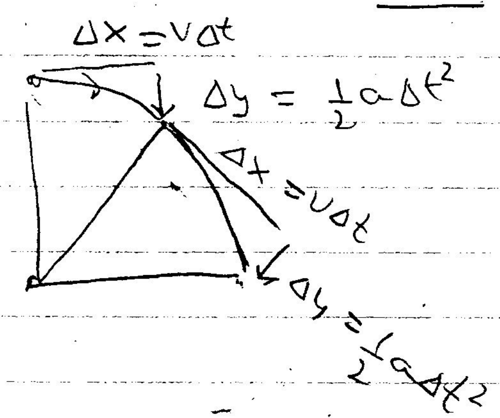
One of
Newton's Great idea

- The moon is falling around the earth



- See slides

- If this picture is going to work:

then the amount you move forward

$\Delta x = V\Delta t$, and the amount you fall

$\Delta y = \frac{1}{2}a\Delta t^2$ must be related to the curvature

of the circular arc -- see slides

$$\boxed{a_m = \frac{V_m^2}{R_m}}$$ ← acceleration of moon falling around the earth

And the object's speed v_m

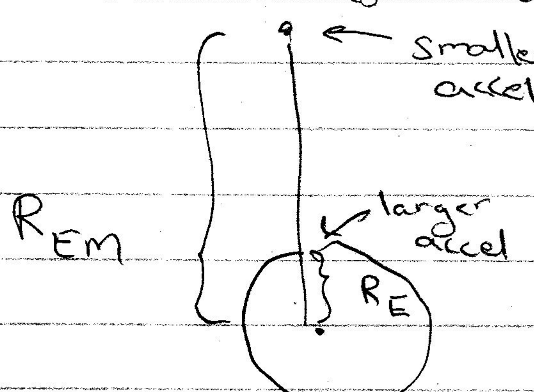
From this analysis

$$a_m = \frac{v_m^2}{R_{Em}} = \frac{\left(\frac{2\pi R_{Em}}{27\,\text{days}}\right)^2}{R_{em}}$$

$$a_m = \quad \simeq \frac{1}{3700}\, g$$

acceleration due on earth

- Newton anticipates that whatever "Force" causes the acceleration of the moon must decrease like $\frac{1}{r^2}$



$$\left(\frac{R_E}{R_{Em}}\right)^2 \simeq \frac{1}{3600}$$

Newton's Laws

- Read Law 1

(1) – a body in motion remains in momotion
Gallileo (and projectiles)

- Newtons second Law

(2) $F = ma \Rightarrow a = \frac{F}{m}$

- A force causses acceleration in the the direction in which it acts (again projectiles)
- The acceleration is inversely porportional to mass

(3) • Newtons Third Law

- For every action force there is an equal and opposite reaction (force

→ Often subtle to use in practice

Gravitation

- Force Decreases like $\frac{1}{r^2}$ $\qquad a \propto \frac{1}{r^2}$

- $F = ma$, but $a = g = \frac{F}{m}$ on earth

is <u>independent</u> of Mass (Gallileo)

So consistency says that $F \propto m$

Thus, $F_{\substack{\text{earth}\\\text{on book}}} \propto \frac{m_{\text{book}}}{r^2}$

- However if the earth pulls on the book, book pulls on earth

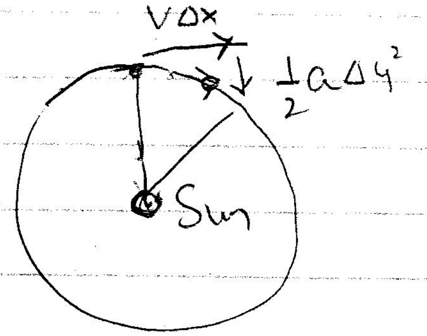
$$F_{\substack{\text{earth}\\\text{Book}}} \propto \frac{M_{\text{earth}} M_{\text{Book}}}{r^2} \qquad \overset{m_1}{\odot} \longleftrightarrow \overset{m_2}{\odot}$$

- The const is the Gravitational const $\equiv G$

$$\boxed{\begin{aligned} F &= G\,\frac{M_{\text{earth}} M_{\text{Book}}}{r^2} \\ G &= 6.6 \times 10^{-11} \ \text{m}^3/\text{kg}\,\text{s}^{-2} \end{aligned}}$$

Force Law of Gravitation

Kepler & the third law of Newton:



$$G\frac{m_\odot m_p}{r^2} = m_p a$$

$$G\frac{m_\odot m_p}{R^2} = m_p \frac{V^2}{R}$$

Now $V = \frac{2\pi R}{T}$

period of orbit

plug in here

So plugging in V and a bit of algebra

$$G\frac{m_\odot \cancel{m_p}}{R^2} = \cancel{m_p}\frac{\left(2\pi R\right)^2}{T^2 R}$$

$\xrightarrow{\text{algebra}}$ $\boxed{\left(\frac{G m_\odot}{4\pi^2}\right) T^2 = R^3}$

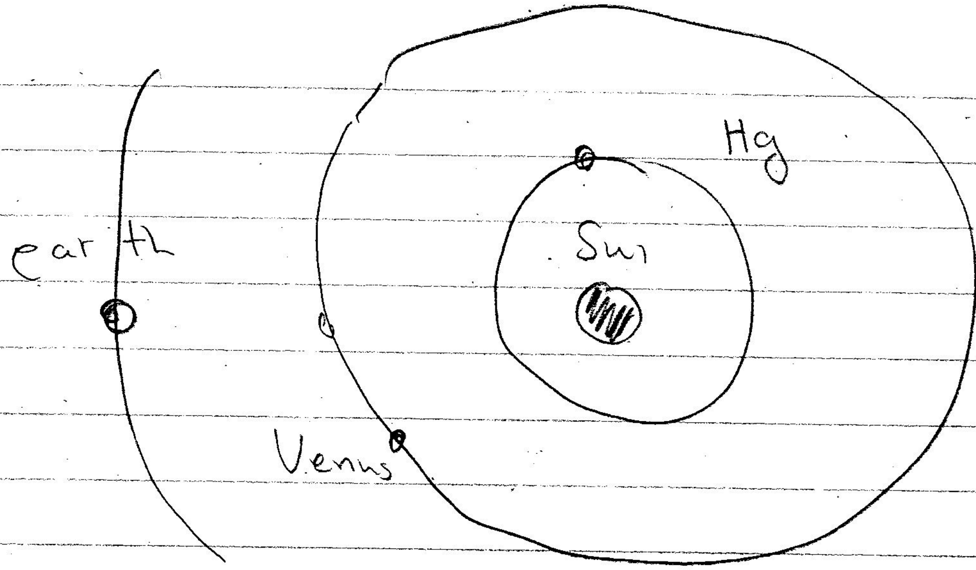
i.e. $\boxed{T^2 \propto R^3}$

Relation between period (T) and radius (R). $m_\odot$ is the mass of the sun

<u>If</u>

$F \propto \frac{m_1 m_2}{R^{2.5}}$ then find $T^2 \propto R^{3.5}$

Thus Kepler's Laws give strong constraints on the motion of the planets.



List all the sorts of corrections you can think of that change Keplers simple rules