- All objects with mass have a Schwarzchild Radius, its just that the object is usually much bigger than R_{sch}.

For the sun

$$R_{sch} \text{ for Sun} = \frac{2 \overbrace{G}^{6{,}67 \times 10^{-11}\ \mathrm{km}^3/\mathrm{kg\,s}^2} \overbrace{M_\odot}^{2 \times 10^{30}\ \mathrm{kg}}}{c^2}$$

$c = 3 \times 10^8 \mathrm{m/s}$

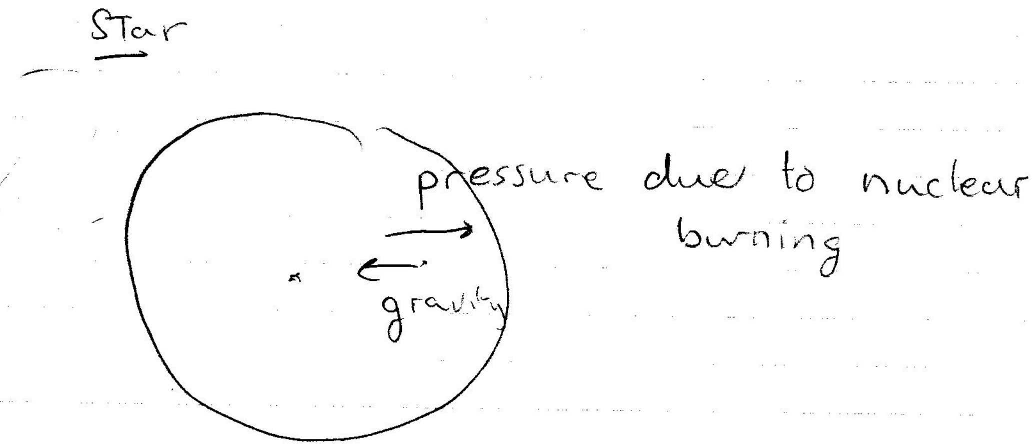
$$R_{sch} = 3\ \mathrm{km}$$

Compare this to the radius of Sun

$$R_\odot \simeq 700{,}000\ \mathrm{km}$$

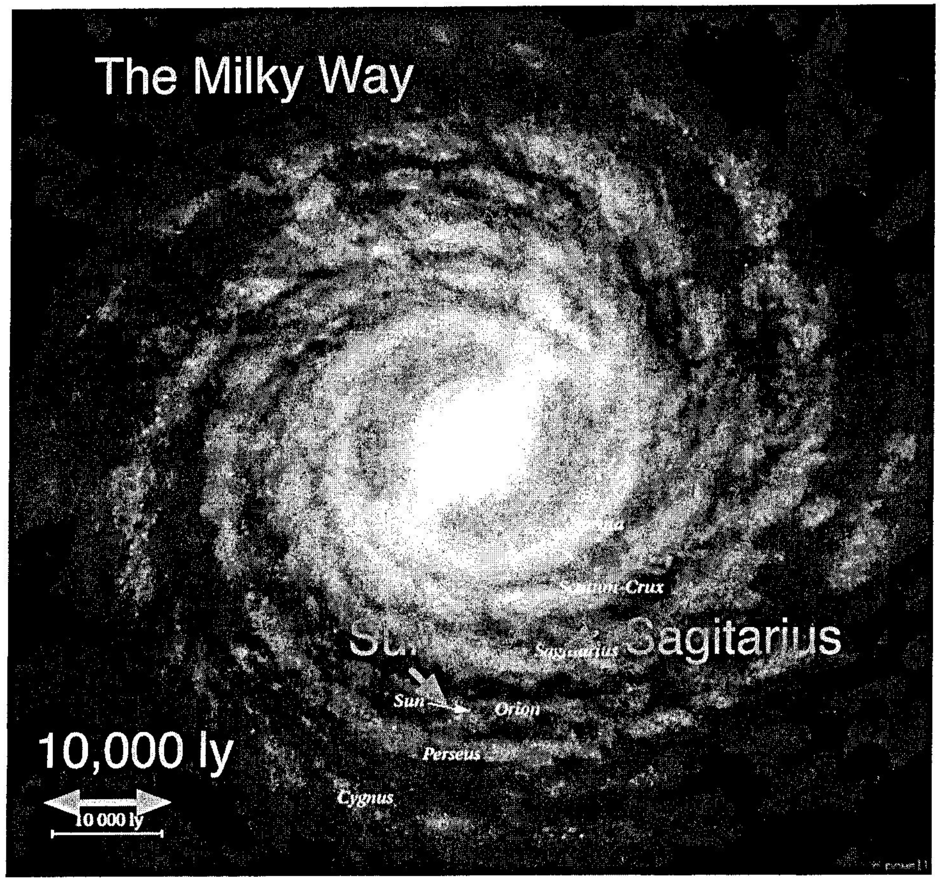
Forming Black Holes

Now How are black hole's formed?
Need to compress the mass into a very small point.



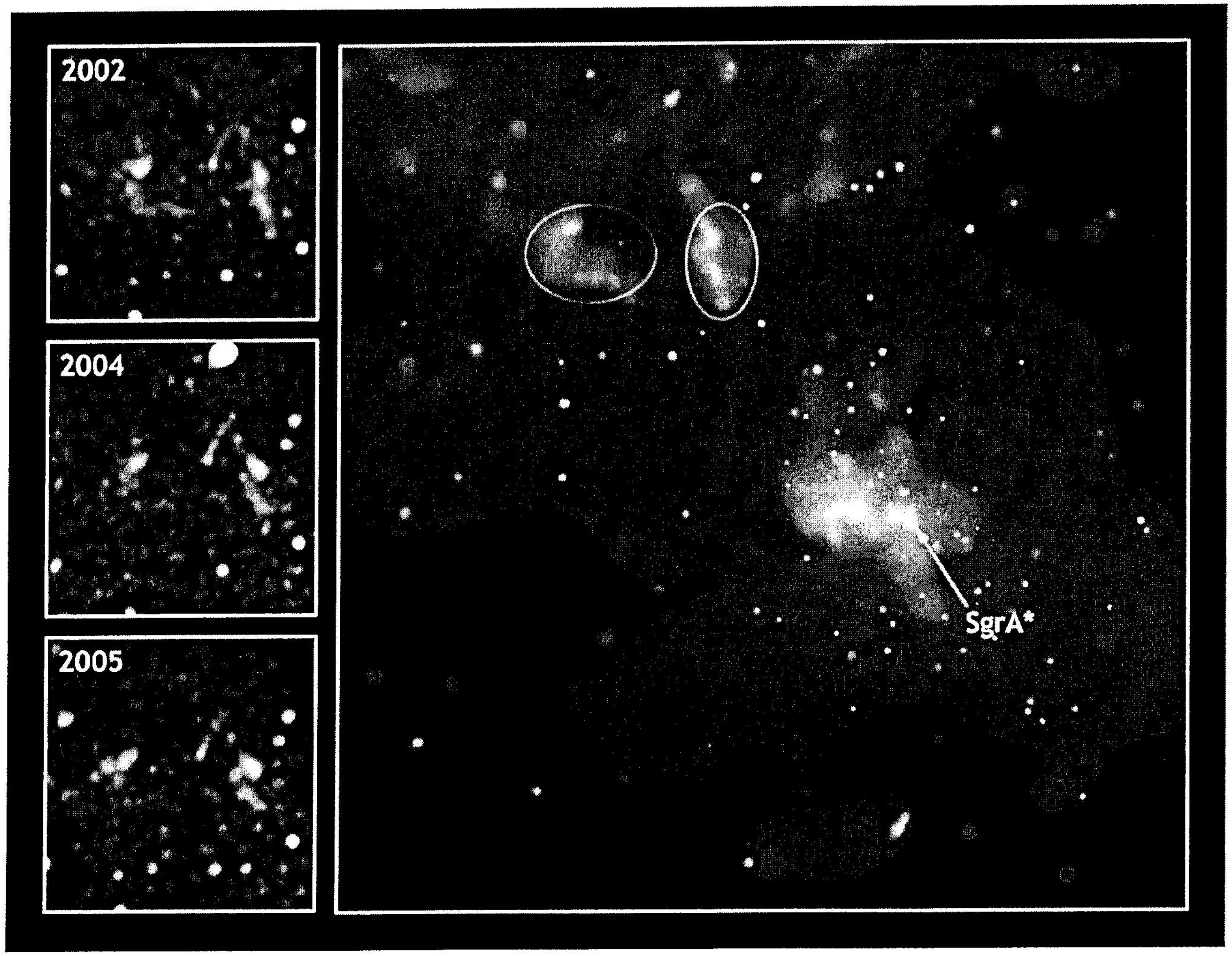
If the mass is sufficently large, then as the nuclear fuel runs out, the star will undergo gravitational collapse driving all of the material into a small radius

Sagittarius A^* (in our galaxy) contains a suppermassive black hole



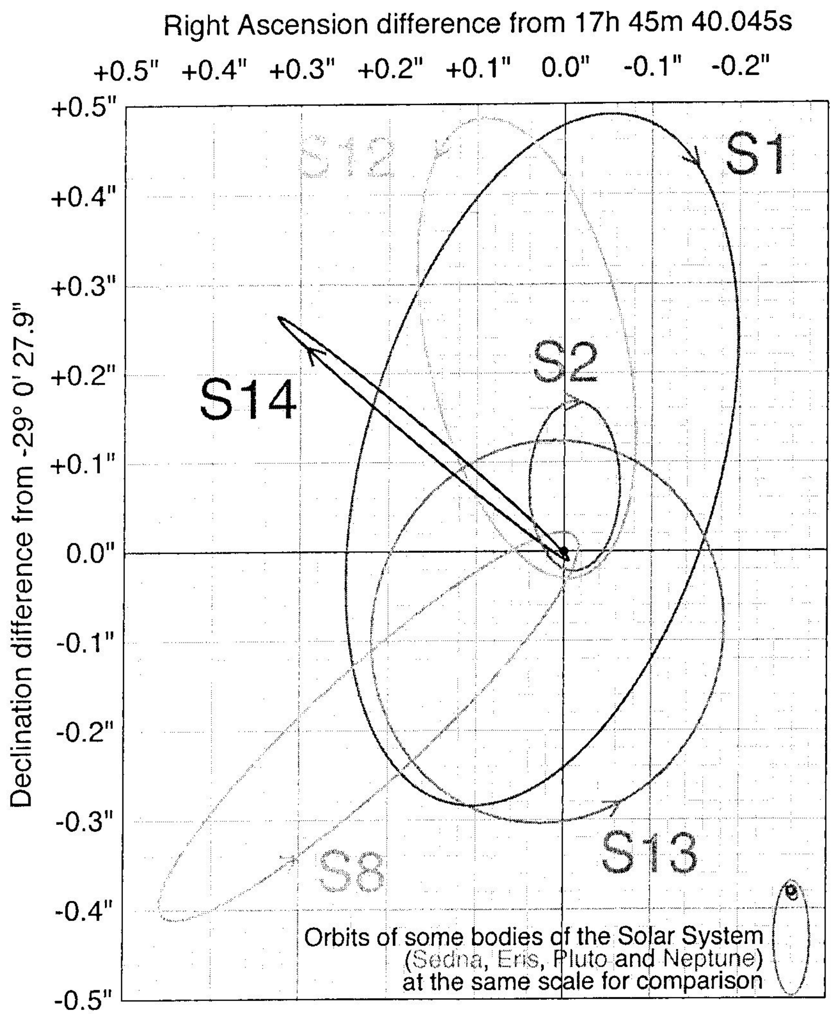
- We will look at the elliptic orbits of a few stars close to Sagittarius A
- The orbital radius of these stars is $R \sim 1000 AU \sim 0.01$ light years

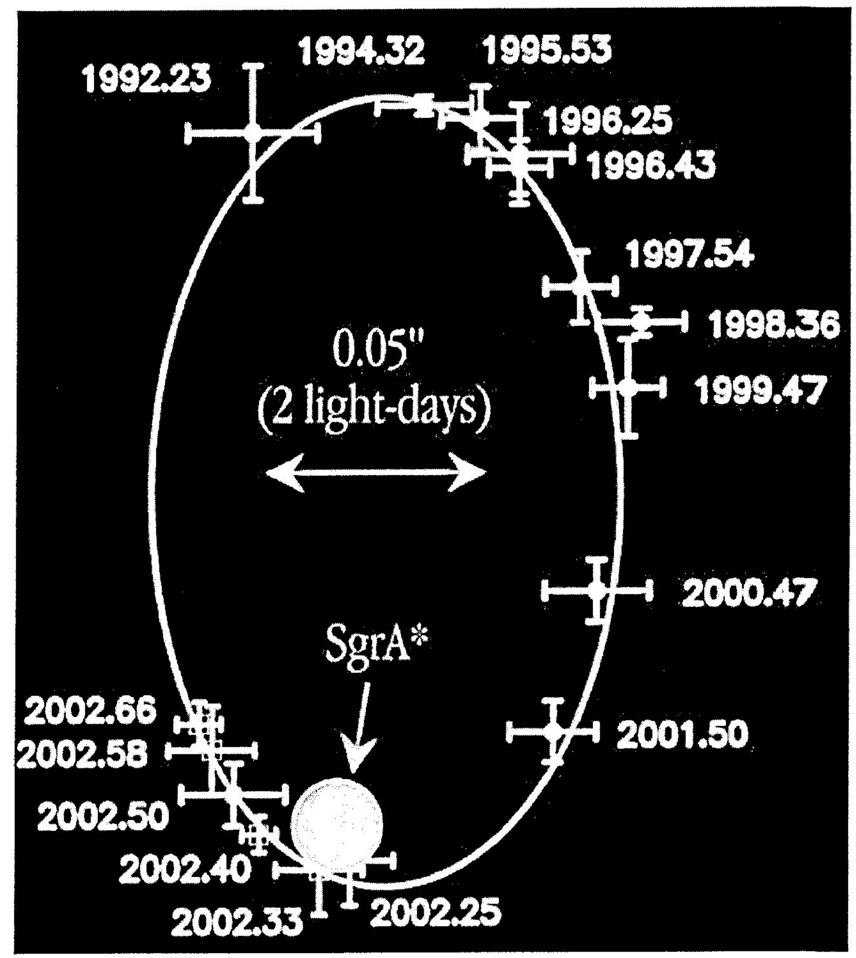
Sagittarius A^* (in our galaxy) contains a suppermassive black hole



- Look at the change in the image as a function of time (years) and look in different frequency bands (x-rays)

Orbits of six stars around the Sagittarius A^* in the center of our galaxy





From the orbital period and radius of the star, use Kepler's laws to find the mass of the SgrA^*

Note: 2 light days = 350 AU

Sagittarius A*

- The period of orbit of S2 arround Sagittarius A* is 15.2 years, the eccentricity of the elliptical orbit is $e = 0.88$, The average radius (the-semi-major axis) is 980 ± 20 AU

→ Calculate the mass of Saggittarius A*, calculate the distance of closest approach of S2 to the center of mass (SGr A*,

→ Calculate the Schwarzchild Radius of Sagitarius A

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Solution

$$\frac{GM}{(2\pi)^2}\,\overset{\text{period}}{\overset{\downarrow}{T^2}} = \overset{\text{semi-major axis}}{\overset{\downarrow}{a^3}}$$

$$M = \frac{(2\pi)^2}{G}\, a^3/T^2$$

$a = 980$ AU $= 1.4 \times 10^{14}$ m

$T = 15.2$ y $= 3.1 \times 10^7$ s

$G = 6.67 \times 10^{-11}$ m^3/kg s^2

Find

$$M = 8.2 \times 10^{36} \text{ kg}$$

Note

$$M_\odot = \text{mass of sun} = 2 \times 10^{30} \text{ kg}$$

Or

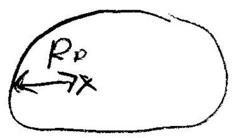
$$\frac{M}{M_\odot} = 4.1 \times 10^6$$

i.e. the mass of Sgr A* is 4.1 <u>million</u> solar masses.

So

$$R_p = a(1-\overset{0.88}{\overset{\downarrow}{e}}) = 980 \text{ AU} (0.12)$$

$$R_p = 117.6 \text{ AU}$$



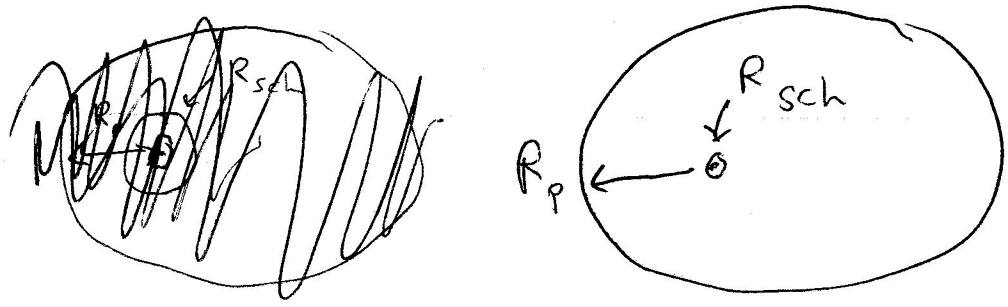
Compare

$$R_{sch} = \frac{2Gm}{c^2} = 0.08 \text{ AU}$$

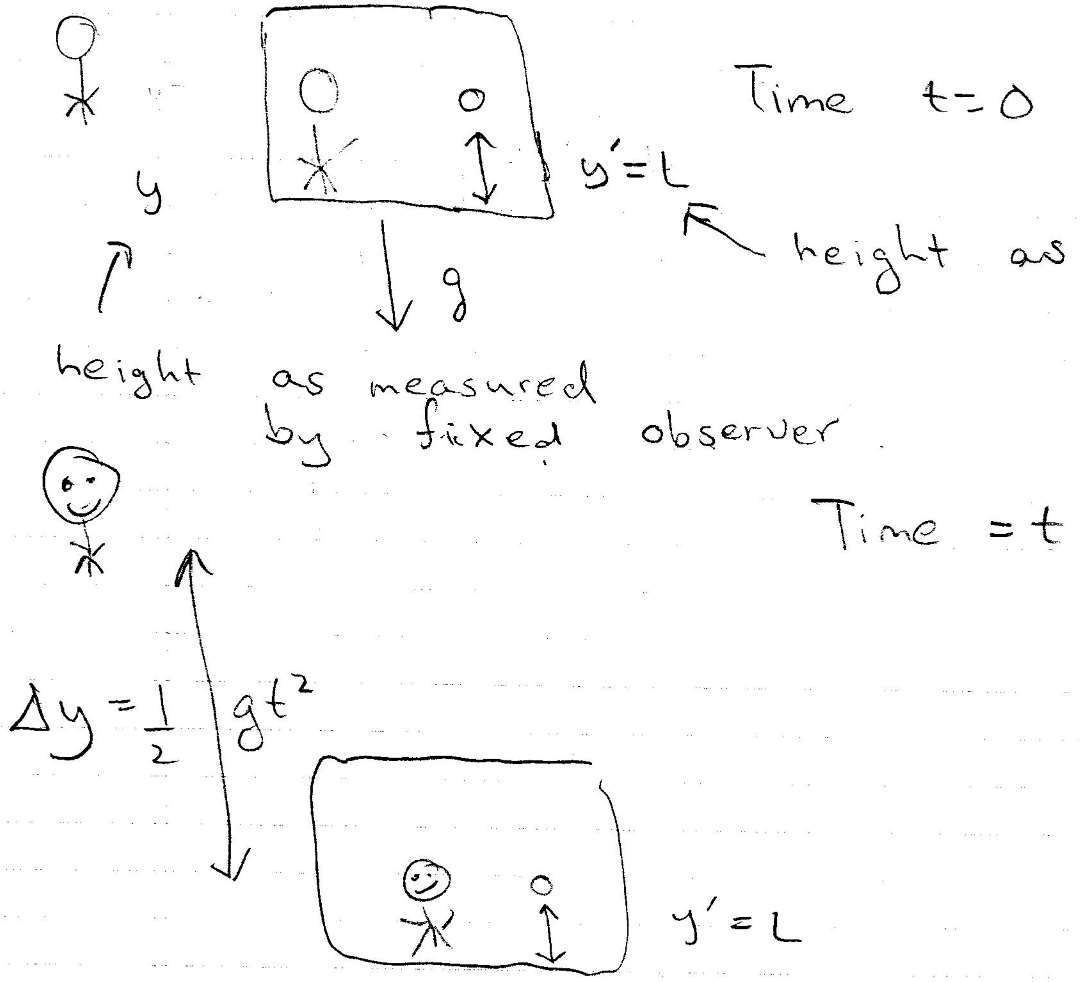
<u>Summary</u>: there is a <u>very</u> massive object with a radius $R < 117.6$ AU

The Schwarzchild Radius is 0.08 AU

Picture:



Description of Gravity in Terms of Curvature



So instead of describing the motion of objects with the t, y coordinate system can use the t', y' coordinate's

$$t' = t$$

$$y' = y - \frac{1}{2} g t^2$$ coordinate system.

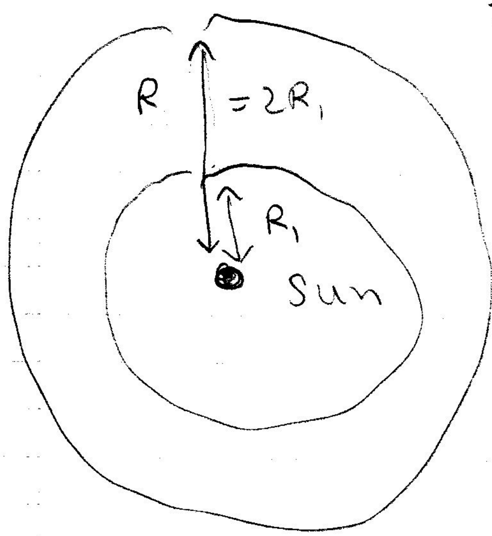
This is the coordinate system of the free falling observer

In this coordinate system the apple has no forces and its motion is fixed

$$m\,a' = 0 \implies y' = \text{constant} = L$$

If I know the free fall coord's, I know effect of gravity

Now recognize that we must choose a different coordinate system for every radius



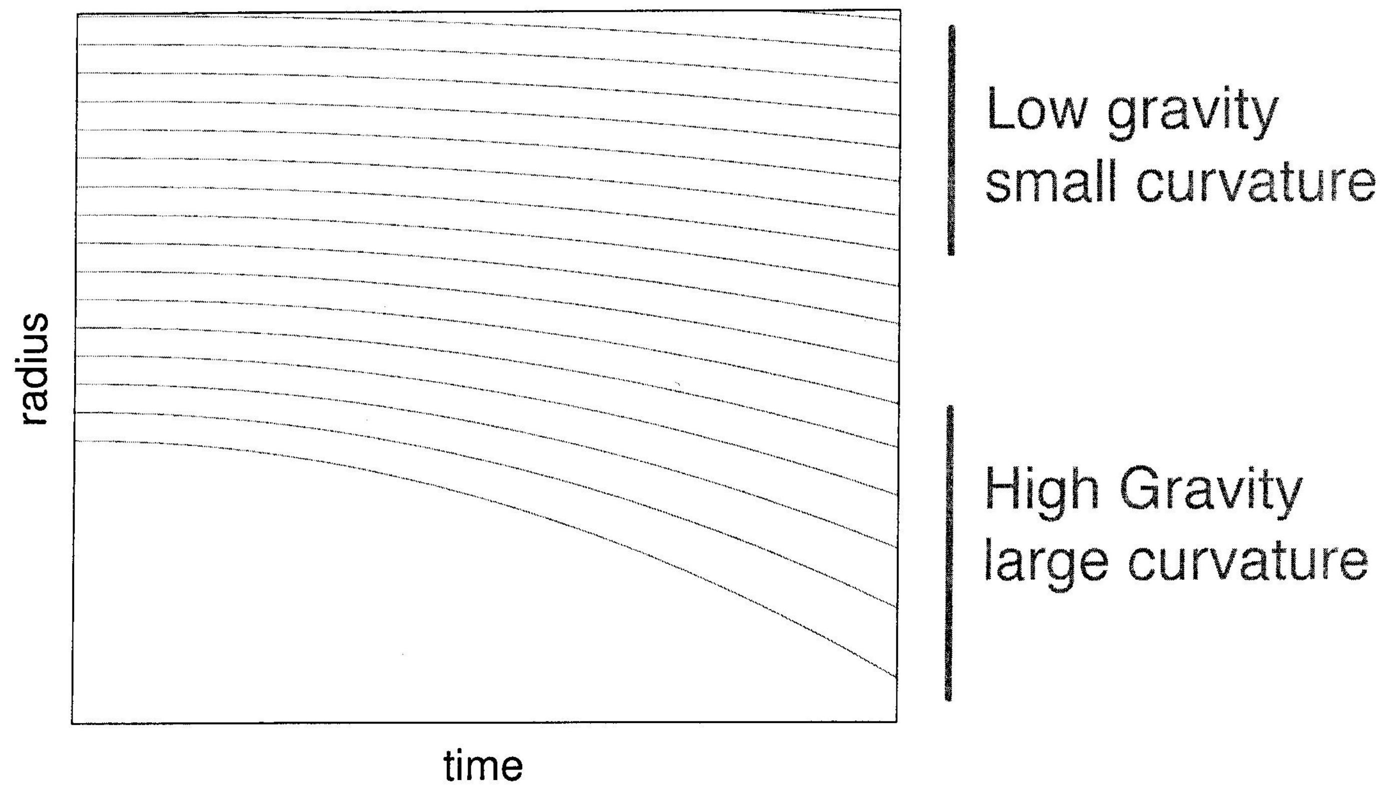
$$t' = t$$

$$y' = y - \frac{1}{2} g(R)\, t^2$$

$$g(r) = \frac{G M_\odot}{R^2}$$

(see slide)

So we can phrase the problem of finding the forces of gravity as finding how the coordinates of free falling observers change in space and time



Can view the problem of determining the forces of gravity, as a problem of finding how the coordinates of free falling observers change from point to point

Newton's Law

determines the coordinates of free fall observers

$$a(r) = G\frac{M}{r^2}$$

acceleration

caused by mass

Einstein:

$$\underbrace{\cancel{R_{\mu\nu}}\ G_{\mu\nu}}_{\text{"Curvature"}} = \underbrace{8\pi G\, T_{\mu\nu}}_{\text{mass (energy density)}}$$

how the local coordinates systems of free falling observers are changing from point-to point