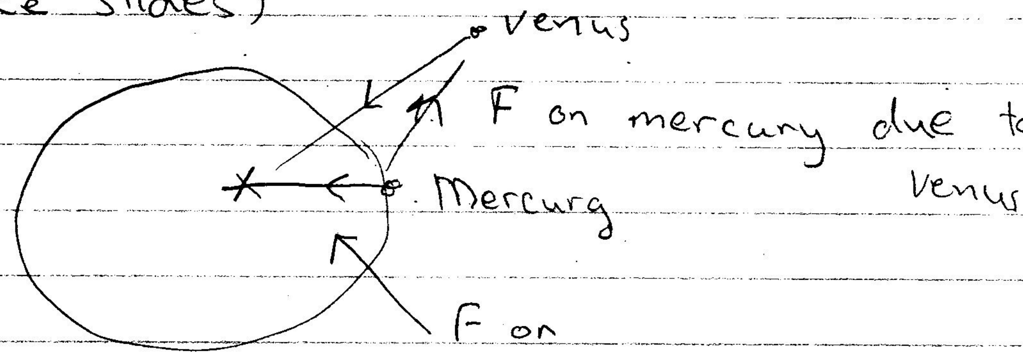
Estimate of the Effect of other Planets on Mercury

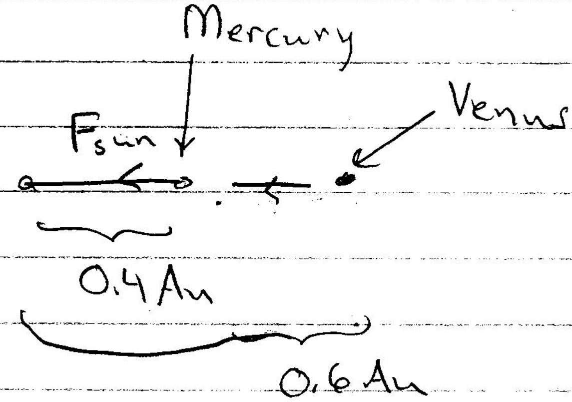
Consider the effect of venus on an mercury

- The effect of all the planets is to cause the precession of the perihelion of mercury (see slides)



- Then venus pulls on mercury disturbing the ellipse

Estimate:



Then

mass of sun

$$F_{sun} \propto \frac{M_{\odot} \, m_{Hg}}{r_{Hg}^2}$$

mass of mercury

distance from sun and mercury

$$F_{venus} \propto \frac{M_{venus} \, m_{Hg}}{(\Delta r)^2}$$

distance between venus and mercury

$r_{Hg} \sim 0.4$ AU

$\Delta r \sim 0.2$ Au

same order of magnitude

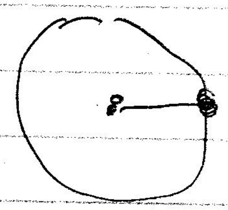
$M_{\odot} \sim 2 \times 10^{30}$ kg

$M_{venus} \sim 5 \times 10^{24}$ kg

different order of magnitude

$$\frac{F_{venus}}{F_{sun}} \sim \frac{m_{venus}}{m_{\odot}} \overbrace{\left(\frac{r_{Hg}^2}{(\Delta r)^2}\right)}^{\sim 1} \sim \frac{m_{venus}}{m_{\odot}} \sim 10^{-6}$$

So the force of the sun causes Hg to orbit the sun in 88 days



$$\frac{\Delta\theta}{\Delta t} = \frac{360^\circ}{88\text{ days}}$$ ← degrees turned per day

Then each orbit we find a correction.

$$\frac{\Delta\phi}{\Delta t} = \frac{360^\circ}{88\text{ days}} \cdot \left[1 + O\left(\frac{F_{venus}}{F_{sun}}\right) + \dots\right]$$

So the rate of precession

$$\text{rate of precession} = \left(\frac{\Delta\phi}{\Delta t} - \frac{360^\circ}{88\text{ days}}\right) \sim \frac{360^\circ}{88\text{ days}} \cdot \left(\frac{F_{venus}}{F_{sun}}\right)$$

So expect

$$\text{precession rate} \sim 0.0036 \ \frac{\text{degrees}}{\text{year}}$$

$$\sim 1310 \ \frac{\text{arc sec}}{\text{century}}$$

$$1' = \frac{1}{60} 1^\circ = 1 \text{ arc min}$$

$$1'' = \frac{1}{60}\frac{1}{60} 1^\circ = 1 \text{ arc sec} = \frac{1}{3600} \text{ deg}$$

A real calculation shows:

$$\text{precession rate} \sim 300 \ \frac{\text{arc sec}}{\text{century}}$$

Our estimate gives within a factor of four of the correct result