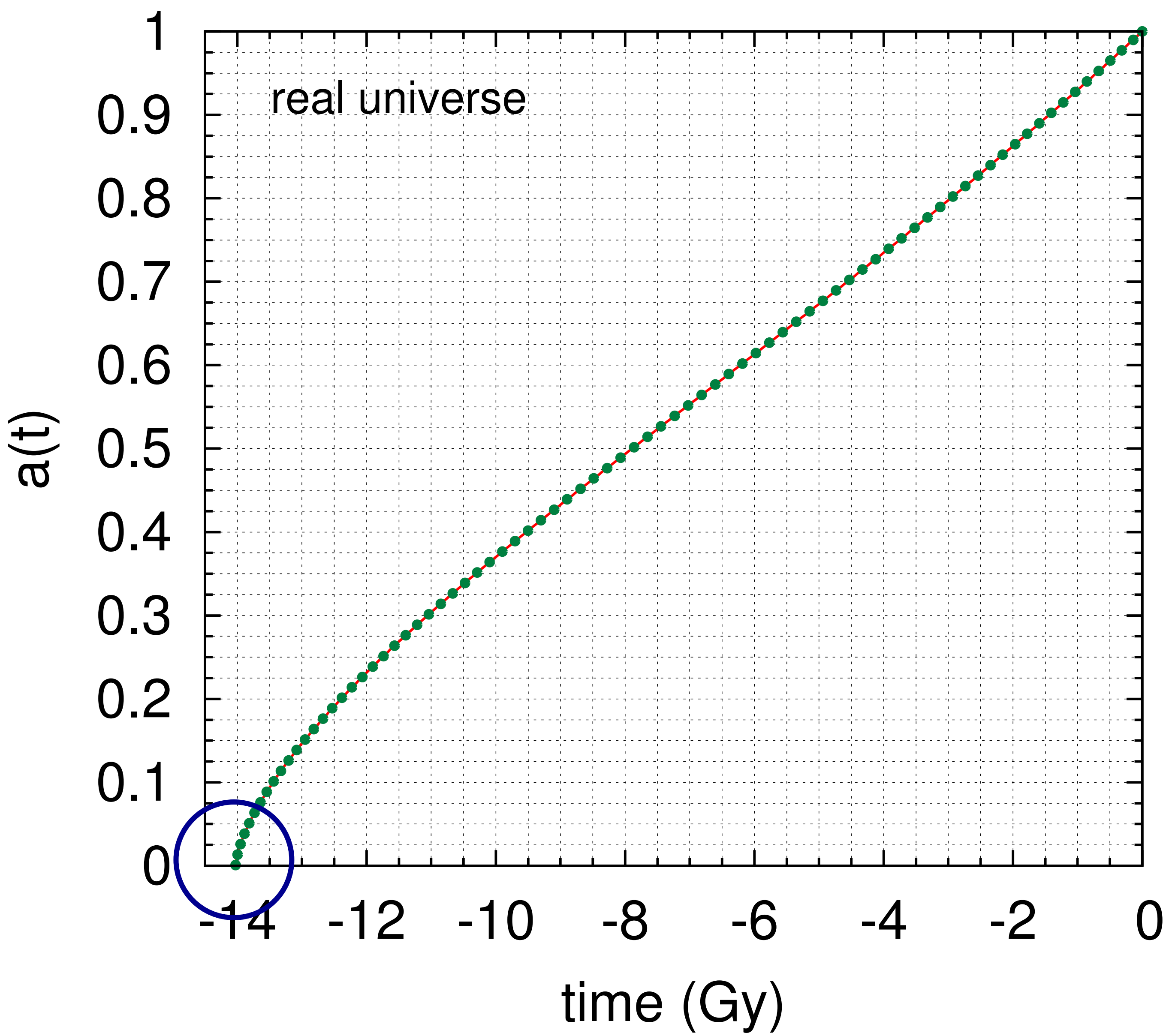
(See slides, 2+3 from now)

The red shift (scale factor) at recombination:



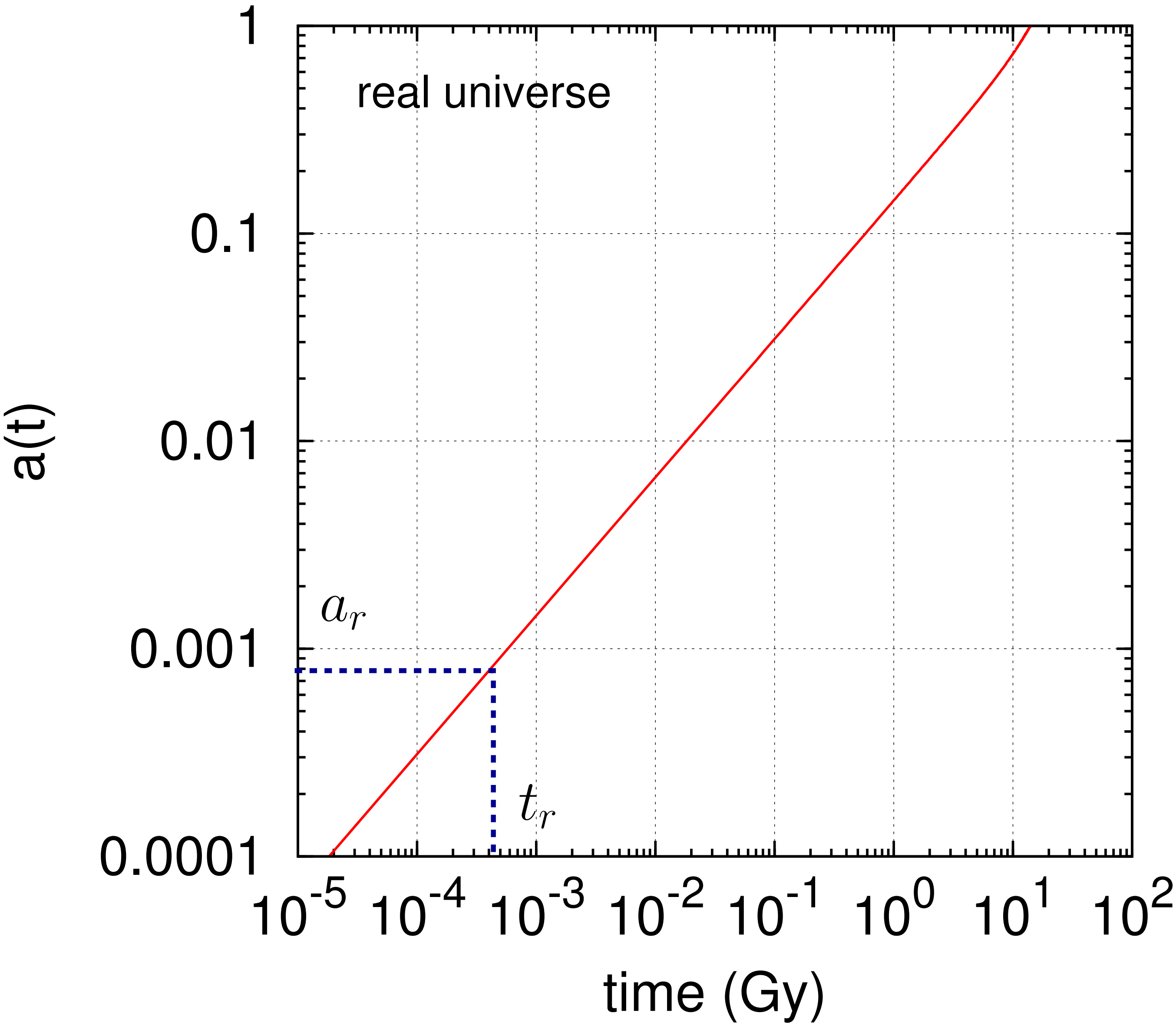
$$\frac{1}{a_r} = (1 + z_r) = \frac{T_r}{T_o} = \frac{3400\,^o\mathrm{K}}{2.72\,^o\mathrm{K}} \simeq 1250$$

Given $a(t)$ can estimate the time (after the big bang) of recombination



We are looking for $a(t) \simeq 1/1250$. This graph wont work

Given $a(t)$ (on a log scale) can estimate the time after the big bang of recombination



For $a_r \simeq 1/1250$, the time is $t_r \simeq 4 \times 10^{-4}\,\mathrm{Gy} \simeq 400,000\,\mathrm{years}$

The uniformity of the Cosmic Microwave Background

- The microwave background is remarkable uniform throughout the sky

$\overline{\overline{T}} = 2.728$ K (see slide)

~~In~~ You subtract the average, and look at the differce:

$T - \overline{\overline{T}} \simeq 3.353$ mK

Last digit of 2,728 K

$\pm\ 0.00353$ mK

Then you begin to see structure to CMB:

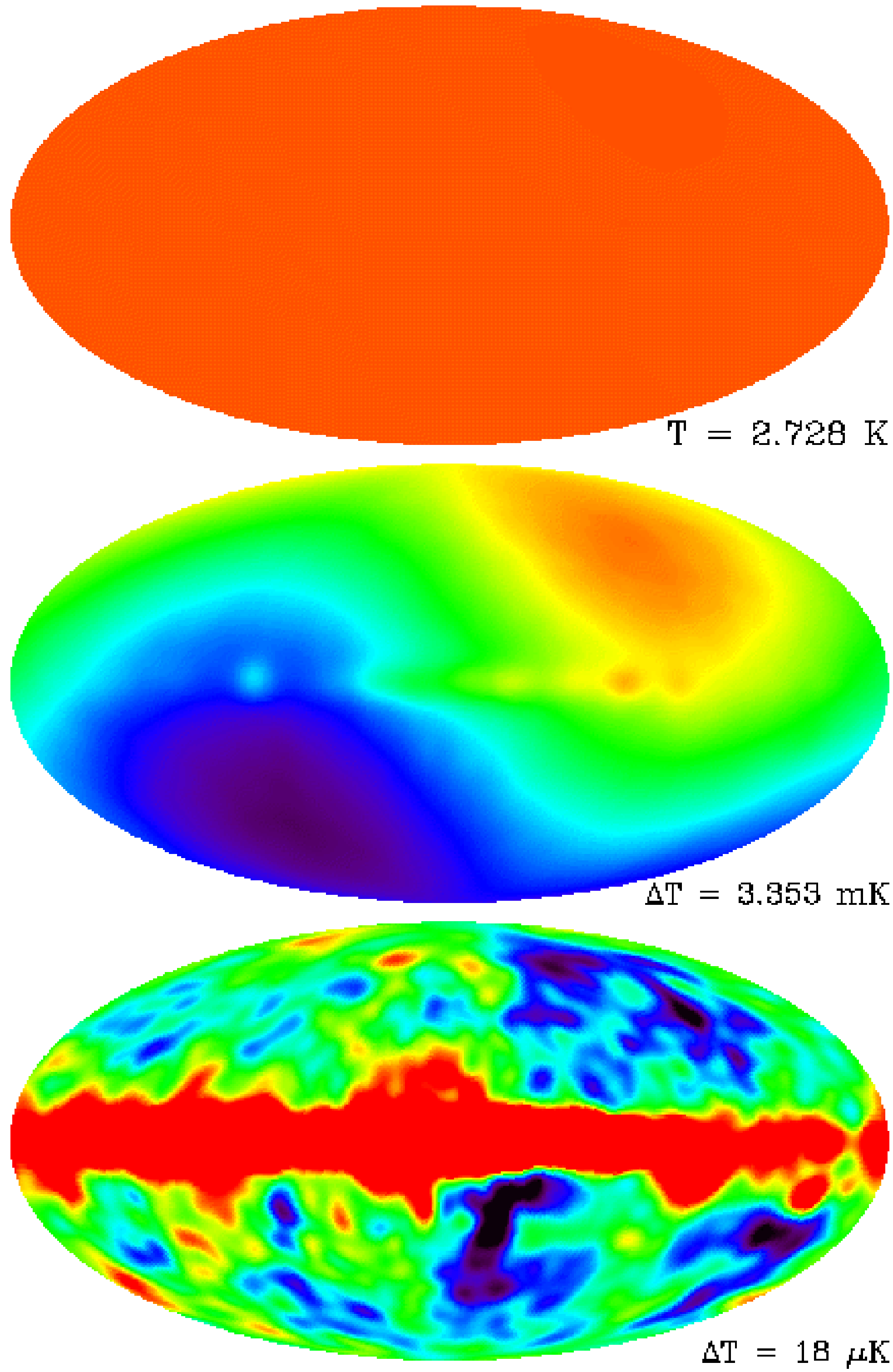
This uniformity is another example of the cosmic principle:

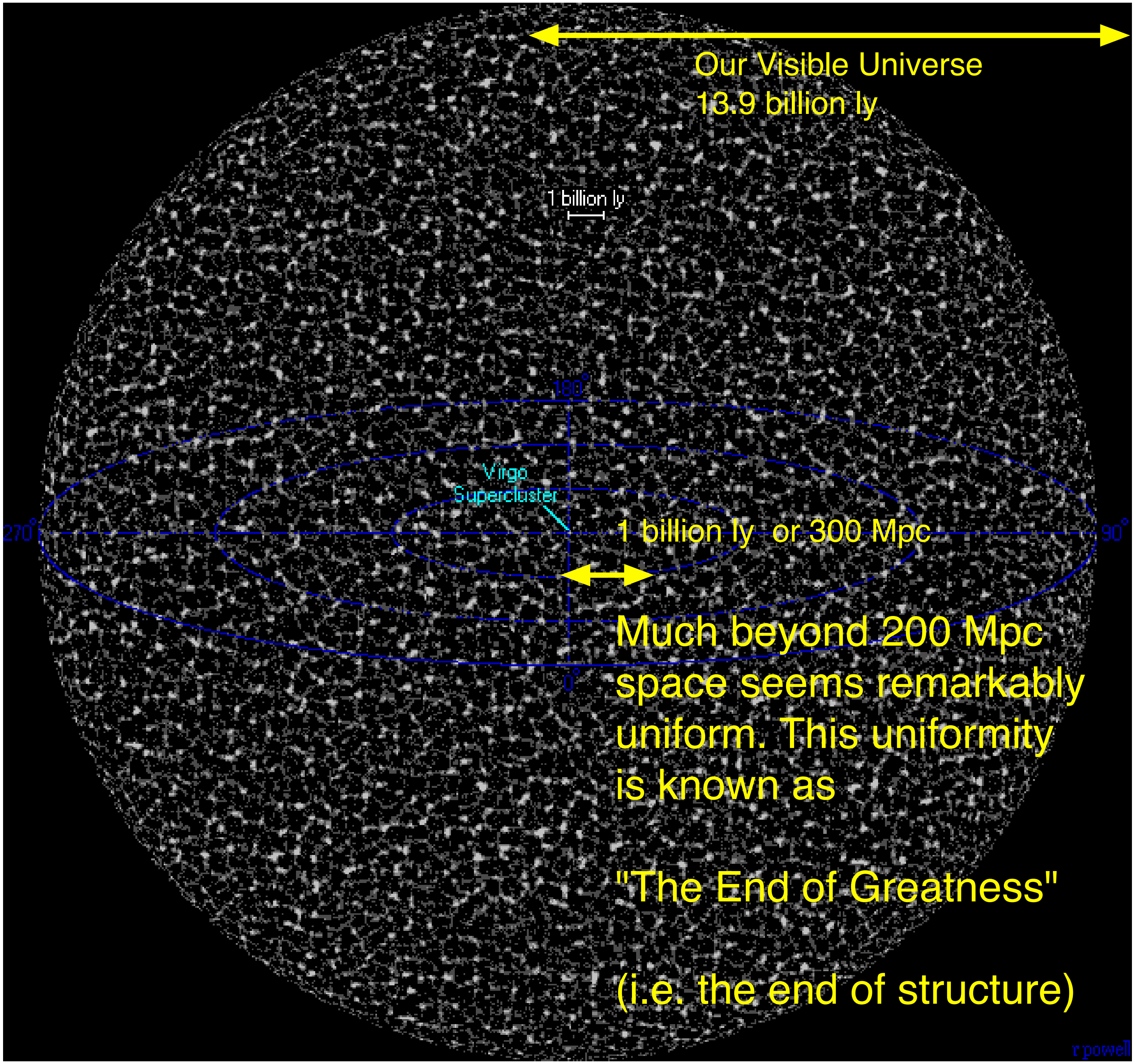
"On the largest distance scales, everythi is the same. All observers measure the same thing"

Two Examples:

① Cosmic Microwave Background

② The "end of greatness" in the distribution of galaxies


T = 2.728 K
ΔT = 3.353 mK
ΔT = 18 μK



Our Visible Universe
13.9 billion ly
1 billion ly
180°
Virgo
Supercluster
270°
1 billion ly or 300 Mpc
90°
0°
Much beyond 200 Mpc
space seems remarkably
uniform. This uniformity
is known as
"The End of Greatness"
(i.e. the end of structure)
r powell