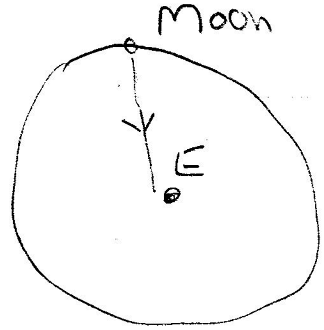
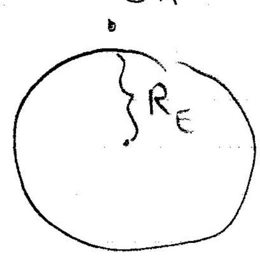
Last Time

(1) Newton's Law of Gravitation



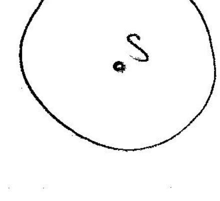
$$F = \frac{G M_E m_{moon}}{r^2}, \quad \text{and} \quad \frac{F_{moon}}{m_{moon}} = \boxed{a_{moon} = \frac{G M_E}{r^2_{Em}}}$$

Similarly, the acceleration of a stone on the surface of earth is



$$g = 10 m/s^2 = \frac{G M_0}{R_E^2}$$

(2) Used Newton's Law's to Show Kepler's Laws



$$\frac{G M_{\odot}}{(2\pi)^2} T^2 = \bar{R}^3$$

mass of sun

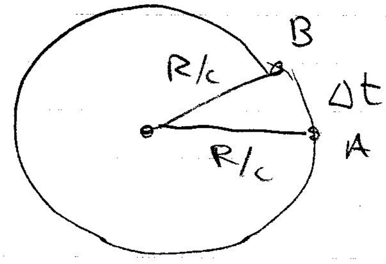
period of orbit

ave radius of semi-major axis

(3) Then we started to think about correction to the otbits of Mercury. We made three points

A. The other planets exert forces on mercury

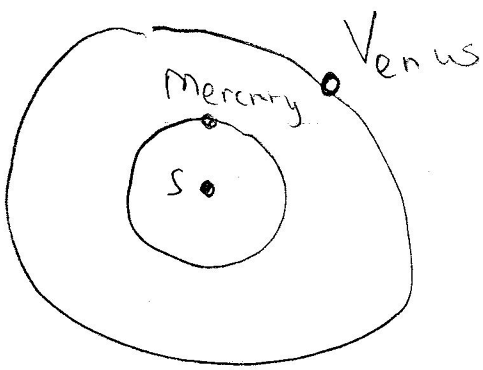
B. The Newton's Law involves a bizarre action at a distance. The force of a planet on one side of the universe is instantly transmitted accross the universe to another planet. Today we understand that this is an approximation, where the transit time of light is short compared to the time of orbit, i.e the force at B reflects the motion at A.



C. These effects cause the precession of the ellipse

Last Time

(4) Then we started to work this out quantitati



Starting with the effect of venus on mercury

the perturbatio

$$\text{correction} = \frac{F_{venus}}{F_{sun}} \sim \frac{M_{venus}}{M_{sun}} \sim 10^{-6} \equiv X$$

So

$$\text{correction} = \frac{\left(\frac{Deg}{day}\right)_{\substack{with\\ Venus}} - \left(\frac{Deg}{day}\right)_{w/out}}{\left(\frac{Deg}{day}\right)_{w/o\ venus}} \sim 10^{-6}$$

$$\text{Precession Rate} = \left(\frac{Deq}{day}\right)_{\substack{with\\ Venus}} - \left(\frac{Deq}{day}\right)_{w/out} \sim \left(\frac{Deg}{day}\right)_{w/out} 10^{-6}$$

$$\left(\frac{Deg}{day}\right)_{w/out} = \frac{360^\circ}{88\ days} = \text{orbital frequency}$$

$$\text{precession rate} \sim \left(\frac{360^\circ}{88\ days}\right) \times 10^{-6} \sim 500\ \frac{arcsec}{century}$$

This number is the same order of magnitude as the real calculation (see slides)

$$\text{Precession of Mercury due to Venus} \simeq 277.42 \ \frac{\text{arcsec}}{\text{century}}$$

Planet

Calculation Today

Calculation in 1912

Table I. The ratio of the sun's mass to the planet's mass, the semimajor axis a_p, and the contribution to the precession of the perihelion of Mercury are given for each planet.

Planet	M/M_P	a_P (AU)	$\delta\gamma$ (arcsec/century) from Eq. (3.5)	$\delta\gamma$ (arcsec/century) from Eq. (4.14)	Doolittle[a] (arcsec/cent.)
Mercury	6 023 600	0.387 098 93	…	…	…
Venus	408 523.5	0.723 331 99	292.84	277.42	277.37
Earth+Moon	328 900.55	1.000 000 11	95.89	90.88	90.92
Mars	3 098 710	1.523 662 31	2.38	2.48	2.48
Jupiter	1 047.350	5.203 363 01	156.94	153.95	154.09
Saturn	3 498.0	9.537 070 32	7.57	7.32	7.32
Uranus	22 960	19.191 263 93	0.14	0.14	0.14
Neptune	19 314	30.068 963 48	0.04	0.04	0.04
Total			555.80	532.23	532.36

[a]Reference 1, p. 179, but corrected for current values of M/M_P.

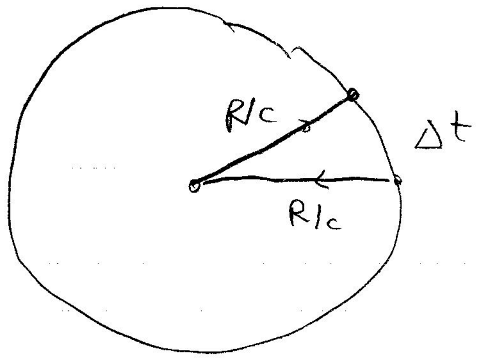
Total = 532" per century, Observed value = 575" per century

500" per century = 0.0014 degrees / year

Start of new material

Estimate of First Relativistic Correction to the precession of Mercury

• If the speed of light were infinite, the force would be instantaneous as newton says



$$\text{Correction} = \frac{\text{Light Transit Time}}{\text{Time of orbit}} \sim \frac{2R/c}{(2\pi R/V)}$$

$$\text{correction} \sim \frac{2}{\pi}\frac{V}{c} \sim \frac{V}{c}$$

In fact all relativistic corrections start as $(V/c)^2$ (Justify later)

$$\frac{\left(\frac{\text{Deg}}{\text{day}}\right)_{\text{w relativity}} - \left(\frac{\text{Deg}}{\text{day}}\right)_{\text{w/out}}}{\left(\frac{\text{Deg}}{\text{day}}\right)_{\text{w/out}}} \sim \left(\frac{V}{c}\right)^2$$

$V = 47 \frac{km}{s}$ $\quad c \sim 3\times10^8 \, m/s$ $\quad V/c \sim 2\times10^{-4}$

$$\text{precession rate} \sim \left(\frac{\text{Deg}}{\text{day}}\right)_{\text{with relativity}} - \left(\frac{\text{Deg}}{\text{day}}\right)_{\text{w/out}} \sim \left(\frac{\text{Deg}}{\text{day}}\right)_{\text{w/out}} \left(\frac{v}{c}\right)^2$$

So

$$\text{pecession rate} \sim \left(\frac{360^\circ}{88 \text{ days}}\right) \times 4 \times 10^{-8}$$

$$\sim 20 \ \frac{\text{arc sec}}{\text{century}}$$

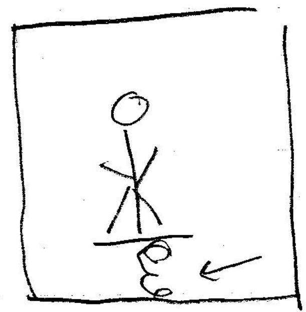
This estimate can be compared to the real calculation by Einstein

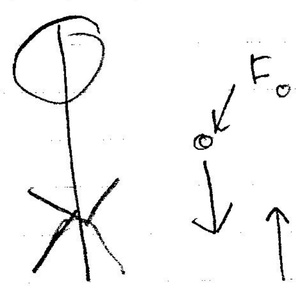
$$\text{pecession rate} = 43 \ \frac{\text{arcsec}}{\text{century}}$$

What is weight?

- Weight is the force that the floor (or spring scale) exerts on you:

No Acceleration





Force of Gravity $= F_g$

Force of Scale $= F_s$

$$F_g - F_s = m\cancel{a}^{\,0} \qquad \text{So } F_g = F_s$$

So since the force of gravity is $mg = F_g$, the spring scale measures mg in this case

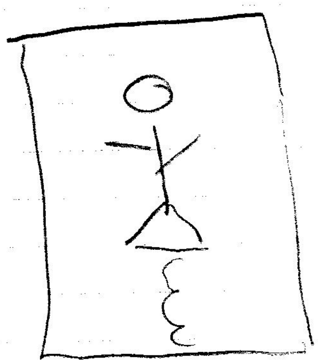
- Now when the elevator starts to move up you feel heavier,

- When the elevator slows down to stop you feel lighter

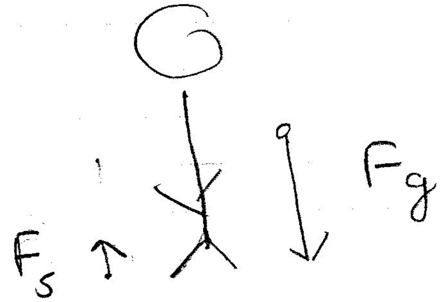
- Now imagine "Free Fall"

  You feel no weight at all

$a = g = 10\,m/s$



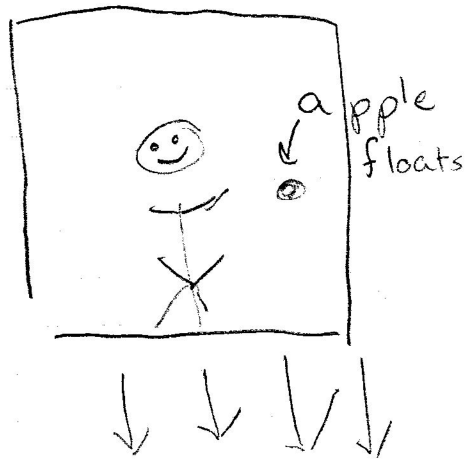
Formally this is because the spring scale exerts no force at all.



$$F_g + F_s = mg$$

$$mg + F_s = mg \Rightarrow F_s = 0$$

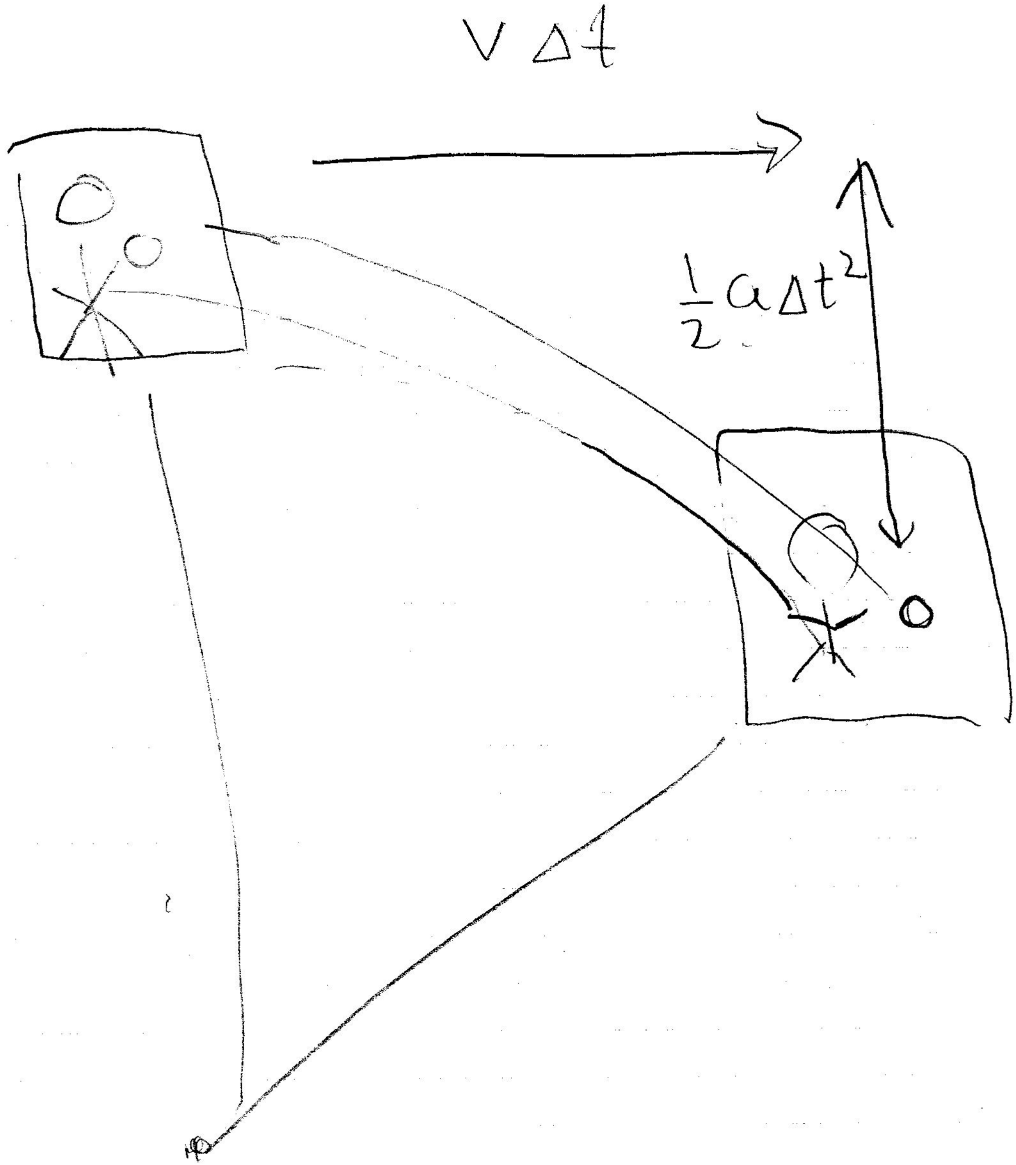
- In particular a free faller, that drops an apple in does not see it fall towards the floor.



Because both he and the apple are falling towards the earth.

Weightless In Space

- Weightless in space (for sattelites) has nothing to do with gravity being weaker



Both the apple and the person are in free fall, they fall together around the earth. Thus, the weightless in space is completely analagous to the free-falling elevator

Einstein's Principle of Equivalence

1. A free falling observer (the elevator guy) will (in a sufficiently small volume) experience the world as if there were no gravitational forces at all.

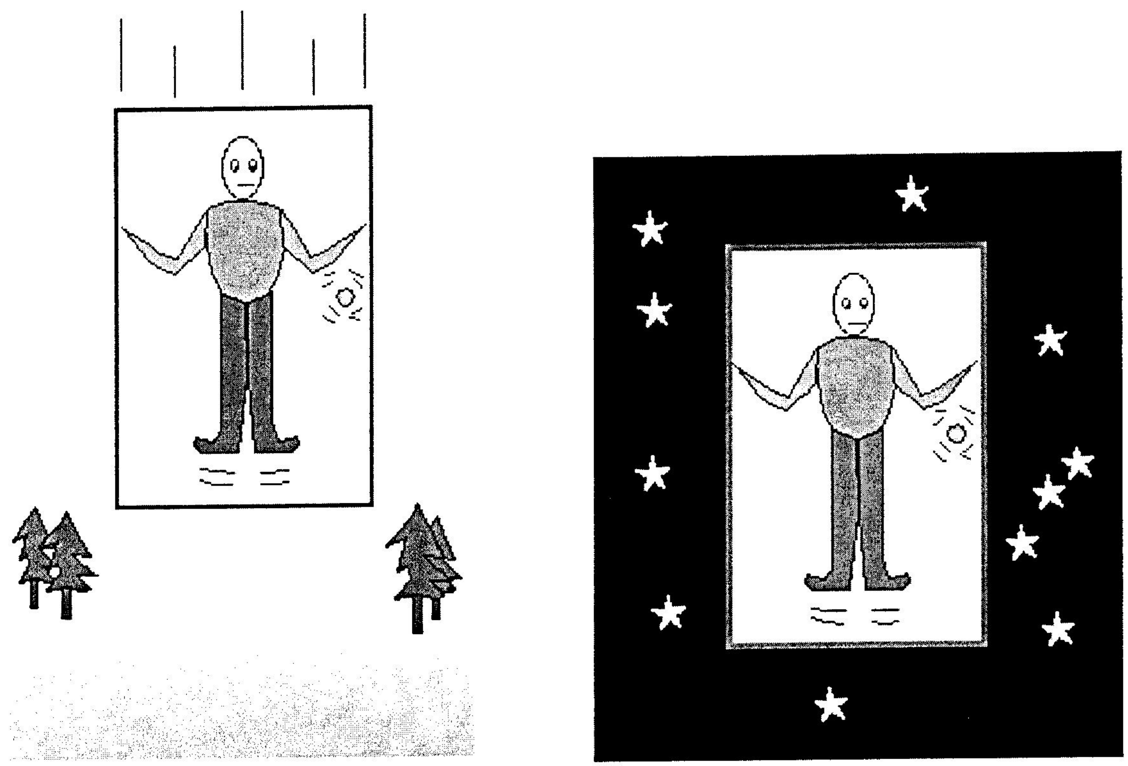
2. There is no experiment a person could conduct in a small volume of space that could distinguish between gravity and uniform acceleration

- See Slides

Today

Equivalence Principle (Part 1)

- A free falling observer, the elevator guy, will (in a small volume) experience the as if there were no gravitational forces.



Things falling freely in a gravity field all accelerate by the same amount, so they move the same way as if they were in a region of zero gravity — ``weightlessness''!

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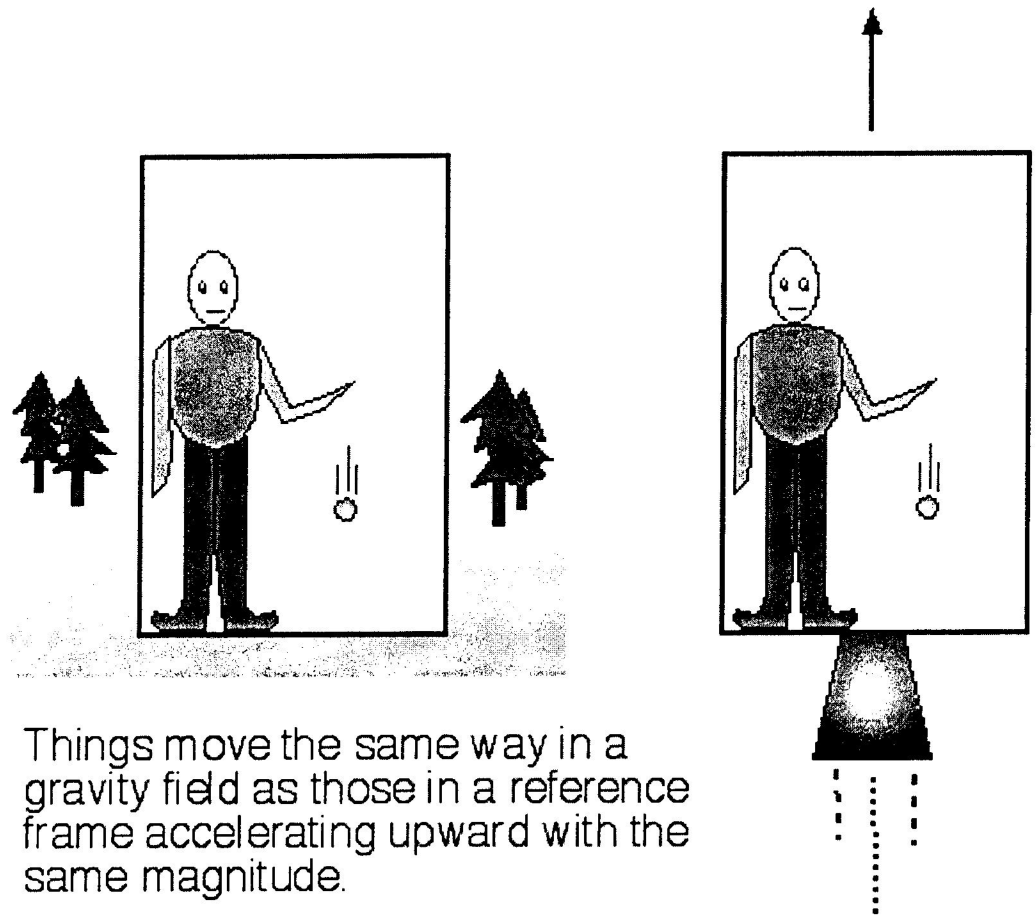
Equivalence Principle (Part 2)

- There is no experiment a person could conduct (in a small volume) that can distinguish gravitational forces from accelerated motion.



Things move the same way in a gravity field as those in a reference frame accelerating upward with the same magnitude.

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