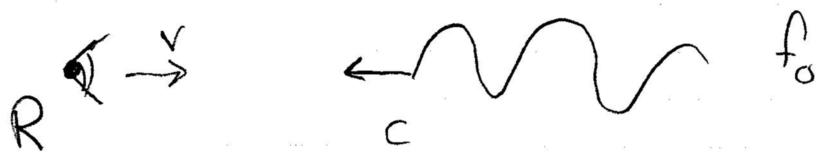
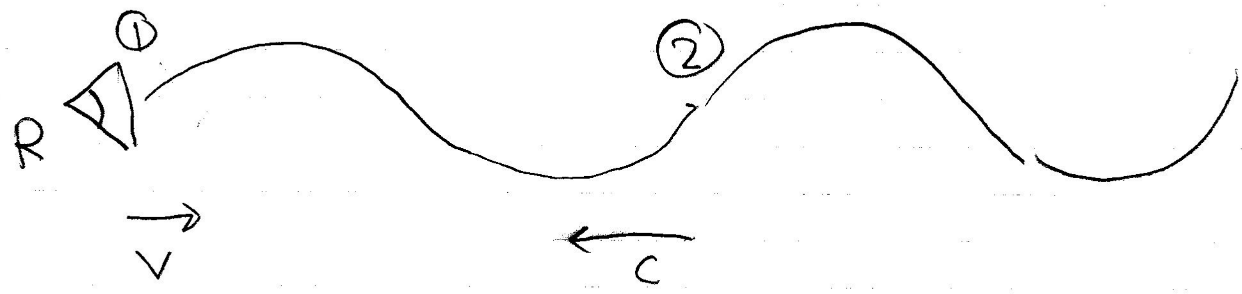
Dopler Shifts



- When the receiver moves toward the source the frequency is higher
- When the receiver moves away from the source the frequency is lower

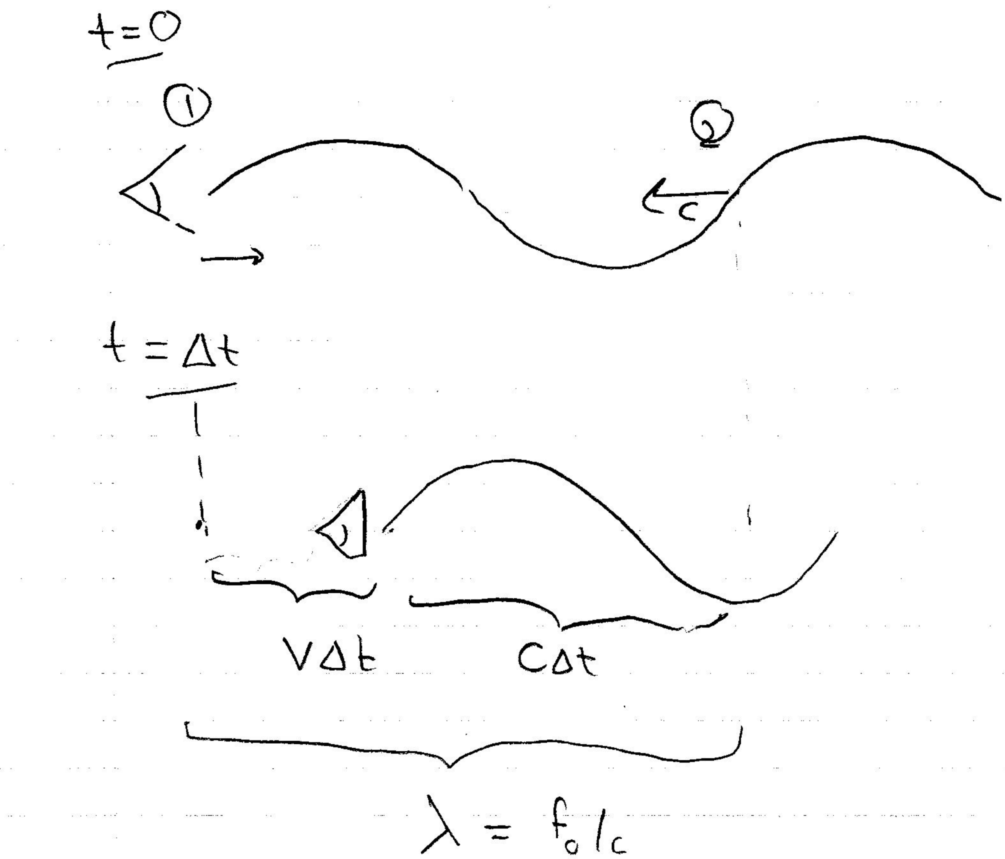
$$\boxed{f_R = \left(1 + \frac{v}{c}\right) f_0}$$ ← v = velocity towards source

Derivation (Skip if pressed for time)



- Then $c = \lambda f_0 \Rightarrow \lambda = c / f_0$
- After a time Δt the signal that the receiver see's repeats

$$f_R = \frac{1}{\Delta t}$$



So

$$V\Delta t + c\Delta t = c/f_0$$

Or $$(V+c)\Delta t = c/f_0$$

$$(V+c)\frac{1}{f_R} = \frac{c}{f_0}$$

$$f_R = \left(\frac{V+c}{c}\right) f_0$$

$$\boxed{f_R = \left(1 + \frac{V}{c}\right) f_0}$$

Then Consider two Equivalent situations

an emitter and a receiver in an accelerating elevator

an emitter and receiver in gravity

- Then consider the left situation.
  The emitter emits light @ frequency
  f_B = frequency at bottom at $t=0$.

- The time it takes for the light to fly up is

$$\Delta t_{fly} = \frac{h}{c}$$

- During this time the elevator and receiver got faster

$$V = \Delta t_{fly}\, g = g \frac{h}{c}$$

- Thus the receiver will be moving away from the source with speed v and therefore see a dopler-shifted frequency

frequency at top

$$f_T = \left(1 - \frac{v}{c}\right) f_B = \left(1 - \frac{gh}{c^2}\right) f_B$$

Or the time between light pulses

$$\Delta t_T = \frac{1}{1 - \frac{gh}{c^2}} \Delta t_B$$

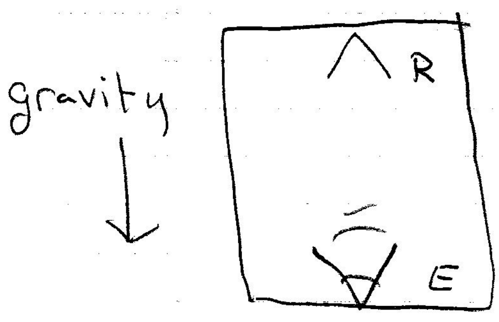
$\frac{1}{f_B}$

$\Delta t_T = \frac{1}{f_T}$ => if the frequency is (one pulse)/two secs then the time between pulses is two secs

Or

$$\Delta t_T \simeq (1 + gh/c^2)\, \Delta t_B$$

So by principle of Equivalence :



If emmitter sends 1 pulse/sec

Then the receiver will register

$$\Delta f_{Top} = \underbrace{\left(1 - \frac{gh}{c^2}\right)}_{\text{smaller than one}} (1\ \text{pulse/sec})$$

Something slightly less than one/sec

So

A measures his own ticks as 1 pulse/sec. (Say 1000 pulse

A ○ But measures B's ticks at less than one/sec (say 999 pulses)

A ○
B ○

↑
Synchronize Clocks so each measures 1 pulse sec

B ○ ← B measures one pulse/se

↑
separate them

○ A
○ B

↑
When you bring them back together they won't be synchronized

A will be older (i.e. more pulses of his clock) than B.

For th

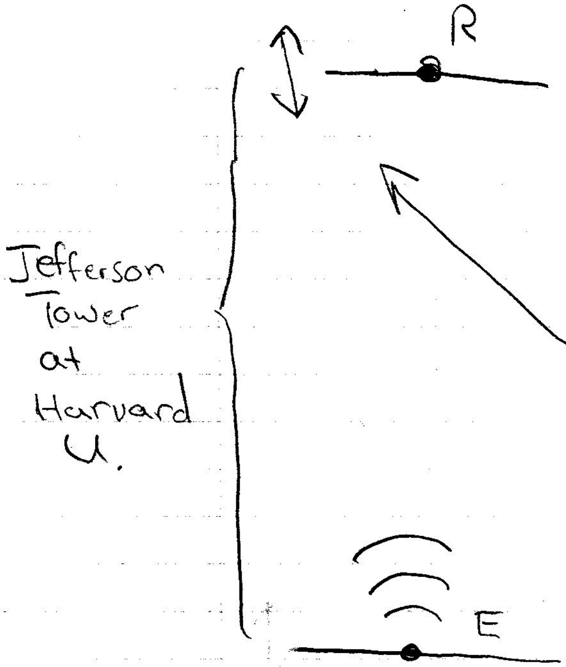
• The effect is small:

fractional
$$\nu_{diff} = \frac{\Delta t_T - \Delta t_B}{\Delta t_B} = \frac{gh}{c^2}$$

For $h \sim 22\,m$ $g = 10\,m/s$ $c = 3 \times 10^8\,m/s$

fractional difference $= 2.4 \times 10^{-15}$

The Pound-Rebka Experiment



Another sample of ^{57}Fe. If the frequency of incoming light matches the frequency of decay the sample can absorb the light

Sits on a vibrating platform moving up and down.

Spontaneous Decay of ^{57}Fe emitting γ-rays at frequency f_B

Then the frequency at the top, when the platform is moving down is

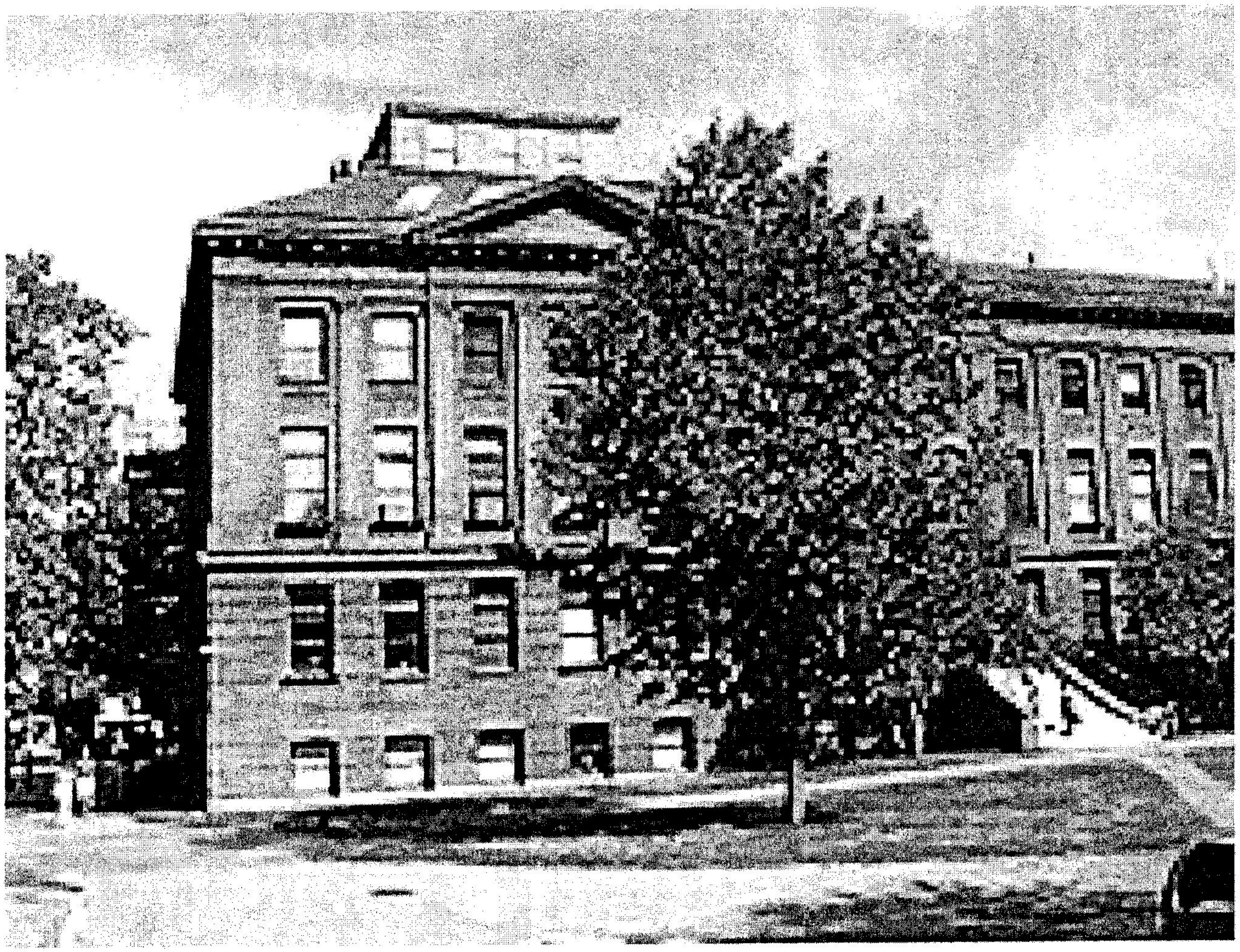
$$f_{top} = \underbrace{\underbrace{\left(1 + \frac{v}{c}\right)}_{\text{Shift due to dopler}}}_{\text{greater than one}} \underbrace{\underbrace{\left(1 - gh/c^2\right)}_{\text{Shift due to gravity}}}_{\text{less than one}} f_B$$

- For certain values of v the combined effect of doppler and gravity will cancel and the receiver system will absorb the gamma rays.

Thus by knowing the dopler shift. The experiment determined the gravitational shift, which was compared w Einstein's formula: $1 - gh/c^2$

The two agreed to 1%

Pound and Rebka experiment conducted in Jefferson Towaer at Harvard U.



GPS - System

- The GPS system is based on very accurate atomic clocks which are synchronized.

- Encoded in the radio waves sent by Boston is a precise record of when the signal was sent.

- Your receiver can determine your distance to the satellite from the send and receive time.

- Now if you have three or more such receivers you can pin-point your location on earth.

  (see slides)

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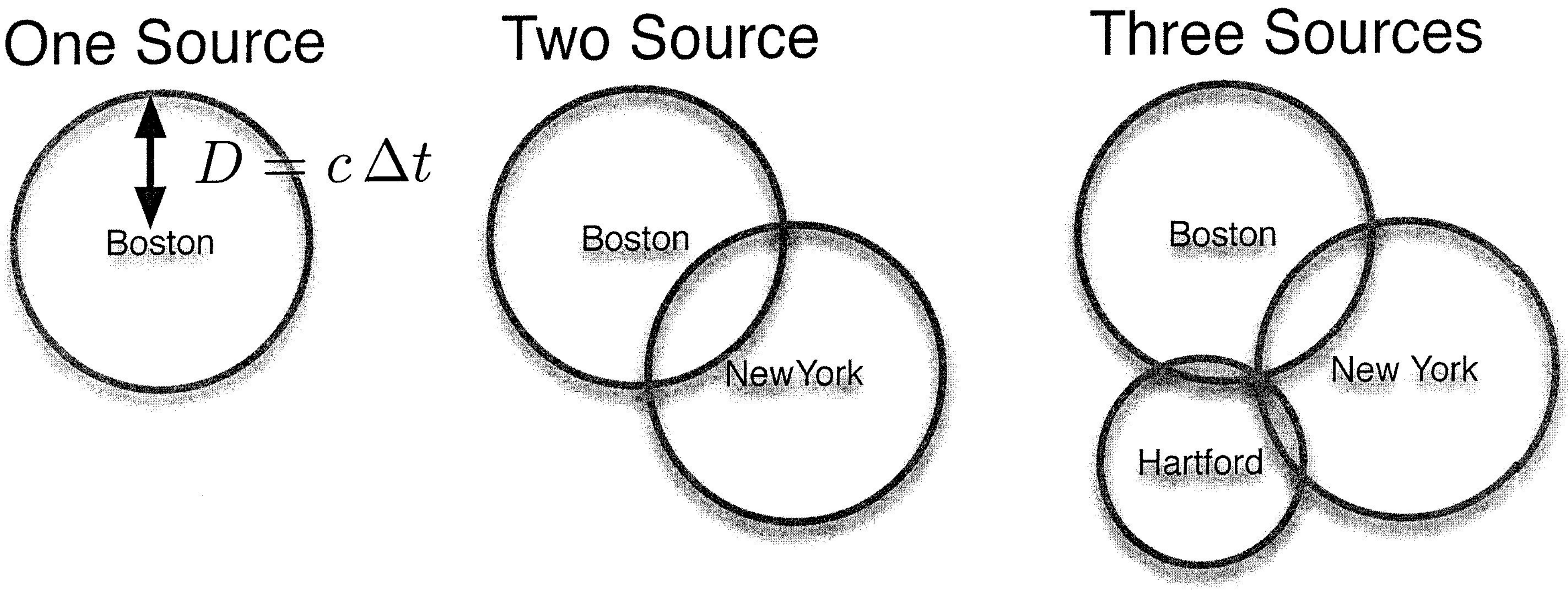
GPS and Triangulation



Basic Facts about GPS

- The GPS sattelites are in a high orbit

$$R \simeq 26000 \text{ km} \simeq 4.5 R_e$$

- The orbit in 12 hours

$$V \simeq \frac{2\pi R}{12 \text{ hours}} = 3.7 \text{ km/s} \simeq 14,000 \text{ km/h}$$

If the clock times are off by $\Delta t \simeq 1 \times 10^{-6}$

then the error will be off by 300m

Now There are two effects

(1) The sattelites are moving and run slower

$$\Delta t_{earth} = \gamma \, \Delta\tau_{sattellite}$$

compared to earth

(2) The gravity is less in the sattellite, so the sattellite clocks run faster compared to earth.

Helpful movie about the GPS system

- See this useful moview

Estimate

Now lets estimate these effects

$$\Delta t_{earth} = \gamma \, \Delta\tau_{satellite}$$

So

you need a lot of digits to calculate this.

$$(\Delta t_{earth} - \Delta\tau_{sattelite}) = \overbrace{(\gamma - 1)}\, \Delta\tau_{sattelite}$$

So if $\Delta\tau_{sattellite} \simeq 1$ day

With $\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}}$ $\quad v = 3.7$ km/s

$c = 3 \times 10^8$ m/s

$$\gamma - 1 \simeq 8 \times 10^{-11}$$

So

$$\Delta t_{earth} - \Delta\tau_{sattelite} = 7 \times 10^{-6}\ \text{s every day}$$

This if not corrected for leads to a distance error of

$$c\,\Delta t = (3 \times 10^8\ \text{m/s})(7 \times 10^{-6}\ \text{s}) = 2\ \text{km every day}$$

Now the time difference due to Gravity

$$\Delta t_{earth} \simeq (1 - gh/c^2)\, \Delta t_{sattelite}$$

Recall that:

$$f_T = (1 - gh/c^2)\, f_B \swarrow \text{bottom (earth)}$$

$$\Delta t_T = \frac{1}{f_T} = \frac{1}{(1-gh/c^2)} \frac{1}{f_B} = \frac{1}{(1-gh/c^2)} \Delta t_B$$

So

$$\Delta t_B = (1 - gh/c^2)\, \Delta t_T$$

$\uparrow$ earth $\qquad \nwarrow$ sattelite

Then for $\Delta t_{sattelite} \simeq 1\,\text{day}$

$$\Delta t_{earth} - \Delta t_{sat} \sim -gh/c^2)\,(1\text{ day})$$

(This is only an estimate since in our derivation we assumed h to be much smaller than R_E). Take $h = 20{,}000$ km $g = 10\,m/s^2$

$$\Delta t_{earth} - \Delta t_{sattelite} = -192 \times 10^{-6}\,s \quad \text{every day}$$

So if not corrected for this would lead to an error of

$$c(\Delta t_{earth} - \Delta t_{sattelite}) \sim 57\,km \text{ every day}$$

Time Near Black Holes

We had

$$f_T \simeq (1 - gh/c^2)\, f_B$$

This is an approximate formula when h is small and gravity is weak

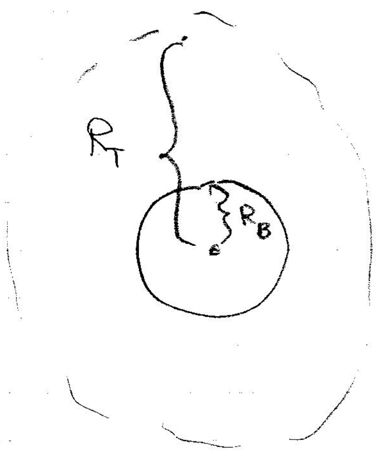
$$g = \frac{GM}{R^2} \quad \text{so} \quad gh/c^2 \ll 1$$

As the mass increases one could imagine that $f_T \longrightarrow 0$

$$gh = \frac{\text{Gravitational PE}}{m} \longrightarrow \frac{GM}{R^2}$$

The more general form is

$$\boxed{f_T(R_T) = \left(\frac{1 - \frac{2GM}{c^2 R_B}}{1 - \frac{2GM}{c^2 R_T}} \right)^{1/2} f_B(R_B)}$$



for simplicity consider very large $R_T \to \infty$

$$f_T = \sqrt{1 - \frac{2GM}{c^2 R_B}}\; f_B(R_B)$$

fequency where there is no gravity, i.e. very far away $R_T \to \infty$

Now Recall that

$$R_{sch} = \frac{2GM}{c^2}$$ ← The event horizon or Schwarzchild Radii

So

$$\boxed{f_T(\infty) = \sqrt{1 - \frac{R_{sch}}{R_B}}\; f_B}$$

~~Thus any finite frequency source f_B will ~~appear~~ be ~~seen~~ measured at lower and lower frequency when ∞ meas~~

Then a high frequency source f_B very close to the event horizon will appear at much lower frequencies since $\sqrt{1 - R_{sch}/R} \simeq$ very small

- Thus you will be only be able to communicate very slowly (poorly) with a person close to the event horizon (= Schwarzchild Radius)

- Eventually the person falls though the schwarzchild radius and can't communicate at all.